AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

AS
MATHEMATICS
Unit Further Pure 1

Wednesday 15 June 2016   Morning   Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The quadratic equation $x^2 - 6x + 14 = 0$ has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

(b) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

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2 A curve C has equation $y = (2 - x)(1 + x) + 3$.

(a) A line passes through the point $(2, 3)$ and the point on C with x-coordinate $2 + h$.

Find the gradient of the line, giving your answer in its simplest form.

[3 marks]

(b) Show how your answer to part **(a)** can be used to find the gradient of the curve C at the point $(2, 3)$. State the value of this gradient.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 2

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3 The variables y and x are related by an equation of the form

$$y = a(b^x)$$

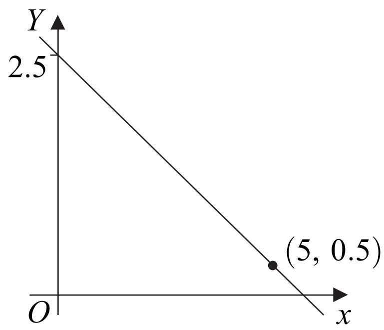
where a and b are positive constants.

Let $Y = \log_{10} y$.

(a) Show that there is a linear relationship between Y and x.

[2 marks]

(b) The graph of Y against x, shown below, passes through the points $(0, 2.5)$ and $(5, 0.5)$.



(i) Find the gradient of the line.

[1 mark]

(ii) Find the value of a and the value of b, giving each answer to three significant figures.

[4 marks]

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4 (a) Given that $\sin\frac{\pi}{3} = \cos\frac{\pi}{k}$, state the value of the integer k.

[1 mark]

(b) Hence, or otherwise, find the general solution of the equation

$$\cos\left(2x - \frac{5\pi}{6}\right) = \sin\frac{\pi}{3}$$

giving your answer, in its simplest form, in terms of π.

[4 marks]

(c) **Hence**, given that $\cos\left(2x - \frac{5\pi}{6}\right) = \sin\frac{\pi}{3}$, show that there is only one finite value for $\tan x$ and state its exact value.

[2 marks]

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5 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $\sum_{r=1}^{n} (6r-3)^2 = 3n(4n^2-1)$.

[5 marks]

(b) Hence express $\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n} (6r-3)^2$ as a product of four linear factors in terms of n.

[4 marks]

QUESTION PART REFERENCE

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6 A parabola with equation $y^2 = 4ax$, where a is a constant, is translated by the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ to give the curve C. The curve C passes through the point $(4, 7)$.

(a) Show that $a = 2$.

[3 marks]

(b) Find the values of k for which the line $ky = x$ does **not** meet the curve C.

[6 marks]

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7 (a) Solve the equation $x^2 + 4x + 20 = 0$, giving your answers in the form $c + d\mathrm{i}$, where c and d are integers.

[3 marks]

(b) The roots of the quadratic equation

$$z^2 + (4 + \mathrm{i} + q\mathrm{i})z + 20 = 0$$

are w and w^*.

(i) In the case where q is real, explain why q must be -1 .

[2 marks]

(ii) In the case where $w = p + 2\mathrm{i}$, where p is real, find the possible values of q.

[5 marks]

QUESTION PART REFERENCE

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8 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

(a) (i) Find the matrix $\mathbf{A}^2$.

[1 mark]

(ii) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}^2$.

[1 mark]

(b) Given that the matrix $\mathbf{B}$ represents a reflection in the line $x + \sqrt{3}y = 0$, find the matrix $\mathbf{B}$, giving the exact values of any trigonometric expressions.

[2 marks]

(c) Hence find the coordinates of the point P which is mapped onto $(0, -4)$ under the transformation represented by $\mathbf{A}^2$ followed by a reflection in the line $x + \sqrt{3}y = 0$.

[6 marks]

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9 A curve C has equation $y = \dfrac{x-1}{(x-2)(2x-1)}$.

The line L has equation $y = \frac{1}{2}(x-1)$.

(a) Write down the equations of the asymptotes of C.

[2 marks]

(b) By forming and solving a suitable cubic equation, find the x-coordinates of the points of intersection of L and C.

[3 marks]

(c) Given that C has no stationary points, sketch C and L on the same axes.

[3 marks]

(d) Hence solve the inequality $\dfrac{x-1}{(x-2)(2x-1)} \geqslant \frac{1}{2}(x-1)$.

[3 marks]

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END OF QUESTIONS