A- LEVEL

Mathematics

Further Pure 1 – MFP1
Mark scheme

6360
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Version 1: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\alpha+\beta=-3;\quad \alpha\beta=\frac{7}{2}\quad(=3.5)$	B1; B1	2	If LHS is missing look for later evidence before awarding the B1s.
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\quad(=9-7)$	M1		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ seen or used. PI
	$(S=)\ \alpha^2+\beta^2-2=2-2=0$	A1		Ft on wrong sign for $\alpha+\beta$
	$(P=)\ \alpha^2\beta^2-(\alpha^2+\beta^2)+1=\frac{45}{4}\ (=11.25)$	A1		Ft on wrong sign for $\alpha+\beta$
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.
	Quadratic is $4x^2+45=0$	A1	5	CSO. ACF of the equation, but must have integer coefficients
(c)	(Vals of α^2-1 and β^2-1 are) $\pm \mathrm{i}\sqrt{\frac{45}{4}}$	M1		PI Ft on c's quadratic provided roots are not real.
	Values of α^2 and β^2 are $1\pm \mathrm{i}\sqrt{\frac{45}{4}}$	A1	2	OE Must see evidence of answer to **(b)** having been used.
	Total		9	
(b)	Altn for first M1: $2(\alpha^2+\beta^2)=-6(\alpha+\beta)-7-7$			
(b)	Altn: A subst. of $y=x^2-1$ attempted in $2x^2+6x+7=0$ (**M1**); $2(y+1)+6x+7=0$ (**A1**); $2y+9=-6x,\quad (2y+9)^2=36x^2=36(y+1)$ (**m1** full substitution); $4y^2+36y+81=36y+36$ (**A1** correct eqn with no brackets or fractions) $4y^2+45=0$ (**A1**CSO as in main scheme)			

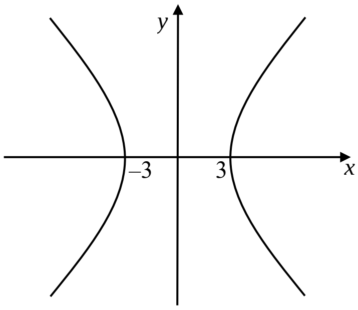
Q2	Solution	Mark	Total	Comment
(a)	Integrand is not defined at $x=0$	E1	1	OE
(b)	$\int \frac{x-4}{x^{1.5}}\ (\mathrm{d}x)=\int (x^{-0.5}-4x^{-1.5})\ (\mathrm{d}x)$	M1		Split into two terms with at least one term correct and in the form ax^n. PI by correct integration of $\int \frac{x-4}{x^{1.5}}\mathrm{d}x$ condoning one slip.
	$=\frac{x^{0.5}}{0.5}-\frac{4x^{-0.5}}{-0.5}\quad(+c)$	A1		ACF
	$\int_0^4 \frac{x-4}{x^{1.5}}\mathrm{d}x$ does NOT have a finite value since	B1		OE Dep. on at least one term after integration being of the form x^k, where k is negative, OE.
	as $x\to 0^{(+)},\quad x^{-0.5}\to\infty$	E1	4	OE explanation. Dep. on no accuracy errors seen.
	Total		5	
(b)	Accept OE wording for '$\rightarrow$' eg 'tends to' 'approaches' 'goes to' etc but NOT '='			

Q3	Solution	Mark	Total	Comment
(a)	$(2+\mathrm{i})^3 = 2^3 + 3(2)^2\mathrm{i} + 3(2)\mathrm{i}^2 + \mathrm{i}^3$	M1		OE Three of the 4 terms correct.
	$= 2^3 + 3(2)^2\mathrm{i} + 3(2)(-1) + (-1)\mathrm{i}$	M1		$\mathrm{i}^2 = -1$ used at least once
	$= 2 + 11\mathrm{i}$	A1	3	NMS 0/3
(b)(i)	$(2+\mathrm{i})^3 + p(2+\mathrm{i}) + q = 0$	M1		May see $2 + b\mathrm{i}$ OE in place of $(2+\mathrm{i})^3$
	Re: $2 + 2p + q = 0$; Im: $b + p = 0$	m1		Equating Re parts **and** equating Im parts attempted. OE
	$2 + 2p + q = 0$; $11 + p = 0$	A1F		Two correct ft (on c's b value in **(a)**) equations
	$p = -11$, $q = 20$	A1	4	CSO both required; AG for p.
(b)(ii)	$[z-(2+\mathrm{i})][z-(2-\mathrm{i})]$	B1		Either $[z-(2+\mathrm{i})][z-(2-\mathrm{i})]$ OE or $(2+\mathrm{i})(2-\mathrm{i}) = 5$ seen or used at any stage in (b)(ii) or (b)(iii).
	(Quadratic factor) $z^2 - 4z + 5$	B1	2	$z^2 - 4z + 5$, terms in any order
(b)(iii)	$z^3 - 11z + 20 = (z^2 - 4z + 5)(z + 4)$	M1		OE method to find factor (z+4) or root −4 Examples: Showing f(−4)=0; Using 2+i + 2−i + $\alpha = 0$
	(Real root is) −4	A1	2	Eg $z^3 - 11z + 20 = 0$, (Real root) −4 **2/2**
	Total		**11**	
(b)(ii)(iii)	May see these answered holistically eg by starting with $z^3 - 11z + 20 = (z^2 - 4z + 5)(z + 4)$ **(M1)(B1)** followed by the two correct answers (Quadratic factor) $z^2 - 4z + 5$, **(B1)** (real root) −4 **(A1)** order of answers can be reversed.			

Q4	Solution	Mark	Total	Comment
(a)	$\sin(3x + 45^\circ) = \sin 30^\circ$	B1		OE value in degrees for $\sin^{-1}(1/2)$ $(=\alpha)$ used PI by later work
	$3x + 45^\circ = 360n^\circ + 30^\circ$, $3x + 45^\circ = 360n^\circ + 180^\circ - 30^\circ$	M1		OE At least one of $3x + 45 = 360n + \alpha$ $3x + 45 = 360n + 180 - \alpha$ ft c's $\sin^{-1}(1/2)$ Condone $2n\pi$ for $360n$
	$x = \dfrac{360n^\circ + 30^\circ - 45^\circ}{3}$ $x = \dfrac{360n^\circ + 180^\circ - 30^\circ - 45^\circ}{3}$ {*}	m1		OE At least one correct rearrangement to $x = \ldots\ldots$ of $3x + 45 = 360n + \alpha$, $3x + 45 = 360n + 180 - \alpha$ ft c's $\sin^{-1}(1/2)$ Condone $2n\pi$ for $360n$
	$x = 120n^\circ - 5^\circ$, $x = 120n^\circ + 35^\circ$	A2,1,0	5	OE full set of correct solutions in degrees written with like terms combined and no fractions. (A1 if correct but unsimplified) (A0 if rads present in answer)
(b)	$n = 2$ in $x = 120n^\circ - 5^\circ$ gives 235°, the solution closest to 200°	B1	1	235 but only award this mark if at least 4 of the previous 5 marks have been scored
	Total		**6**	
	Condone missing degree symbols			
(a)	Lots of different forms of full sets of solutions can score full marks. Eg $3x + 45 = 180n + (-1)^n 30$ **(B1M1),** $x = 60n + (-1)^n 10 - 15$ **(m1A2)**			
(a)	Example, a cand. stops at {*} scores B1M1m1A1. A cand. who simplifies {*} incorrectly also scores 4/5			

Q5	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix}-2 & c\\ d & 3\end{bmatrix}\begin{bmatrix}5\\2\end{bmatrix}=\begin{bmatrix}-2\\1\end{bmatrix}$	M1		PI, allowing for recovery, by at least one correct element in evaluation of LHS or by at least one correct linear equation
	$\begin{bmatrix}-10+2c\\ 5d+6\end{bmatrix}=\begin{bmatrix}-2\\1\end{bmatrix}$;			
	$-10+2c=-2,\quad 5d+6=1$	M1		At least one correct equation
	$c=4$	A1		$c=4$
	$d=-1$	A1	4	$d=-1$
(b)(i)	$\mathbf{B}^2=\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$	B1		
	$\mathbf{B}^4=\begin{bmatrix}-16 & 0\\ 0 & -16\end{bmatrix}$			
	$=-16\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=-16\,\mathbf{I}$	B1	2	Accept either form or ‘ $=k\mathbf{I},\ k=-16$’ after seeing $\begin{bmatrix}-16 & 0\\ 0 & -16\end{bmatrix}$.
(b)(ii)	$\mathbf{B}=\begin{bmatrix}\sqrt{2} & \sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}=2\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$	M1		Sight of $2\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$ OE in trig form PI by award of at least B1B1 below
	$=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}\cos 315 & -\sin 315\\ \sin 315 & \cos 315\end{bmatrix}$	A1		OE eg $\begin{bmatrix}\cos(-45) & -\sin(-45)\\ \sin(-45) & \cos(-45)\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ PI by award of B1B2 below
	(ie combination of an) enlargement and (a) rotation	B1		‘Enlargement’ and ‘rotation’ OE with no extra transformation
	Enlargement with scale factor 2 and rotation through 315º (about *O*)	B2,1,0	5	OE eg Enlargement sf 2, clockwise rotation 45º If not B2 then B1 for ‘enlargement sf ±2 and angle of rotation ± an odd multiple of 45º.’
	Altn for M1A1 in (b)(ii)			
	$\begin{bmatrix}\sqrt{2} & \sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}\begin{bmatrix}0&1&0&1\\0&0&1&1\end{bmatrix}=\begin{bmatrix}0 & \sqrt{2} & \sqrt{2} & 2\sqrt{2}\\ 0 & -\sqrt{2} & \sqrt{2} & 0\end{bmatrix}$	(M1)		Attempting to find the image of vertices of a square under **B**, with at least two non-origin images correct. (Same PI as above)
		(A1)		Correct image of square under **B** seen or used. (Same PI as above)
(b)(iii)	$\mathbf{B}^{17}=[k^2]^2\,\mathbf{I}\,\mathbf{B}$	M1		An appreciation that $\mathbf{B}^8=k^2\mathbf{I}$ OE eg $\mathbf{B}^{17}=(\text{c's sf})^{17}\begin{bmatrix}\cos(17\alpha) & -\sin(17\alpha)\\ \sin(17\alpha) & \cos(17\alpha)\end{bmatrix}$, where $\alpha=$ c’s angle of rotation
	$=65536\begin{bmatrix}\sqrt{2} & \sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}$	A1	2	ACF, no trig., eg $2^{16}\begin{bmatrix}\sqrt{2} & \sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}$
	Total		**13**	

(b)(iii)	Example: $\mathbf{B}^{17}$ represents ‘enlargement sf 2^{17} and rotation through angle 17×315º ’ OE scores **M1**

Q6	Solution	Mark	Total	Comment
(a)		B1		hyperbola with the two branches covering the correct quadrants and no zero gradients
		B1	2	Only intercepts are on x-axis at 3 and -3. Condone correct coordinates in place of values of intercepts
(b)	$k=-3$	B1F		Seen or used. Ft on minus c's intercept with +'ve x-axis
	Asymptotes of C_1 are $\frac{x}{3}=\pm\frac{y}{4}$ so asymptotes of C_2 are $\frac{x+3}{3}=\pm\frac{y}{4}$	M1		Either $\frac{x-k}{3}=\pm\frac{y}{4}$ or $\frac{x+k}{3}=\pm\frac{y}{4}$ OE If not in terms of k, ft c's k value.
		A1	3	CSO $\frac{x+3}{3}=\pm\frac{y}{4}$ OE
	Total		**5**	

Q7	Solution	Mark	Total	Comment
(a)(i)	$f(x)=2x^3+5x^2+3x-132000$ $f(39)=-5640\ (<0)$; $f(40)=4120\ (>0)$;	M1		f(39) and f(40) both considered.
	Since sign change (and f continuous), $39<\alpha<40$	A1	2	All values and working correct plus relevant concluding statement involving 39 and 40.
(a)(ii)	$f'(x)=6x^2+10x+3$	B1		PI by eg $f'(40)=10003$
	$(x_2=)\ 40-\dfrac{f(40)}{f'(40)}$.	M1		Seen or used to indicate NR applied
	$=39.59$ (to 2 dp)	A1	3	Must be 39.59 Answer only, NMS scores **0**/3
(b)	$\sum_{r=1}^{n}2r(3r+2)=\sum_{r=1}^{n}(6r^2+4r)$ $=6\sum_{r=1}^{n}r^2+4\sum_{r=1}^{n}r$	M1		$\sum_{r=1}^{n}(\alpha r^2+\beta r)=\alpha\sum_{r=1}^{n}r^2+\beta\sum_{r=1}^{n}r$ PI by the next line.
	$=\left\{6\frac{n}{6}(n+1)(2n+1)+4\frac{n}{2}(n+1)\right\}$	m1 A1		OE Either term inside { } correct OE Both terms inside { } correct
	$=n(n+1)[2n+1+2]$	m1		$\{\ \}=n(n+1)[......]$ Taking out factor $n(n+1)$
	$=n(n+1)(2n+3)$	A1	5	CSO form of AG . $n(n+1)(2n+3)$ convincingly obtained Answer only, NMS scores **0**/5
(c)(i)	$(\log_8 4^r=)\ \frac{2}{3}r$	B1	1	$\frac{2}{3}r$. (Condone $\lambda=\frac{2}{3}$)
(c)(ii)	$g(r)=(3r+2)\log_8 4^r=\frac{1}{3}\times 2r(3r+2)$ $\sum_{r=k+1}^{60}g(r)=\sum_{r=1}^{60}g(r)-\sum_{r=1}^{k}g(r)$	M1		$\sum_{r=k+1}^{60}......=\sum_{r=1}^{60}.....-\sum_{r=1}^{k}.....$ seen or attempted.
	$\sum_{r=1}^{60}g(r)=\frac{60}{3}\times 61\times 123\ (=150060)$	B1F		OE Ft on c's values for λ, p and q in $30\lambda(60+p)(120+q)$
	Need greatest integer k such that $150060-\frac{k}{3}(k+1)[2k+3]>106060$ $\frac{k}{3}(k+1)[2k+3]<44000$ $2k^3+5k^2+3k-132000<0$	A1		A correct 'cubic' inequality for k obtained correctly
	(Required greatest value of) k is 39	A1	4	CSO. (NMS k=39 scores 0/4)
	Total		**15**	
(a)	Condone 'root', 'solution', 'x', 'it' in place of α.			

Q8	Solution	Mark	Total	Comment
(a)	$y=1$	B1	1	OE eg $y-1=0$. If more than one asymptote then **B0**
(b)	$k=\dfrac{x(x-3)}{x^2+3}$	M1		Elimination of y
	$k(x^2+3)=x(x-3)$ $(k-1)x^2+3x+3k=0$ **(*)**	A1		A correct quadratic equation in the form $Ax^2+Bx+C=0$, PI by later work
	$y=k$ intersects C so roots of (*) are real $b^2-4ac=3^2-4(k-1)(3k)$	M1		b^2-4ac in terms of k; ft on c's quadratic provided a and c are both in terms of k
	$3^2-4(k-1)(3k)\geq 0$	A1		A correct inequality where k is the only unknown.
	$9-12k^2+12k\geq 0,\ 12k^2-12k-9\leq 0$ ie $4k^2-4k-3\leq 0$	A1	5	CSO AG Be convinced
(c)	$(2k+1)(2k-3)\quad(\leq 0)$	M1		Method to find critical values from printed quadratic in **(b)**. PI by correct critical values stated
	Critical values are -0.5 and 1.5	A1		
	Sub $k=-0.5$ in (*) gives $x^2-2x+1=0$ Sub $k=1.5$ in (*) gives $x^2+6x+9=0$	m1		Subst of either -0.5 or 1.5 into quadratic eq to reach a quadratic in x with equal roots
	So $(1, -0.5)$ is a stationary point So $(-3, 1.5)$ is a stationary point	A1 A1	5	Correct coordinates Correct coordinates NMS scores 0/5
	Total		**11**	

(b)	For final A1CSO must see intermediate step between $9-12k^2+12k\geq 0$ and printed answer eg either $12k^2-12k-9\leq 0$ (as in soln above) or $3-4k^2+4k\geq 0$.
(b)	SC for $(k-1)x^2-3x+3k=0$, ie sign of coefficient of x incorrect, a max of M1A0M1A1A0.