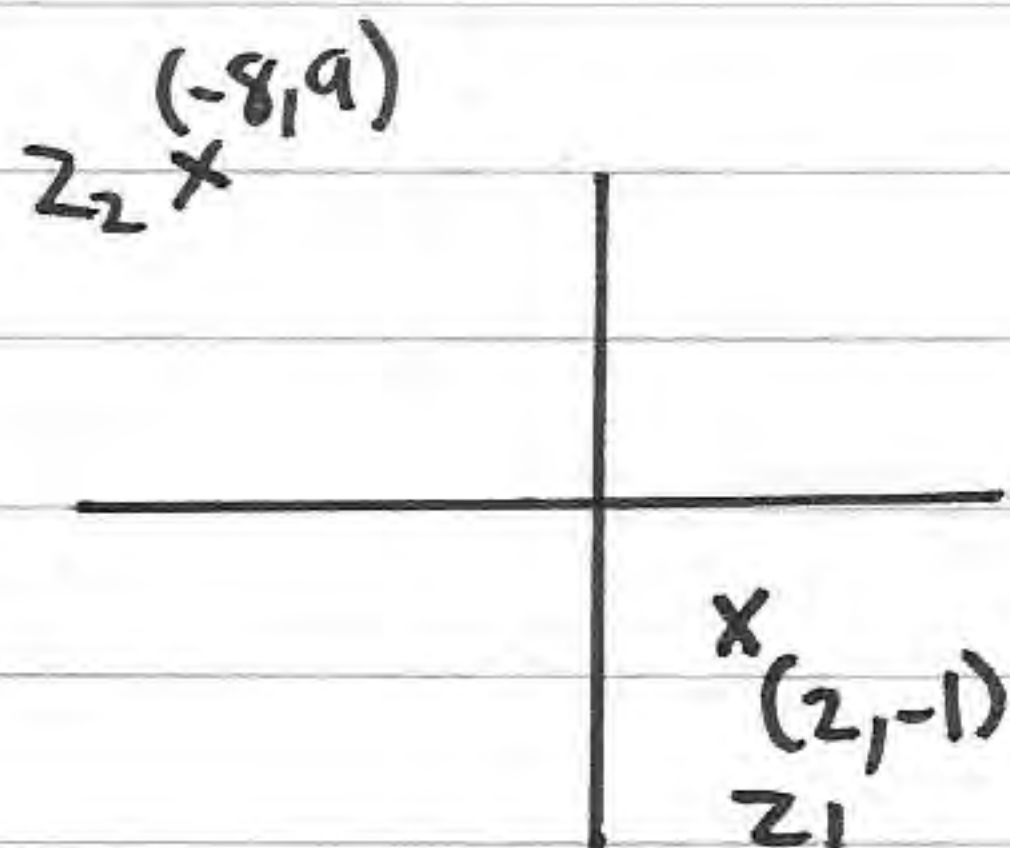


FPI June 2009

1) $z_1 = 2 - i$
 $z_2 = -8 + 9i$



b) $|z_1| = \sqrt{2^2 + 1^2} = \sqrt{5}$

c) $\arg(z_1) = -\tan^{-1}\left(\frac{1}{2}\right) = \underline{-0.46^\circ}$

d) $\frac{z_2}{z_1} = \frac{-8+9i}{2-i} \times \frac{2+i}{2+i} = \frac{-16-8i+18i+9i^2}{4-i^2} = \frac{-25+10i}{5}$
 $= \underline{-5+2i}$

2) $\sum r(r+1)(r+3) = \sum r^3 + 4r^2 + 3r = \sum r^3 + 4\sum r^2 + 3\sum r$

$$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$$

$$= \frac{3}{12}n^2(n+1)^2 + \frac{8}{12}n(n+1)(2n+1) + \frac{18}{12}n(n+1)$$

$$= \frac{1}{12}n(n+1)(3n(n+1) + 8(2n+1) + 18)$$

$$= \frac{1}{12}n(n+1)(3n^2 + 19n + 26)$$

$$= \frac{1}{12}n(n+1)(n+2)(3n+13) \Rightarrow \underline{k=13}$$

b) $\sum_{21}^{40} r(r+1)(r+3) = \frac{1}{12}(40)(41)(42)(133) - \frac{1}{12}(20)(21)(22)(73)$

$$= 707210$$

$$3) f(x) = (x^2 + 4)(x^2 + 8x + 25)$$

$$x^2 = -4 \Rightarrow x = \pm 2i$$

$$x^2 + 8x = -25 \Rightarrow (x+4)^2 - 16 = -25 \Rightarrow (x+4)^2 = -9$$

$$\Rightarrow x+4 = \pm 3i \Rightarrow x = -4 \pm 3i$$

$$x = 2i, -2i, -4+3i, -4-3i$$

$$b) 2i - 2i - 4 + 3i - 4 - 3i = \underline{-8}$$

$$4) f(x) = x^3 - x^2 - 6 \quad f(2.2) = -0.192 < 0$$

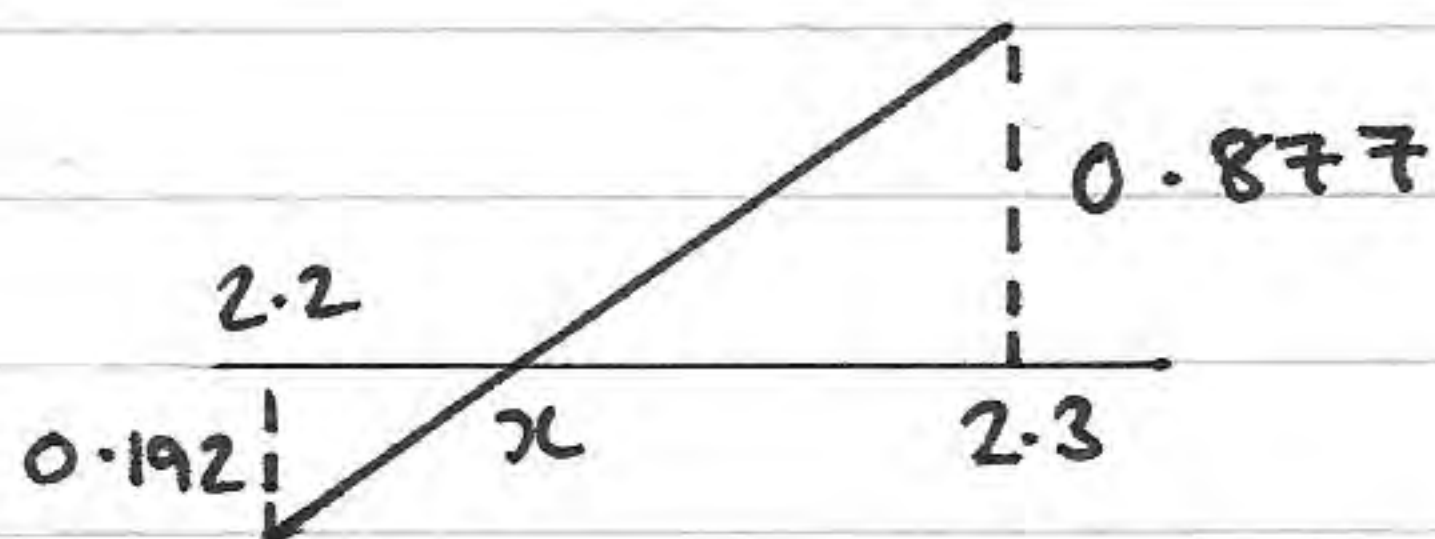
$$f(2.3) = 0.877 > 0$$

$$\text{Sign change} \Rightarrow x \in [2.2, 2.3]$$

$$b) f'(x) = 3x^2 - 2x$$

$$x_1 = 2.2 - \frac{f(2.2)}{f'(2.2)} = \underline{2.219}$$

c)



$$\frac{x - 2.2}{0.192} = \frac{2.3 - x}{0.877}$$

$$0.877x - 1.9294 = 0.4416 - 0.192x$$

$$1.069x = 2.371$$

$$x = \underline{2.218}$$

$$5) R = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix} \quad R^2 = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix} \begin{pmatrix} a & 2 \\ a & b \end{pmatrix} = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$$

$$R^2 = \begin{pmatrix} 15 & 0 \\ 0 & 15 \end{pmatrix} \Rightarrow a^2+2a=15 \Rightarrow a^2+2a-15=0 \\ (a+5)(a-3)=0 \\ \Rightarrow \underline{a=3} \quad \text{Since } a >$$

$$2a+2b=0 \Rightarrow \underline{b=-3}$$

$$6) y^2=16x=4ax \Rightarrow a=4 \quad \text{focus } S(4,0) \\ \text{directrix } x+4=0$$

$$a) x=4t^2 \Rightarrow y^2=(4t^2) \times 16 = 64t^2 \\ y = \sqrt{64t^2} = 8t \quad \#$$

$$b) S(4,0)$$

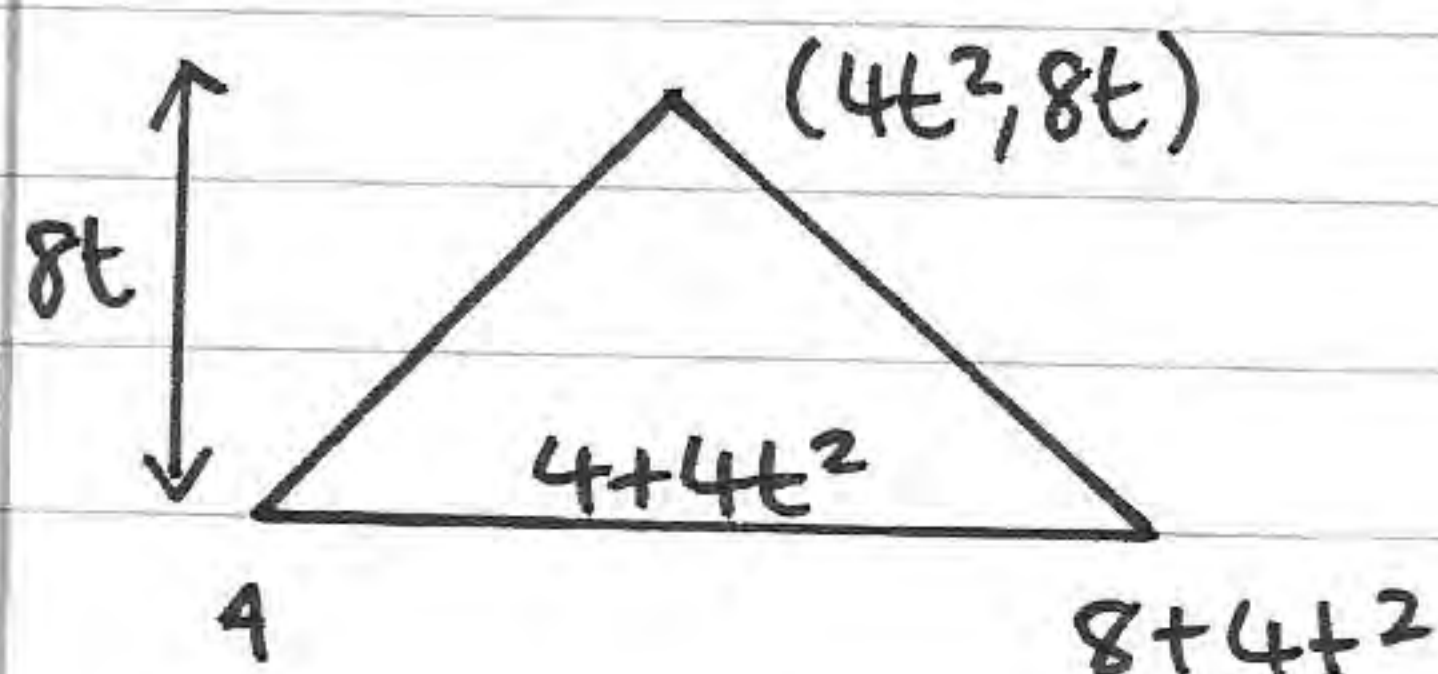
$$c) y^2=16x \Rightarrow \frac{d}{dx} y^2 = \frac{d}{dx} 16x \Rightarrow 2y \frac{dy}{dx} = 16$$

$$\Rightarrow \frac{dy}{dx} = \frac{8}{y} = Mt \Rightarrow Mn = \frac{-y}{8} = \frac{-8t}{8} = -t$$

$$\text{alt } y^2=16x \Rightarrow y=4x^{\frac{1}{2}} \quad \frac{dy}{dx} = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{4t^2}} = \frac{2}{2t}$$

$$y-8t = -t(x-4t^2) \Rightarrow y-8t = -tx + 4t^3 \\ \Rightarrow y+tx = 8t+4t^3 \quad \#$$

$$c) \text{ meets } x \text{ when } y=0 \Rightarrow tx = 8t+4t^3 \\ \Rightarrow x = 8+4t^2$$



$$\text{Area} = \frac{1}{2}(4+4t^2) \times 8t \\ = \underline{16t+16t^3}$$

$$7) A = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix} \quad \text{Singular} \Rightarrow \det A = 0 \Rightarrow 4a - 2 = 0 \Rightarrow \underline{a = \frac{1}{2}}$$

$$b) B = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix} \quad \det B = 12 - 2 = 10 \quad B^{-1} = \frac{1}{10} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

$$c) BP = Q \quad B^{-1}BP = B^{-1}Q \Rightarrow P = B^{-1}Q$$

$$P = \frac{1}{10} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} u-6 \\ 3u+12 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 4u-24+6u+24 \\ u-6+9u+36 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 10u \\ 10u+30 \end{pmatrix}$$

$$P = \begin{pmatrix} u \\ u+3 \end{pmatrix} \quad P(u, u+3) \quad \begin{array}{l} y = x + 3 \\ \text{if } x = u \quad y = u + 3 \text{ as required.} \end{array}$$

$$8) n=1 \quad f(1) = 5^1 + 8(1) + 3 = 16 = 4 \times 4 \Rightarrow \text{divisible by 4}$$

$$n=k \quad f(k) = 5^k + 8k + 3 \quad \text{assume is divisible by 4}$$

$$n=k+1 \quad f(k+1) = 5^{k+1} + 8(k+1) + 3 \\ \Rightarrow f(k+1) = 5(5^k) + 8k + 11$$

$$\begin{aligned} f(k+1) - f(k) &= (5(5^k) + 8k + 11) - (5^k + 8k + 3) \\ &= 4(5^k) + 8 \\ &= 4[(5^k) + 2] \end{aligned}$$

$$\Rightarrow f(k+1) = 4[(5^k) + 2] + f(k)$$

true for $n=1$

true for $n=k+1$ if true $n=k$

\therefore by induction true for all $n \in \mathbb{Z}^+$

$$b) \quad n=1 \quad \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \quad \text{true for } n=1$$

$$n=k \quad \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \quad \text{assume true}$$

$$n=k+1 \quad \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -1-2k \end{pmatrix}$$

$$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k$$

$$= \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$$

$$= \begin{pmatrix} 6k+3-4k & -6k-2+4k \\ 4k+2-2k & -4k-1+2k \end{pmatrix}$$

$$= \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -1-2k \end{pmatrix} \quad \text{as required.}$$

true for $n=1$, true for $n=k+1$ if true for $n=k$
 \therefore by induction true for all $n \in \mathbb{Z}^+$