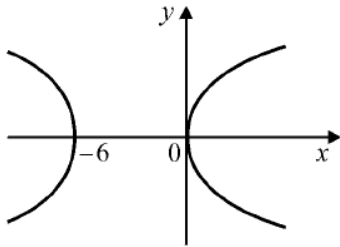
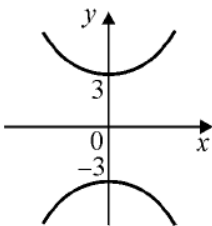


Further Pure 1 Past Paper Questions Pack B: Mark Scheme

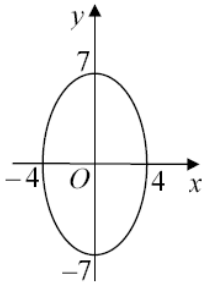
Taken from MBP1, MBP3, MBP4, MBP5

Parabolas, Ellipses and Hyperbolas

Pure 3 January 2002

| | | | | |
|--------------|---|-----------|----------|---|
| 1(a) | G1 | B1 | 1 | |
| (b)(i) |  | M1 A1✓ | 2 | Idea of translation to left (ft their graph) Correct intercepts marked |
| (ii) |  | M1 A1 | 2 | ✓ reflected in $y = x$ Correct with intercepts marked on y -axis |
| Total | | | 5 | |

Pure 3 January 2003

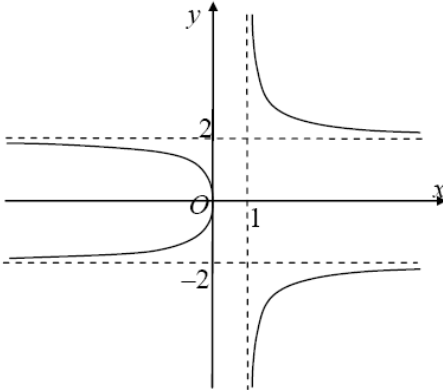
| | | | | |
|--------------|---|----------------------|----------|--|
| 3(a) |  | M1 A1 B1 | 3 | Ellipse Symmetrical with major axis in y -direction Intercepts 4 and 7 |
| (b) | One way stretch in x -direction Scale factor $\frac{1}{2}$ Translation in y -direction 3 units | M1 A1 M1 A1 | 4 | $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ |
| Total | | | 7 | |

Pure 3 January 2004

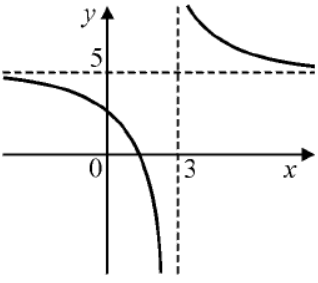
| | | | | |
|--------------|--|----------|----------|--|
| 2(a) | c - shaped parabola Vertex at O , good sketch, symmetry obvious | M1 A1 | 2 | <i>Essentially</i> all correct |
| (b) | $x^2 = 8y$ or equivalent | M1 A1 | 2 | M1 for general idea |
| (c) | Translation; by vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ | M1 A1 | 2 | sc: B1 for correct description without "translation" |
| Total | | | 6 | |

Rational Functions and Asymptotes

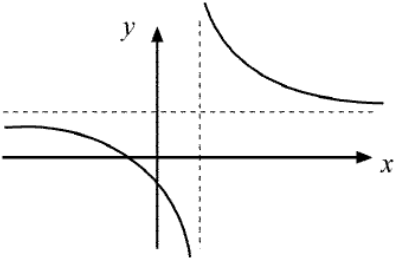
Pure 3 January 2004

| | | | | |
|---|---|-------------------------|----------|---|
| 3(a) | $a = 4$ and $b = 1$ | B1 B1 | 2 | |
| (b) | Asymptotes $x = 1$, $y = 2$, $y = -2$ Graph: Correct for $y > 0$ Symmetry in x -axis All correct | B1 B1 B1 B1 B1 | 5 | One correct; second correct Or B1 for each correct region E.g. 4/5 for all correct graph but with asymptotes $x = 1, y = \pm 4$ |
|  | | | | |
| Total | | | 7 | |

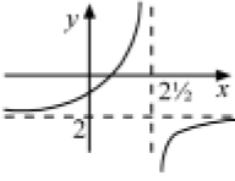
Pure 3 January 2002

| | | | | |
|------|--|----------|---|--|
| 3(a) |  <p style="text-align: center;">$y = 5$ $x = 3$</p> <p style="text-align: center;">$(\frac{7}{5}, 0)$ and $(0, \frac{7}{3})$</p> | M1 A1 | | Rectangular hyperbola, one branch Good sketch |
| (b) | $x > 3$ $x < \frac{7}{5}$ | B1 B1 | 5 | Asymptotes equations stated Allow single B1 if values only shown on graph (no equations stated). Condone as values marked on graph |
| | | B1 B1 | 2 | |

Pure 3 June 2002

| | | | |
|---|--|----------|---|
| <p>2(a)</p> <p>$(0, -2)$ accept $x = 0, y = -2$</p> <p>$\left(-\frac{4}{3}, 0\right)$ accept $y = 0, x = -\frac{4}{3}$</p> <p>Asymptotes $x = 2$</p> <p>$y = 3$</p>  | <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1✓</p> | <p>6</p> | <p>'x asymptote is 2, y asymptote is 3' - allow B1 only</p> <p>$x \rightarrow 2, y \rightarrow 3$ B1 only</p> <p>One branch of hyperbola ft asymptotes</p> |
| <p>(b)</p> <p>Appropriate method</p> <p>Consideration of graph</p> <p>$y = 1 \Rightarrow 3x + 4 = x - 2$</p> <p>$\Rightarrow x = -3$</p> <p>Solution: $x < -3$</p> <p>$x > 2$</p> | <p>M1</p> <p>A1</p> <p>A1</p> | <p>3</p> | <p>Multiply both sides by $(x - 2)^2$</p> <p>$\frac{3x + 4}{x - 2} - 1 > 0$</p> <p>Considering $(x - 2) > 0$ and $(x - 2) < 0$</p> <p>$3(x + 4) > (x - 2) \Rightarrow x > -3$ only M0</p> |
| Total | | 9 | |

Pure 3 January 2003

| Q | Solution | Marks | Total | Comments |
|--|--|----------|---|----------|
| <p>1(a)</p> <p>$(0, -\frac{3}{5})$ accept $x = 0, y = -\frac{3}{5}$</p> <p>$\frac{3}{4}, 0$ accept $y = 0, x = \frac{3}{4}$</p> <p>Asymptotes $x = 2\frac{1}{2}$</p> <p>$y = -2$</p>  | <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1✓</p> | <p>6</p> | <p>One branch of hyperbola</p> <p>ft asymptotes</p> <p>Condone lack of symmetry to show second branch</p> | |
| <p>(b)</p> <p>Appropriate method</p> <p>Consideration of graph</p> <p>$y = 0 \Rightarrow 3 - 4x = 0 \Rightarrow x = \frac{3}{4}$</p> <p>Solution: $x < \frac{3}{4}$</p> <p>$x > 2\frac{1}{2}$</p> | <p>M1</p> <p>A1</p> <p>B1</p> | <p>3</p> | <p>Multiply both sides by $(2x - 5)^2$</p> <p>Considering $(2x - 5) > 0$ and $(2x - 5) < 0$</p> <p>$3 - 4x < 0 \Rightarrow x > \frac{3}{4}$ scores M0</p> | |
| Total | | 9 | | |

Pure 5 June 2002

| | | | | |
|--------------|--|----------------------------------|-----------|--|
| 7(a) | {Vert. Asym....} $x = 2$ $x = 1.5$ {Horiz. Asym....} $y = 1.5$ | B1 B1 B1 | 3 | sc If 0/3 give B1 for all three values seen |
| (b)(i) | $(2x - 3)(x - 2) = 2x^2 - 7x + 6$ $(2y - 3)x^2 + (9 - 7y)x + 6y - 7 \{=0\}$ $\Delta = (9 - 7y)^2 - 4(2y - 3)(6y - 7)$ $y^2 + 2y - 3$ $(y + 3)(y - 1)$ For real x , $\Delta \geq 0 \Rightarrow y \geq 1$ or $y \leq -3$ \Rightarrow no real values of x for which $-3 < y < 1$ | B1 M1 A1 m1 A1 m1 | | Can be gained in part (a) or (b)(i) Attempt to form quadratic in x Correct quadratic in x Considers $b^2 - 4ac$ Attempt to factorise or solve |
| (b)(ii) | $y = 1 \Rightarrow -x^2 + 2x - 1 = 0 \Rightarrow x = 1$ $y = -3 \Rightarrow -9x^2 + 30x - 25 = 0 \Rightarrow x = 5/3$ Turning points are $(1, 1)$ and $(5/3, -3)$ | M1 m1 A2,1 | 7 4 | ag For subst $y = 1$ or $y = -3$ to form a valid quadratic in x For a good attempt to solve a quadratic equation in x Allow A1 for one point correct sc If method implied by 'hence' not used then max 2/4 B2 for $(1, 1)$ and $(5/3, -3)$ B1 for any 2 of these 4 coordinates |
| Total | | | 14 | |

Pure 5 January 2003

| | | | | |
|--------------|--|----------|----------|---|
| 2(a) | Asymptote $x = 0$ Asymptote $y = 2$ | B1 B1 | 2 | |
| (b) | | B2 | 2 | Fully correct shape with horizontal asymptote clearly above the x -axis (B1 if either only 1 branch correctly drawn or correct shape with horizontal asymptote the x -axis) |
| Total | | | 4 | |

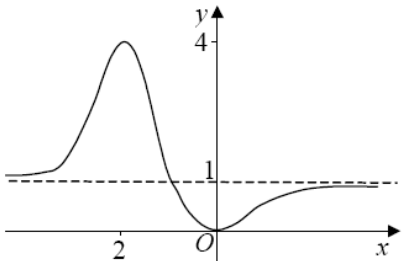
Pure 5 June 2003

| | | | | |
|--------------|--|----------|----------|---|
| 3 | $x^2 - 2yx + 2 - y (= 0)$ $\Delta = (-2y)^2 - 4(1)(2 - y)$ $4(y^2 + y - 2)$ $4(y + 2)(y - 1)$ For real x , $\Delta \geq 0 \Rightarrow y \geq 1$ or $y \leq -2$ \Rightarrow no real values of x for which $-2 < y < 1$ | M1 | 6 | Attempt to form quadratic in x with y involved Condone one sign error. Consider $b^2 - 4ac$ with y involved and no x 's. oe eg '4' can be missing if linking 0 Attempt to factorise or solve a quadratic in y only cao. Need $b^2 - 4ac$ linked to an inequality ag Only award if no previous errors |
| | | A1 | | |
| | | m1 | | |
| | | A1 m1 | | |
| Total | | | 6 | |

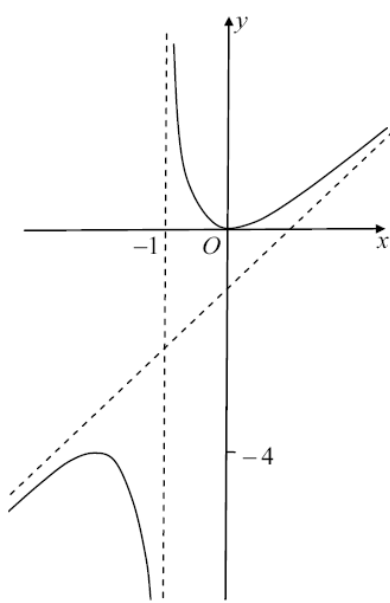
Pure 3 June 2004

| | | | | |
|--------------|--|----------|---|--|
| 2(a)(i) | (0,4) and $\left(-\frac{4}{3}, 0\right)$ | B1 | 2 | |
| | | B1 | | |
| (ii) | Asymptote at $x = \frac{1}{2}$ and at $y = -1\frac{1}{2}$ | B1 B1 | 2 | |
| (iii) | | M1 A1 | 2 | One branch roughly correct Good graph |
| | | (b) | $3x + 4 = 1 - 2x \Rightarrow 5x = -3$ $\Rightarrow x = -\frac{3}{5}$ | M1 A1 |
| (c) | Use of value from (b) $\Rightarrow x \leq -\frac{3}{5}$ Also $x > \frac{1}{2}$ | M1 A1 | | If algebraic method – must be sound eg simply multiplying up to give $3x + 4 \leq 1 - 2x \Rightarrow M0$ |
| | | B1 | 3 | |
| Total | | | 11 | |

Pure 5 January 2004

| | | | | |
|--------|--|----------|-----------|--|
| 5(a) | $y = 1$ | B1 | 1 | Must be the equation |
| (b)(i) | $(y-1)x^2 + 3yx + 3y \quad \{=0\}$ | M1 A1 | | Attempt to form quadratic in x Correct quadratic in x |
| | $\Delta = (3y)^2 - 4(y-1)(3y)$ | m1 | | Considers $b^2 - 4ac$ |
| | $-3y^2 + 12y$ | A1 | | |
| | $-3y(y-4)$ | m1 | | Attempt to factorise or solve |
| | For real x , $\Delta \geq 0 \Rightarrow 0 \leq y \leq 4$ | A1 | 6 | ag cso |
| (ii) | $y = 4 \Rightarrow 3x^2 + 12x + 12 = 0$ | M1 | | Substitute $y = 4$ to form a 'valid' quadratic in x . (PI) |
| | $\Rightarrow x = -2$, turning point $(-2, 4)$ | A1 | | If not using 'hence' then $(-2, 4)$ is B1 max. |
| | $\{y = 0 \Rightarrow -x^2 = 0 \Rightarrow x = 0\}$ | | | |
| | Turning point $(0,0)$ | B1 | 3 | |
| (c) |  | B3,2,1 | 3 | B1 for shape B1 for origin as only point where graph meets the axes B1 for correct behaviour at the 'end-points' |
| | Total | | 13 | |

Pure 5 June 2004

| | | | | |
|--------------|---|--------------------|----------|--|
| 5(a) | Asymptote $x = -1$ $y = x - 1 + \frac{1}{x+1}$ Asymptote $y = x - 1$ | B1 M1 A1 | 3 | Full attempt to divide out |
| (b) | Turning point (0,0) When $y = -4$, $x^2 + 4x + 4 = 0$ Turning point $(-2, -4)$ | B1 M1 A1 | 3 | Alternative Valid method to find $y'(x)$ and then puts $y'(x) = 0$ [M1] $x^2 + 2x = 0 \Rightarrow$ TPs (0,0) [A1] and $(-2, -4)$ [A1] |
| (c) |  | B1 B1 B1 | 3 | Single upper branch; shape and y not < 0 Single lower branch; shape and y not > -4 Dependent on previous two Bs. Asymptotic behaviour on both branches; through the origin |
| Total | | | 9 | |

Complex Numbers

Pure 3 June 2002

| | | | | |
|------|---|---------------------|---|---|
| 4(a) | $z^2 = 4 - 12 - 8\sqrt{3}i$ $= -8 - 8\sqrt{3}i$ $4z = -8 + 8\sqrt{3}i$ $\Rightarrow z^2 + 4z = -16$ oe | M1 A1 A1✓ | 3 | 3 terms or $-8 + ki$ Answer is real or imaginary part shown to be zero ft provided it is real number |
|------|---|---------------------|---|---|

Pure 3 January 2004

| | | | | |
|---------|--|-----------|---|----|
| 6(a)(i) | $-5 + 12i$ | M1 A1 | 2 | |
| (ii) | Squaring their answer to (i) or use of the binomial theorem: $-119 - 120i$ | M1 A1✓ | 2 | ft |

Roots of Quadratic Equations

Pure 3 January 2002

| | | | | |
|----------------|--|---|-----------|--|
| <p>9(a)(i)</p> | $\alpha + \beta = -4; \alpha\beta = 13$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $= -64 + 156 = 92$ | <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>4</p> | <p>or $(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ good attempt, correct in terms of $\alpha + \beta$ and $\alpha\beta$</p> |
| <p>(ii)</p> | $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^2}$ $= \frac{92}{169}$ | <p>M1</p> <p>A1✓</p> | <p>2</p> | <p>fit 'their' $\alpha^3 + \beta^3$</p> |
| <p>(b)</p> | <p>product of roots = $\frac{1}{\alpha\beta} = \frac{1}{13}$</p> $x^2 - \frac{92}{169}x + \frac{1}{13} = 0$ $169x^2 - 92x + 13 = 0$ | <p>B1</p> <p>M1</p> <p>A1</p> | <p>3</p> | <p>their values – any variable</p> |
| <p>(c)</p> | $(x + 2)^2 = -9$ $x + 2 = \pm 3i$ | <p>M1</p> <p>M1</p> | | <p>or use of formula dealing with $\sqrt{-36}$</p> |
| Total | | | 12 | |

Pure 3 June 2002

| | | | | |
|--------------|--|------------------------|-----------|--|
| 1(a)(i) | $\alpha + \beta = -4; \quad \alpha\beta = -3$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 16 + 6 = 22$ | B1 M1 A1 | | Likely to be earned in (ii) oe |
| (ii) | $\alpha^2\beta^2 + 2(\alpha + \beta) + \frac{4}{\alpha\beta}$ $9 - 8 - \frac{4}{3}$ $= -\frac{1}{3}$ | B1 M1 A1 | 6 | Substitution into similar form as above |
| (b) | <p>Sum of roots</p> $= \alpha^2 + \beta^2 + \frac{2}{\alpha} + \frac{2}{\beta}$ $= \alpha^2 + \beta^2 + \frac{2}{\alpha\beta}(\alpha + \beta)$ $= 22 + \frac{2}{-3} \times -4 = \frac{74}{3}$ <p>New equation</p> $y^2 - (\text{sum of new roots})y + \text{product} = 0$ $\Rightarrow y^2 - \frac{74}{3}y - \frac{1}{3} = 0$ $\Rightarrow 3y^2 - 74y - 1 = 0$ | M1 A1 M1 A1ft | 4 | (ft any variable fractional values) Must have = 0 |
| Total | | | 10 | |

Pure 3 January 2003

| | | | | |
|--------------|--|----------|-----------|--|
| 7(a) | $\alpha + \beta = -3; \quad \alpha\beta = -2$ | B1 | 1 | |
| (b)(i) | $\frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2 \beta^2}$ | M1 ml | | |
| | $= \frac{13}{4}$ | A1✓ | 3 | ft their (a) values |
| (ii) | $\alpha\beta - \frac{3}{\alpha} - \frac{3}{\beta} + \frac{9}{\alpha^2 \beta^2}$ | M1 | | Good attempt at multiplying out |
| | $= \alpha\beta - \frac{3(\alpha + \beta)}{\alpha\beta} + \frac{9}{\alpha^2 \beta^2}$ | ml | | In a form ready for substitution |
| | $= -\frac{17}{4}$ | A1✓ | 3 | ft their (a) values |
| (c) | Sum of roots | | | |
| | $= \alpha + \beta - 3\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)$ | M1 | | |
| | $= -3 - \frac{39}{4} = -\frac{51}{4}$ | A1 | | |
| | New equation | | | Condone single sign error or missing = 0 |
| | $y^2 - (\text{sum of new roots})y + \text{product} = 0$ | M1 | | $\Rightarrow y^2 + \frac{51}{4}y - \frac{17}{4} = 0$ |
| | $\Rightarrow 4y^2 + 51y - 17 = 0$ | A1 | 4 | Must have = 0 |
| Total | | | 11 | |

Pure 3 June 2003

| | | | | |
|--------------|--|---|----------------------------|---|
| 9(a)(i) | $\alpha + \beta = 3; \quad \alpha\beta = 1$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 9 - 2 = 7$ | B1 M1 A1 | 3 | Withhold if obviously incorrect in (ii) ag However, condone $(-3)^2$ |
| (ii) | $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= 18$ | M1 A1 A1 | 3 | Good attempt at any equivalent Correct formula |
| (b)(i) | $(\alpha^2 + \beta^2)^2 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4$ $\Rightarrow \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$ | B1 | 1 | ag Be generous here. |
| (ii) | $\alpha^4 + \beta^4 = 49 - 2$ $= 47$ | M1 A1 | 2 | Substitute candidate's $\alpha\beta$ |
| (c) | Sum of roots $= \alpha^3 + \beta^3 - (\alpha + \beta)$ $= 15$ Product $= (\alpha\beta)^3 + \alpha\beta - (\alpha^4 + \beta^4)$ $= 1 + 1 - 47 = -45$ New equation $y^2 - 15y - 45 = 0$ | M1 A1 M1 A1 B1 \checkmark | 2 5 | Condone one slip ft any variable, integer coefficients Must have = 0 |
| Total | | | 14 | |

Pure 3 January 2004

| | | | | |
|--------------|--|-----------------------------|-----------|--|
| 1(a)(i) | $\alpha + \beta = -2, \quad \alpha\beta = 3$ | B1 B1 | 2 | |
| (ii) | $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $\Rightarrow \alpha^3 + \beta^3 = 10$ | M1 A1 A1 | 3 | Or $(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ & $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ag |
| (iii) | $\frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \frac{10}{27}$ | M1 A1 | 2 | |
| (b) | New product of roots $= \frac{1}{(\alpha\beta)^3} = \frac{1}{27}$ $x^2 - [\text{cand's (a) (iii)}]x + [\text{cand's product}]$ $\Rightarrow 27x^2 - 10x + 1 = 0$ | B1 M1 A1 \checkmark | 3 | ft Must have integer coefficients and be an equation |
| Total | | | 10 | |

Pure 3 June 2004

| | | | | |
|--------------|---|----------------|----------|----------------------------------|
| 3(a)(i) | $\alpha + \beta = -(7 + p)$ $\alpha\beta = p$ | B1 B1 | 2 | |
| (b) | $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= (7 + p)^2 - 2p$ | M1 A1 | 2 | oe $p^2 + 12p + 49$ |
| (c)(i) | $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$ $= p^2 + 12p + 49 - 2p = p^2 + 10p + 49$ | M1 A1 | 2 | ag |
| (ii) | $(\alpha - \beta)^2 = 25$ $p^2 + 10p + 49 = 25 \Rightarrow p^2 + 10p + 24 = 0$ $p = -4, p = -6$ | B1 M1 A1 | 3 | May be using 5 etc instead of 25 |
| Total | | | 9 | |

Series

Pure 1 June 2003

| | | | | |
|--------------|---|----------|----------|--|
| 3(a)(i) | 25 502 500 | B1 | 1 | |
| (ii) | Attempt to find $S_{100} - S_{50}$ using $\sum r^3$ $= 23\,876\,875$ | M1 A1 | 2 | Formula in booklet. Condone $S_{100} - \begin{cases} S_{51} \\ S_{49} \end{cases}$ |
| (b) | $S_n = \frac{1}{2}n(2a + (n-1)d)$ formula attempted (condone one slip) | M1 | | Or $\frac{n(\text{first} + \text{last})}{2}$ attempted |
| | correct values substituted, candidate's 25 (51+100) | m1 | | Or $S_{100} - S_{50/51/49}$ using $\sum r = \frac{1}{2}n(n+1)$ |
| | $= 3775$ | A1 | 3 | Or candidate's $50 \times 101 - 25 \times 51$ |
| (c) | Use of (a)(ii) – 6325 (b) $= 0$ | M1 A1 | 2 | sc B3 for correct answer without working sc B2 for correct answer without working |
| Total | | | 8 | |

Pure 1 June 2004

| | | | | |
|--------------|--|----------------|----------|--|
| 6 (a) | Use of $\frac{n}{6}(n+1)(2n+1)$ $= 8\,555$ | M1 A1 | 2 | $\frac{29}{6} \times 30 \times 59$ |
| (b) (i) | common difference, $d = 4$ Use of $a + (r-1)d$ $u_r = 4r - 1$ | B1 M1 A1 | 3 | Condone $a + (n-1)d$ Condone $4n - 1$ |
| (ii) | Upper limit 200 and lower limit 1 on \sum $\sum_{r=1}^{200} 4r - 1$ | B1 B1✓ | 2 | Or equivalent ft their u_r (ignore limits) Two B marks are independent |
| Total | | | 7 | |

Calculus

Pure 1 June 2004

| | | | | |
|---------|---|----------------|---|---|
| (d) (i) | $y(1+h) = 1 + 2h + h^2 - 6 - 6h + 10$ Gradient = $\frac{y(1+h) - y(1)}{h}$ $= \frac{h^2 - 4h}{h} = h - 4$ | M1 m1 A1 | 3 | Subs $1+h$ and attempt to multiply out $y(1) = 5$ ag |
| (ii) | As $h \rightarrow 0$, gradient at $P = -4$ | B1 | 1 | Must use limit and not calculus rule |

Linear Laws

Pure 3 January 2002

| | | | | |
|--------------|---|--------------|-----------|--------------------------|
| 6(a) | $\ln E = \ln K + \alpha \ln B$ | B1 | 1 | |
| (b) | $\ln B$ 1.151 2.258 2.907 | | | 3.367 3.723 |
| | $\ln E$ 0 0.693 1.099 | B2 | | 1.386 1.609 |
| | plotting points – roughly correct | (-1ee) M1 | 3 | |
| (c) | straight line of reasonable fit | B1 | 1 | |
| (d)(i) | $B = 25.5 \Rightarrow \ln B = 3.2387$ | M1 | | |
| | From graph $\ln E \approx 1.31$ | M1 | | |
| | $\Rightarrow E = 3.7$ | A1 | 3 | Condone 3.6 to 3.8 |
| (ii) | gradient = $\alpha = \frac{\Delta \ln E}{\Delta \ln B}$ | M1 | | |
| | $= \frac{1.792}{2.865} \approx 0.63$ | A1 | | Condone 0.62 to 0.64 |
| | Intercept used/or 2 points | M1 | | full attempt to find k |
| | $k \approx 0.48 / 0.49$ | A1 | 4 | |
| Total | | | 12 | |

Pure 3 January 2003

| | | | | |
|--------------|---------------------------------------|----|----------|--|
| 5(a) | $\ln 1.43 = 0.358\dots$ | M1 | | |
| | From graph $\ln P = 2.4\dots$ | m1 | | Expected in range 2.43 to 2.45 |
| | Hence $P = 11.4/5/6$ | A1 | 3 | Follow through their values within range |
| (b)(i) | $\ln P = \ln k + \alpha \ln x$ | B1 | 1 | |
| (ii) | $\ln k$ is intercept on vertical axis | M1 | | $\ln k = 1.9$ (or use of formula) |
| | $k = 6.7$ (to 2 SF) | A1 | | |
| | Gradient of graph gives α | M1 | | M0 if further wrong calculation using exponentials |
| | $\alpha = 1.5$ (to 2 SF) | A1 | 4 | |
| Total | | | 8 | |

Pure 3 June 2003

| | | | | |
|--------------|---|----------------------|-----------|--|
| 7(a) | 2.52 $N = 10^{2.52}$ $= 331$ | B1 M1 A1 | 3 | Seen (even if log of this value taken) Accept 300 or 330 following correct logs |
| (b)(i) | $\log_{10} N = \log_{10} a + t \log_{10} b$ | B2 | 2 | B1 if ln used or $\log_{10} b^t$ not simplified |
| (ii) | $\log_{10} a$ is intercept on $\log_{10} N$ axis $a = 251$ Gradient is $\log_{10} b = \frac{0.12}{5}$ etc $b = 1.06$ | M1 A1 M1 A1 | 4 | $\log_{10} a = 2.4$ Must be 3sf or better Must be 3sf or better May score M1 for setting up 2 equations M1 for solving one or two equations A2, 1 for correct answers |
| (c) | Growth limited by test tube; some die etc | E1 | 1 | |
| Total | | | 10 | |

Pure 3 January 2004

| | | | | |
|--------------|---|-------------------|-----------|---|
| 5(a) | $\ln Q = \ln a + b \ln x$ | B1 | 1 | |
| (b)(i) | $\ln x$: -0.92 -0.69 -0.51 -0.36 -0.22 $\ln Q$: 0.54 1.11 1.56 1.94 2.28 Points plotted on graph provided | B1 B1 B1 | 3 | Most correct At most one error Reasonably accurately |
| (ii) | “Good” line of best fit drawn | B1 | 1 | |
| (c)(i) | $\ln Q = 1.29 - 1.30 \Rightarrow Q \approx 3.6 - 3.7$ | M1 A1 | 2 | |
| (ii) | Method for finding gradient: $b = 2.5$ Reading off y-intercept: $\ln a \approx 2.8$ $a = 16 - 17$ | M1 A1 M1 A1 | 4 | ± 0.1 Give M marks for simultaneous equations approach |
| Total | | | 11 | |

Pure 3 June 2004

| | | | | |
|--------------|--|----------------------|----------|---|
| 6(a) | $\ln 3 = 1.0986\dots$ $\ln y = 1.33$ $y = 3.8$ | M1 m1 A1 | 3 | Condone 1.30 to 1.35 Accept 3.7 to 3.9 |
| (b)(i) | $\ln y = \ln A + n \ln x$ | B1 | 1 | |
| (ii) | $\ln A = 0.80$ (intercept on $\ln y$ -axis) $A = 2.2$ $n =$ gradient of line $= 0.48$ | M1 A1 M1 A1 | 4 | Condone value rounding to this Accept value rounding to 0.47, 0.48 or 0.49 |
| Total | | | 8 | |

Numerical Methods

Pure 4 January 2002

| | | | | |
|--------------|--|----|----------|--|
| 2(a) | $p\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{8}\right) - 5\left(\frac{1}{4}\right) + 2$ $= 0.25 \Rightarrow \text{Remainder} = 0.25$ | M1 | | must attempt $p\left(-\frac{1}{2}\right)$ or long division to remainder. |
| | | A1 | 2 | |
| (b) | $p'(x) = 12x^2 - 10x$ $-0.5 - p(-0.5) / p'(-0.5)$ $= -0.5 - \frac{0.25}{8} = -0.531$ | B1 | | denominator 8 may imply B1 |
| | | M1 | | |
| | | A1 | 3 | |
| Total | | | 5 | |

Pure 4 January 2003

| | | | | |
|--------------|--|----|----------|---|
| 2(a) | $x \ln 2 = \ln 7$ $\Rightarrow x = 2.81$ | M1 | | May use \log_{10} 2.80735... Accept more than 3 SF |
| | | A1 | 2 | |
| (b) (i) | $f(x) = 2^x - 7 + x ;$ $f(2.0) = -1 ; f(2.4) = 0.678...$ $\Rightarrow \text{root lies in interval } (2.0, 2.4)$ | B1 | 1 | Or equivalent considering both sides but must contain a valid conclusion M0 if bisection method NOT used |
| (ii) | Considering $f(2.2)$ first $f(2.2) = -0.2052...$ $\Rightarrow \text{root lies in interval } (2.2, 2.4)$ $f(2.3) = 0.224...$ $\Rightarrow \text{root lies in interval } (2.2, 2.3)$ | M1 | | |
| | | A1 | 3 | |
| Total | | | 6 | |

Pure 4 June 2003

| | | | | |
|---------|--|----|---|--|
| 5(a)(i) | $\frac{dy}{dx} = 2 + 2 \cos 2x$ | M1 | | $k \cos 2x$ or $k \cos x$ correct derivative |
| | | A1 | 2 | |
| (ii) | $0.2 - \frac{y(0.2)}{y'(0.2)}$ $= 0.255 \text{ to 3 sig figs}$ | M1 | | Used, since formula in booklet Must be to 3sf |
| | | A1 | 2 | |

Pure 4 June 2004

| | | | | |
|--------|---|----------------|---|--|
| 4(a) | $p(3) = 27 - 54 + 36 - 11$ $= -2$ (is remainder) | M1 | | Must consider $p(3)$ or full long division to remainder |
| (b)(i) | $p(4) = 64 - 96 + 48 - 11 = 5$ [Change of sign] $\Rightarrow \alpha$ lies between 3 and 4 | A1 | 2 | |
| | | B1 | 1 | Both $p(3)$ and $p(4)$ must be correct and there must be some statement/conclusion |
| (ii) | $p(3.5)$ used first ($=0.375$) $p(3.25) = -1.046875$ \Rightarrow root lies between 3.25 and 3.5 | M1 m1 B1 | | \Rightarrow root lies between 3 and 3.5 |
| | | | 3 | |

Matrix Transformations

Pure 3 January 2002

| | | | | |
|--------------|--|----|----------|--|
| 2(a) | Shear invariant line $y = 0$ mapping $(0, 1)$ to $(1, 1)$ o.e | M1 | | |
| | | A1 | 2 | |
| (b) | $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ | M1 | | |
| | | A1 | 2 | |
| Total | | | 4 | |

Pure 3 January 2003

| | | | | |
|--------------|---|----|----------|--|
| 2(a) | $M^2 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ $M^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | M1 | | Attempt to multiply matrices correctly |
| | | A1 | | Correct |
| | | A1 | 3 | |
| (b) | Rotation (about origin) through $\frac{2\pi}{3}$ (anticlockwise) | M1 | | |
| | | A1 | 2 | Or equivalent clockwise turn |
| Total | | | 5 | |