Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics — MFP1

Unit Further Pure 1

Friday 5 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The quadratic equation $2x^2 + 6x + 7 = 0$ has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

(b) Find a quadratic equation, with integer coefficients, which has roots $\alpha^2 - 1$ and $\beta^2 - 1$.

[5 marks]

(c) **Hence** find the values of α^2 and β^2.

[2 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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2 (a) Explain why $\int_0^4 \frac{x-4}{x^{1.5}}\,\mathrm{d}x$ is an improper integral.

[1 mark]

(b) Either find the value of the integral $\int_0^4 \frac{x-4}{x^{1.5}}\,\mathrm{d}x$ or explain why it does not have a finite value.

[4 marks]

QUESTION PART REFERENCE

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3 (a) Show that $(2+\mathrm{i})^3$ can be expressed in the form $2+b\mathrm{i}$, where b is an integer.
[3 marks]

(b) It is given that $2+\mathrm{i}$ is a root of the equation

$$z^3+pz+q=0$$

where p and q are real numbers.

(i) Show that $p=-11$ and find the value of q.
[4 marks]

(ii) Given that $2-\mathrm{i}$ is also a root of $z^3+pz+q=0$, find a quadratic factor of z^3+pz+q with real coefficients.
[2 marks]

(iii) Find the real root of the equation $z^3+pz+q=0$.
[2 marks]

QUESTION PART REFERENCE

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Answer space for question 3

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4 (a) Find the general solution, in degrees, of the equation

$$2\sin(3x + 45^\circ) = 1$$

[5 marks]

(b) Use your general solution to find the solution of $2\sin(3x + 45^\circ) = 1$ that is closest to 200°.

[1 mark]

QUESTION PART REFERENCE

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5 (a) The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} -2 & c \\ d & 3 \end{bmatrix}$.

Given that the image of the point $(5,\ 2)$ under the transformation represented by $\mathbf{A}$ is $(-2,\ 1)$, find the value of c and the value of d.

[4 marks]

(b) The matrix $\mathbf{B}$ is defined by $\mathbf{B} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$.

(i) Show that $\mathbf{B}^4 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix.

[2 marks]

(ii) Describe the transformation represented by the matrix $\mathbf{B}$ as a combination of two geometrical transformations.

[5 marks]

(iii) Find the matrix $\mathbf{B}^{17}$.

[2 marks]

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6 A curve C_1 has equation

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

(a) Sketch the curve C_1, stating the values of its intercepts with the coordinate axes.

[2 marks]

(b) The curve C_1 is translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, where $k < 0$, to give a curve C_2.

Given that C_2 passes through the origin $(0, 0)$, find the equations of the asymptotes of C_2.

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE	Answer space for question 6

7 (a) The equation $2x^3 + 5x^2 + 3x - 132\,000 = 0$ has exactly one real root α.

(i) Show that α lies in the interval $39 < \alpha < 40$.

[2 marks]

(ii) Taking $x_1 = 40$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to two decimal places.

[3 marks]

(b) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} 2r(3r+2) = n(n+p)(2n+q)$$

where p and q are integers.

[5 marks]

(c) (i) Express $\log_8 4^r$ in the form λr, where λ is a rational number.

[1 mark]

(ii) By first finding a suitable cubic inequality for k, find the greatest value of k for which $\sum_{r=k+1}^{60} (3r+2) \log_8 4^r$ is greater than $106\,060$.

[4 marks]

QUESTION PART REFERENCE

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Turn over ▶

8 A curve C has equation

$$y = \frac{x(x-3)}{x^2+3}$$

(a) State the equation of the asymptote of C.

[1 mark]

(b) The line $y = k$ intersects the curve C. Show that $4k^2 - 4k - 3 \leqslant 0$.

[5 marks]

(c) **Hence** find the coordinates of the stationary points of the curve C.

(No credit will be given for solutions based on differentiation.)

[5 marks]

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END OF QUESTIONS

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