FP1 Jan 2014 (IAL) (MA)

Q1a) $f(1) = -0.701511 5\ldots$
$f(1.4) = 1.950164\ldots$

} change in sign between $x=1$ and $x=1.4$ ∴ a root lies between $x=1$ and 1.4.

b)

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	$-0.7015\ldots$	1.4	$1.95016\ldots$	1.2	$0.588\ldots$
1	-0.7015	1.2	$0.588\ldots$	1.1	$-0.06798\ldots$

∴ interval is $\boxed{[1.1, 1.2]}$ //

Q2ai) $\text{Area}_{\text{image}} = \text{Area}_{\text{object}} \times |\det A|$.

$2x = x \cdot |\det A|$.

$\det A = -4k + 30$ //

$\therefore \quad 2\cancel{x} = \cancel{x}\,|30-4k|$

$2 = |30-4k|$

$2 = 30-4k$ or $2 = -(30-4k)$

$4k = 28$ $\qquad$ $4k = 32$

$\boxed{k=7}$ $\qquad$ $\boxed{k=8}$

ii) $BC = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 2+3 & 8-4-6 \\ -4+1 & 10-18 \end{pmatrix}$

2 × 3 $\qquad$ 3 × 2 $\qquad$ 2 × 2

$$= \boxed{\begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}}$$

Q3) $x = 2t$ $\quad y = \frac{2}{t}$

$P\ (1, 4)$ $\quad Q\ (8, \frac{1}{2})$

$$m_{PQ} = \frac{\frac{1}{2} - 4}{8 - 1} = -\frac{1}{2}$$

$\therefore\ m_L = 2$ $\quad \left(-\frac{1}{2} \times 2 = -1\right)$

$\hookrightarrow$ L is $\perp$ to PQ.

$\Rightarrow\ y - 0 = 2(x - 0)$

$\Rightarrow\ \boxed{y = 2x}$

Q4) $f(x) = 2x^{\frac{1}{2}} - 6x^{-2} - 3$

$f'(x) = x^{-\frac{1}{2}} + 12x^{-3}$

$f(3.5) = 0.25186\ldots$

$f'(3.5) = 0.81441\ldots$

$$\Rightarrow\ x_1 = 3.5 - \frac{0.25186\ldots}{0.81441\ldots} = \boxed{3.191}\ \text{3dp.}$$

Q5a) $|w| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \boxed{2}$

b) $zw = (5+\sqrt{3}i)(\sqrt{3}-i) = 5\sqrt{3} - 5i + 3i + \sqrt{3}$

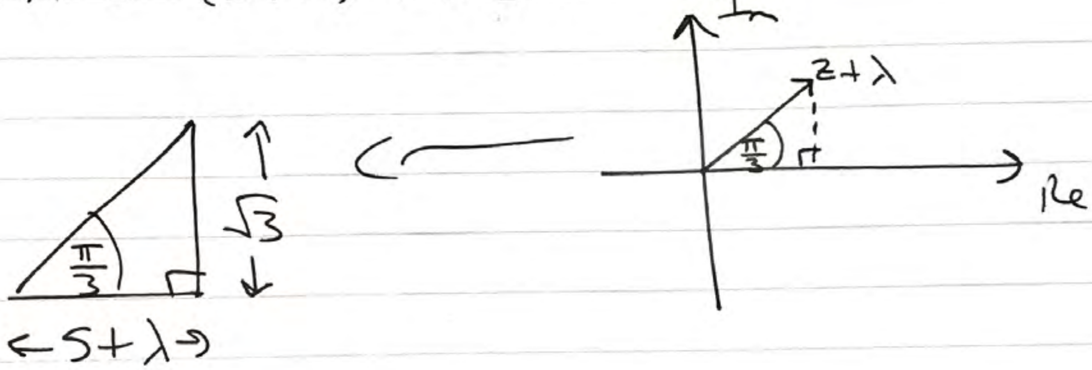
$= \boxed{6\sqrt{3} - 2i}$

c) $\dfrac{z}{w} = \dfrac{5+i\sqrt{3}}{\sqrt{3}-i} = \dfrac{(5+i\sqrt{3})(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$

$= \dfrac{5\sqrt{3} + 5i + 3i - \sqrt{3}}{3 + 1} = \dfrac{4\sqrt{3} + 8i}{4}$

$= \boxed{\sqrt{3} + 2i}$

d) $z + \lambda = (5+\lambda) + i\sqrt{3}$.



$\tan\frac{\pi}{3} = \dfrac{\sqrt{3}}{5+\lambda} = \sqrt{3} \xrightarrow{(\div\sqrt{3})} \dfrac{1}{5+\lambda} = 1.$

$\Rightarrow 5+\lambda = 1$

$\Rightarrow \boxed{\lambda = -4}$

Q6a) $\sum_1^n r(r+1)(r-1) = \sum_1^n r(r^2-1) = \sum_1^n r^3 - r$

$= \sum_1^n r^3 - \sum_1^n r$

$= \frac{n^2}{4}(n+1)^2 - \frac{n}{2}(n+1)$

$= \frac{n}{4}(n+1)\left[n(n+1) - 2\right]$

$= \frac{n}{4}(n+1)(n^2+n-2)$

$= \frac{n}{4}(n+1)(n+2)(n-1)$

$a = 2$

b) $10\sum_1^n r^2 = \frac{10}{6}n(n+1)(2n+1)$

$\Rightarrow \frac{10\cancel{n}}{6}\cancel{(n+1)}(2n+1) = \frac{\cancel{n}}{4}\cancel{(n+1)}(n+2)(n-1)$

$\Rightarrow \frac{5}{3}(2n+1) = \frac{1}{4}(n^2+n-2)$

$\times 12$

$\Rightarrow 20(2n+1) = 3n^2 + 3n - 6$

$\Rightarrow 3n^2 - 37n - 26 = 0$

$\Rightarrow (3n+2)(n-13) = 0 \quad n > 1 \quad \therefore \quad \boxed{n = 13}$

Q7a) $\det P = 3a(2b) - (2ab) = 6ab - 2ab$
$= 4ab$

$\therefore P^{-1} = \boxed{\frac{1}{4ab}\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}}$ //

b) $M = PQ$.

$P^{-1}M = P^{-1}PQ$

$P^{-1}M = Q$ //

$$P^{-1}M = \frac{1}{4ab}\begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$$

$$= \frac{1}{4ab}\begin{pmatrix} -12ab + 4ab & 14ab - 2ab \\ -6ab + 6ab & -3ab + 7ab \end{pmatrix} = \frac{1}{4ab}\begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}}$$

Q8a) $y^2 = 4ax$ [Implicit differentiation]

$2y\frac{dy}{dx} = 4a \quad \therefore \quad \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2ap} = \frac{1}{p}$ //

so at normal, $m = -p$ //

$\Rightarrow y - 2ap = -p(x - ap^2)$

$\Rightarrow y = -px + ap^3 + 2ap$

$\Rightarrow y + px = ap^3 + 2ap$

b) $\underline{(6a, 0)}$: $0 + p(6a) = ap^3 + 2ap$ //

$6\cancel{a}\cancel{p} - 2\cancel{a}\cancel{p} = \cancel{a}p^{\cancel{3}2}$

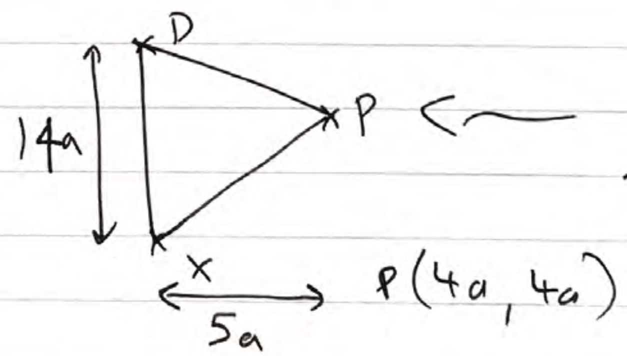
$4 = p^2 \quad \therefore \quad p = 2$ // $(p > 2)$.

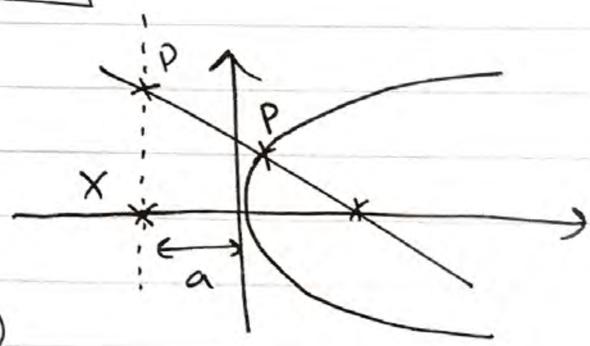
$\underline{\text{at } D, \ x = -a}$: $y - a(2) = a(8) + 2a(2)$

$y = 8a + 4a + 2a = 14a$

so $\boxed{D(-a, 14a)}$

c)





$\therefore$ Area XPD $= \frac{1}{2} \times 5a \times 14a = \boxed{35a^2}$

Q9) $(3-i)z^* + 2iz = 9-i$

$z^* = x - iy$
$z = x + iy$

$(3-i)(x-iy) + 2i(x+iy) = 9-i$

$3x - 3iy - ix - y + 2xi - 2y = 9 - i$

Gather all real / imaginary terms,

$\Rightarrow (3x - y - 2y) + (2x - x - 3y) = 9 - i$

$\Rightarrow (3x - 3y) + (x - 3y) = (9) + (-1)i$

compare real / imaginary terms,

$3x - 3y = 9$ — 1
$x - 3y = -1$ — 2

solve simultaneously,

1 : $x - y = 3 \quad \therefore \quad y - x = -3$

2 : $x - 3y = -1$

1 + 2 : $y + x - x - 3y = -4$

$\hookrightarrow -2y = -4$

$\therefore \boxed{y = 2}$

so $x = y + 3$

$\boxed{x = 5}$