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Mark Scheme (Results)

Summer 2012

GCE Mathematics
6667 Further Pure 1

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Summer 2012
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Summer 2012
6667 Further Pure Maths 1
FP1 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqcap$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|$$, leading to x = ...

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$, leading to x = …

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*), leading to x = …

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:
1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Summer 2012
6667 Further Pure FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
	$f(x) = 2x^3 - 6x^2 - 7x - 4$		
1. (a)	$\underline{f(4) = 128 - 96 - 28 - 4 = 0}$	$\underline{128 - 96 - 28 - 4 = 0}$	B1
	$\underline{\text{Just}}$ $2(4)^3 - 6(4)^2 - 7(4) - 4 = 0$ or $2(64) - 6(16) - 7(4) - 4 = 0$ is B0 But $2(64) - 6(16) - 7(4) - 4 = 128 - 128 = 0$ or $2(4)^3 - 6(4)^2 - 7(4) - 4 = 4 - 4 = 0$ is B1		
	There must be sufficient working to <u>show</u> that f(4) = 0		
			[1]
(b)	$f(4) = 0 \Rightarrow (x - 4)$ is a factor.		
	$f(x) = (x - 4)(2x^2 + 2x + 1)$	M1: $(2x^2 + kx + 1)$ Uses inspection or long division or compares coefficients **and** $(x - 4)$ (**not** $(x + 4)$) to obtain a quadratic factor of this form. A1: $(2x^2 + 2x + 1)$ cao	M1A1
	So, $x = \dfrac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$ $(2)\left(x^2 + x + \frac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \frac{1}{2}\right)^2 \pm k \pm \frac{1}{2}\right)\ k \neq 0 \Rightarrow x =$	Use of correct quadratic formula for their <u>3TQ</u> or completes the square.	M1
	Allow an attempt at factorisation provided the usual conditions are satisfied and proceeds as far as *x* = ..		
	$\Rightarrow x = \dfrac{-2 \pm \sqrt{-4}}{2(2)}$		
	$\Rightarrow x = 4,\ \dfrac{-2 \pm 2\mathrm{i}}{4}$	**All <u>three</u> roots stated somewhere in (b).** Complex roots must be at least as given but apply isw if necessary.	A1
			[4]
			5 marks

Question Number	Scheme	Notes	Marks
2. (a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct (unsimplified) elements in a <u>dimensionally correct</u> matrix. A 2x2 matrix with a number or a calculation at each corner.	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	Correct answer	A1
	A correct answer with no working can score both marks		
			[2]
(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,		
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	An attempt to add C to D. Can be implied by two out of four correct (unsimplified) elements in a <u>dimensionally correct</u> matrix.	M1
	$\mathbf{E}$ does not have an inverse $\Rightarrow \det \mathbf{E} = 0$.		
	$8(6+k) - 12(2k+2)$	Applies "$ad - bc$" to $\mathbf{E}$ where $\mathbf{E}$ is a 2x2 matrix.	M1
	$8(6+k) - 12(2k+2) = 0$	States or applies $\det(\mathbf{E}) = 0$ where $\det(\mathbf{E}) = ad - bc$ or $ad + bc$ only and $\mathbf{E}$ is a 2x2 matrix.	M1
	Note $8(6+k) - 12(2k+2) = 0$ or $8(6+k) = 12(2k+2)$ could score both M's		
	$48 + 8k = 24k + 24$ $24 = 16k$		
	$k = \frac{3}{2}$		A1 oe
			[4]
			6 marks

Question Number	Scheme	Notes	Marks
3.	$f(x) = x^2 + \dfrac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$		
	$f(x) = x^2 + \dfrac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$f'(x) = 2x - \dfrac{3}{8}x^{-\frac{3}{2}} - 3 \{+0\}$	M1: $x^n \to x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$f(4) = -2.625 = -\dfrac{21}{8} = -2\dfrac{5}{8}$ *or* $4^2 + \dfrac{3}{4\sqrt{4}} - 3\times 4 - 7$	$f(4) = -2.625$ A correct <u>evaluation</u> of f(4) or a correct <u>numerical expression</u> for f(4). This can be implied by a correct answer below but in all other cases, <u>f(4) must be seen explicitly evaluated</u> or as an <u>expression</u>.	B1
	$f'(4) = 4.953125 = \dfrac{317}{64} = 4\dfrac{61}{64}$	Attempt to insert $x = 4$ into their $f'(x)$. Not dependent on the first M but must be what they think is $f'(x)$.	M1
	$\alpha_2 = 4 - \left(\dfrac{"-2.625"}{"4.953125"}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454... \quad \left(= \dfrac{1436}{317} = 4\frac{168}{317}\right)$		
	$= 4.53 \;\; (2\,\text{dp})$	4.53 **cso**	A1 **cao**
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \dfrac{f(4)}{f'(4)} = 4.53$ can score full marks.		
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
4. (a)	$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right)$		
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)} - 3n$	M1; An attempt to use at least one of the standard formulae correctly in summing at least 2 terms of $r^3 + 6r - 3$ A1: Correct underlined expression. B1: $-3 \rightarrow -3n$	M1A1B1
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2 + 3n - 3n$		
	If any marks have been lost, no further marks are available in part (a)		
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2$ $= \frac{1}{4}n^2\left((n+1)^2 + 12\right)$	Cancels out the $3n$ and attempts to factorise out at least $\frac{1}{4}n$.	dM1
	$= \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **(AG)**	Correct answer **with no errors seen**.	A1 *
	Provided the first 3 marks are scored, allow the next two marks for correctly showing the algebraic equivalence. E.g. showing that both $\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1) - 3n$ and $\frac{1}{4}n^2\left(n^2 + 2n + 13\right) = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{13}{4}n^2$		
	There are no marks for proof by induction but apply the scheme if necessary.		
			[5]
(b)	$S_n = \sum_{r=16}^{30}\left(r^3 + 6r - 3\right) = S_{30} - S_{15}$		
	$= \frac{1}{4}(30)^2\left(30^2 + 2(30) + 13\right) - \frac{1}{4}(15)^2\left(15^2 + 2(15) + 13\right)$	Use of $S_{30} - S_{15}$ or $S_{30} - S_{16}$	M1
	NB They must be using $S_n = \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **not** $S_n = n^3 + 6n - 3$		
	$= 218925 - 15075$		
	$= 203850$	203850	A1 **cao**
	NB $S_{30} - S_{16}$ =218925 – 19264 = 199661 (Scores M1 A0)		
			[2]
			7 marks

Question Number	Scheme	Notes	Marks
5.	$C: y^2 = 8x \Rightarrow a = \frac{8}{4} = 2$		
(a)	$PQ = 12 \Rightarrow$ By symmetry $y_P = \frac{12}{2} = \underline{6}$	$y = \underline{6}$	B1
			[1]
(b)	$y^2 = 8x \Rightarrow 6^2 = 8x$	Substitutes their y-coordinate into $y^2 = 8x$.	M1
	$\Rightarrow x = \underline{\frac{36}{8}} = \underline{\frac{9}{2}}$ (So P has coordinates $\left(\frac{9}{2}, 6\right)$)	$\Rightarrow x = \underline{\frac{36}{8}}$ or $\underline{\frac{9}{2}}$	A1 oe
			[2]
(c)	Focus $S(2, 0)$	Focus has coordinates $(2, 0)$. Seen or implied. Can score anywhere.	B1
	Gradient $PS = \dfrac{6-0}{\frac{9}{2}-2}$ $\left\{= \dfrac{6}{\left(\frac{5}{2}\right)} = \dfrac{12}{5}\right\}$	Correct method for finding the gradient of the line segment PS. If no gradient formula is quoted and the gradient is incorrect, score M0 but allow this mark if there is a clear use of $\dfrac{y_2 - y_1}{x_2 - x_1}$ even if their coordinates are 'confused'.	M1
	Either $y - 0 = \frac{12}{5}(x-2)$ or $y - 6 = \frac{12}{5}(x - \frac{9}{2})$; - - - - - - - - - - - - **or** $y = \frac{12}{5}x + c$ and $0 = \frac{12}{5}(2) + c \Rightarrow c = -\frac{24}{5}$;	$y - y_1 = m(x - x_1)$ with 'their PS gradient' and their (x_1, y_1) **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).** - - - - - - - - - - - - or uses $y = mx + c$ with 'their gradient' in an attempt to find c. **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).**	M1
	l: $\underline{12x - 5y - 24 = 0}$	$\underline{12x - 5y - 24 = 0}$	A1
	Allow any equivalent form e.g. $k(12x - 5y - 24) = 0$ where k is an integer		
			[4]
			7 marks

Question Number	Scheme	Notes	Marks
6.	$\mathrm{f}(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$		
(a)	$\mathrm{f}(1) = -2.45369751...$ $\mathrm{f}(2) = 1.557407725...$	Attempts to evaluate both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf. Nm	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is between $x = 1$ and $x = 2$.	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -2.453.. < 0 < 1.5574..) and conclusion.	A1
			[2]
(b)	$\frac{\alpha - 1}{"2.45369751..."} = \frac{2 - \alpha}{"1.557407725..."}$ **or** $\frac{"2.45369751..." + "1.557407725"}{1} = \frac{"2.45369751..."}{\alpha - 1}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any “negative lengths” are used, score M0		
	$\alpha = 1 + \left(\frac{"2.45369751..."}{"1.557407725..." + "2.45369751..."}\right)1$ $= \frac{6.464802745}{4.011105235}$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.61.)	A1 $\sqrt{}$
	$= 1.611726037...$	awrt 1.61	A1
			[3]
			5 marks
	Special Case – Use of Degrees		
	$\mathrm{f}(1) = -2.991273132...$ $\mathrm{f}(2) = 0.017455064...$	Attempts to evaluate both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1A0
	$\frac{\alpha - 1}{"2.991273132..."} = \frac{2 - \alpha}{"0.017455064..."}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any “negative lengths” are used, score M0		
	$\alpha = 1 + \left(\frac{"2.99127123..."}{"0.017455064..." + "2.99127123..."}\right)1$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.99.)	A1 $\sqrt{}$
	$= 1.994198523...$		A0

Question Number	Scheme	Notes	Marks
7. (a)	$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	$\tan^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\tan^{-1}\left(\pm\frac{2}{\sqrt{3}}\right)$ seen or evaluated	M1
	Awrt ±0.71 or awrt ±0.86 can be taken as evidence for the method mark. Or ±40.89 or ±49.10 if working in degrees		
	$= -0.7137243789.. = -0.71$ (2 dp)	awrt -0.71 or awrt 5.57	A1
	NB $\tan\left(\frac{\sqrt{3}}{2}\right) = 1.18$ and $\tan\left(\frac{2}{\sqrt{3}}\right) = 2.26$ **and both score M0**		
			[2]
(b)	$z^2 = \left(2 - \mathrm{i}\sqrt{3}\right)\left(2 - \mathrm{i}\sqrt{3}\right)$ $= 4 - 2\mathrm{i}\sqrt{3} - 2\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
	$= 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} - 3\right)$ $= 2 - \mathrm{i}\sqrt{3} + \left(1 - 4\mathrm{i}\sqrt{3}\right)$ $= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	M1: An understanding that $\mathrm{i}^2 = -1$ and an attempt to add z and put in the form $a + b\mathrm{i}\sqrt{3}$ A1: $3 - 5\mathrm{i}\sqrt{3}$	M1A1
	$z + z^2 = 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} + 3\right) = 9 - 5\mathrm{i}\sqrt{3}$ scores M1M0A0 (No evidence of $\mathrm{i}^2 = -1$)		
			[3]
(c)	$\frac{z+7}{z-1} = \frac{2 - \mathrm{i}\sqrt{3} + 7}{2 - \mathrm{i}\sqrt{3} - 1}$	Substitutes $z = 2 - \mathrm{i}\sqrt{3}$ into both numerator and denominator.	M1
	$= \frac{\left(9 - \mathrm{i}\sqrt{3}\right)}{\left(1 - \mathrm{i}\sqrt{3}\right)} \times \frac{\left(1 + \mathrm{i}\sqrt{3}\right)}{\left(1 + \mathrm{i}\sqrt{3}\right)}$	Simplifies $\frac{z+7}{z-1}$ **and** multiplies by $\frac{\textbf{their } \left(1 + \mathrm{i}\sqrt{3}\right)}{\textbf{their } \left(1 + \mathrm{i}\sqrt{3}\right)}$	**d**M1
	$= \frac{9 + 9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3} + 3}{1 + 3}$ $= \frac{12 + 8\mathrm{i}\sqrt{3}}{4}$	Simplifies realising that a real number is needed in the denominator and applies $\mathrm{i}^2 = -1$ in their numerator expression and denominator expression.	M1
	$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3,\ d = 2$.)	$3 + 2\mathrm{i}\sqrt{3}$	A1
			[4]
(d)	$w = \lambda - 3\mathrm{i}$, and $\arg(4 - 5\mathrm{i} + 3w) = -\frac{\pi}{2}$		
	$\left(4 - 5i + 3w = 4 + 3\lambda - 14\mathrm{i}\right)$		
	So real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	States real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	M1
	So, $\lambda = -\frac{4}{3}$	$-\frac{4}{3}$	A1
			[2]
	Allow $\pm\left(\frac{14}{3\lambda + 4}\right) = \pm\infty \Rightarrow 3\lambda + 4 = 0$ M1 $\Rightarrow \lambda = -\frac{4}{3}$ A1		
			11 marks

Question Number	Scheme	Notes	Marks
8.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = kx^{-2}$	
	$xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{c}{t^2}.\frac{1}{c}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -c^2x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ *or* $\frac{dy}{dx} = \frac{-c}{t^2}.\frac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	So, $m_T = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	$-\frac{1}{t^2}$	
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$ $\quad (\times t^2)$	$y - \frac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \frac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of t or c or both c and t.**	M1
	$x + t^2y = 2ct$ (Allow $t^2y + x = 2ct$)	Correct solution.	A1 *
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\frac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
			[4]
(b)	$y = 0 \Rightarrow x = 2ct \Rightarrow A(2ct, 0)$.	$x = 2ct$, seen or implied.	B1
	$x = 0 \Rightarrow y = \frac{2ct}{t^2} \Rightarrow B\left(0, \frac{2c}{t}\right)$.	$y = \frac{2ct}{t^2}$ or $\frac{2c}{t}$, seen or implied.	B1
	Area $OAB = 36 \Rightarrow \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 36$	Applies $\frac{1}{2}(\text{their } x)(\text{their } y) = 36$ where x and y are functions of c or t or both (not x or y) and some attempt was made to substitute both $x = 0$ **and** $y = 0$ in the tangent to find A and B.	M1
	Do not allow the x and y coordinates of P to be used for the dimensions of the triangle.		
	$\Rightarrow 2c^2 = 36 \Rightarrow c^2 = 18 \Rightarrow c = 3\sqrt{2}$	$c = 3\sqrt{2}$	A1
		Do <u>not</u> allow $c = \pm 3\sqrt{2}$	**[4]**
			8 marks

Question Number	Scheme	Notes	Marks
9. (a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	$\underline{-23}$	B1
			[1]
(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	Either, $3(2a-7) + 4(a-1) = 25$ or $2(2a-7) - 5(a-1) = -14$ or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Any one correct equation (unsimplified) inside or outside matrices	A1
	giving $a = 5$	$a = 5$	A1
			[3]
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4);\ = \underline{12}\ (\text{units})^2$	M1: $\frac{1}{2}(6)(\text{Their } a - 1)$ A1: 12 **cao and cso**	M1A1
	Note A(6, 0) is sometimes misinterpreted as (0, 6) – this is the wrong triangle and scores M0 e.g.1/2x6x3 = 9		
			[2]
(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	$\pm \det \mathbf{M} \times$ (their part (c) answer)	M1
		$\underline{276}$ (follow through provided area > 0)	A1 $\sqrt{\ }$
	A method not involving the determinant requires the coordinates of R$'$ to be calculated ((18, 12)) and then a <u>correct</u> method for the area e.g. (26x25 – 7x13 – 9x12 – 7x25) M1 = 276 A1		
			[2]
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1: Rotation, Rotates, Rotate, Rotating (not **turn**) B1: 90° anti-clockwise (or 270° clockwise) about (around/from etc.) $(0, 0)$	B1;B1
			[2]
(f)	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0) - (1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix};\ = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Applies $\mathbf{M}(\text{their } \mathbf{A}^{-1})$	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$		A1
	NB some candidates state **M** = **AB** and then calculate **MA**$^{-1}$ or state **M** = **BA** and then calculate **A**$^{-1}$**M**. These could score M0A0 M1A1ft and M1A1M0A0 respectively.		**[4]**
			14 marks
	Special case		
(f)	$\mathbf{M} = \mathbf{AB}$	$\mathbf{M} = \mathbf{AB}$, seen or implied.	M0
		$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A0
	$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & -4 \end{pmatrix}$	Applies $(\text{their } \mathbf{A}^{-1})\mathbf{M}$	M1A1ft

Question Number	Scheme	Notes	Marks
10.	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^{+}$.		
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	M1: Attempts $\mathrm{f}(k+1) - \mathrm{f}(k)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} - 2^{2k-1} - 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} - 2^{2k-1} - 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) - 2^{2k-1} - 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 3\left(2^{2k-1}\right) + 8\left(3^{2k-1}\right)$		
	$= 3\left(2^{2k-1}\right) + 3\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 3\mathrm{f}(k) + 5\left(3^{2k-1}\right)$		
	$\therefore \mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $4(2^{2k-1} + 3^{2k-1}) + 5\left(3^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end,** at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks
	All methods should complete to f(k + 1) = ... where f(k +1) is clearly shown to be divisible by 5 to enable the final 2 marks to be available.		
Note that there are many different ways of proving this result by induction.			

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.

- ft denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"

Other Possible Solutions

Question Number	Scheme	Notes	Marks
Aliter **4.(a) Way 2**	$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right)$		
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)} - 3n$	An attempt to use at least one of the standard formulae correctly. Correct underlined expression. $-3 \rightarrow -3n$	M1 A1 B1
	If any marks have been lost, no further marks are available in part (a).		
	$= \frac{1}{4}n\left(n(n+1)^2 + 12(n+1) - 12\right)$ $= \frac{1}{4}n\left(n(n+1)^2 + 12n + 12 - 12\right)$ $= \frac{1}{4}n\left(n(n+1)^2 + 12n\right)$	Attempts to factorise out at least $\frac{1}{4}n$ from a <u>correct</u> expression and cancels the constant inside the brackets.	dM1
	$= \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **(AG)**	Correct answer	A1 $*$ **[5]**
			5 marks

Question Number	Scheme	Notes	Marks
***Aliter* 6.(b) Way 2**	$y - f(2) = \frac{f(2) - f(1)}{2-1}(x-2)$ $or\ y - f(1) = \frac{f(2) - f(1)}{2-1}(x-1)$ $or\ y = \frac{f(2) - f(1)}{2-1}x + c$ *with an attempt to find c*	Correct straight line method. It must be a correct statement using their f(2) and f(1). Can be implied by working below.	M1
	NB 'm' = 4.011105235		
	$y = 0 \Rightarrow \alpha = \frac{f(2)}{f(1) - f(2)} + 2$ $or\ \alpha = \frac{f(1)}{f(1) - f(2)} + 1$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.61.)	A1 $\sqrt{}$
	= 1.611726037...	awrt 1.61	A1
			[3]

Question Number	Scheme	Notes	Marks
***Aliter* 7. (b) Way 2**	$z + z^2 = z(1+z)$		
	$= \left(2 - \mathrm{i}\sqrt{3}\right)\left(1 + \left(2 - \mathrm{i}\sqrt{3}\right)\right)$ $= \left(2 - \mathrm{i}\sqrt{3}\right)\left(3 - \mathrm{i}\sqrt{3}\right)$ $= 6 - 2\mathrm{i}\sqrt{3} - 3\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
	$= 6 - 2\mathrm{i}\sqrt{3} - 3\mathrm{i}\sqrt{3} - 3$	M1: An understanding that $\mathrm{i}^2 = -1$ and an attempt to put in the form $a + b\mathrm{i}\sqrt{3}$	M1
	$= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	$3 - 5\mathrm{i}\sqrt{3}$	A1
			[3]

Question Number	Scheme	Notes	Marks
Aliter **9. (b) Way 2**	$\mathbf{M}: \begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} \rightarrow \begin{pmatrix} 25 \\ -14 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$ or $\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}^{-1}\begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	$\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \frac{1}{(-23)}\begin{pmatrix} -5 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 25 \\ -14 \end{pmatrix} = \frac{1}{(-23)}\begin{pmatrix} -125+56 \\ -50-42 \end{pmatrix}$		
	Either, $(2a-7) = 3$ or $(a-1) = 4$	Any one correct equation.	A1
	giving $a = 5$	$a = 5$	A1
			[3]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (c)** **Way 2 Determinant**	$Area\ ORS = \frac{1}{2}\begin{vmatrix} 6 & 3 & 0 & 6 \\ 0 & 4 & 0 & 0 \end{vmatrix}$ $= \frac{1}{2}\left\|(6\times 4 - 3\times 0 + 0 - 0 + 0 - 0)\right\|$	Correct calculation	M1
	= 12		A1
			[2]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (d)** **Way 2 Determinant**	$Area\ ORS = \frac{1}{2}\begin{vmatrix} 18 & 25 & 0 & 18 \\ 12 & -14 & 0 & 12 \end{vmatrix}$ $= \frac{1}{2}\left\|(18\times -14 - 12\times 25 + 0 - 0 + 0 - 0)\right\|$	Correct calculation	M1
	= 276		A1 $\sqrt{}$
			[2]

Question Number	Scheme	Notes	Marks
***Aliter* 9. (f) Way 2**	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, with constants to be found.	A1
	$\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} b & -a \\ d & -c \end{pmatrix}$	$\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$ = their $\begin{pmatrix} b & -a \\ d & -c \end{pmatrix}$ with at least two elements correct on RHS.	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$	Correct matrix for **B** of $\begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$ or a = -4, b = 3, c = 5, d = 2	A1
			[4]

Question Number	Scheme	Notes	Marks
Aliter **10. Way 2**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in$ ¢$^{+}$.		
	$\mathrm{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1}$	M1: Attempts $\mathrm{f}(k+1)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$\mathrm{f}(k+1) = 4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$ or $\mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $\mathrm{f}(k+1) = 9\mathrm{f}(k) - 5\left(2^{2k-1}\right)$ or $\mathrm{f}(k+1) = 9\left(2^{2k-1} + 3^{2k-1}\right) - 5\left(2^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]

Question Number	Scheme	Notes	Marks
***Aliter* 10. Way 3**	$f(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$f(1) = 2^1 + 3^1 = 5,$	Shows that $f(1) = 5$.	B1
	Assume that for $n = k$, $f(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$f(k+1) + f(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1}$	M1: Attempts $f(k+1) + f(k)$. A1: Correct expression for <u>f(*k* + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} + 2^{2k-1} + 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} + 2^{2k-1} + 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 2^{2k-1} + 9\left(3^{2k-1}\right) + 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right)$		
	$= 5\left(2^{2k-1}\right) + 5\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 5f(k) + 5\left(3^{2k-1}\right)$		
	$\therefore f(k+1) = 4f(k) + 5\left(3^{2k-1}\right)$ or $4\left(2^{2k-1} + 3^{2k-1}\right) + 5\left(3^{2k-1}\right)$	Where f(*k* + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
***Aliter* 10. Way 4**	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	Shows that $\mathrm{f}(1) = 5$.	B1
	Assume that for $n = k$, $\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^+$.		
	$\mathrm{f}(k+1) = \mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$		
	$\mathrm{f}(k+1) = 2^{2(k+1)-1} + 3^{2(k+1)-1} + 2^{2k-1} + 3^{2k-1} - (2^{2k-1} + 3^{2k-1})$	M1: Attempts $\mathrm{f}(k+1) + \mathrm{f}(k) - \mathrm{f}(k)$ A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) + 2^{2k-1} + 3^{2k-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 5\left(2^{2k-1}\right) + 10\left(3^{2k-1}\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$=5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \left(2^{2k-1} + 3^{2k-1}\right)$		
	$=5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - \mathrm{f}(k)$ *or* $5\left(\left(2^{2k-1}\right) + 2\left(3^{2k-1}\right)\right) - (2^{2k-1} + 3^{2k-1})$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end**, at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks

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