Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MFP1

Unit Further Pure 1

Tuesday 10 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve passes through the point $(9, 6)$ and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{2 + \sqrt{x}}$$

Use a step-by-step method with a step length of 0.25 to estimate the value of y at $x = 9.5$. Give your answer to four decimal places.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 The quadratic equation

$$2x^2 + 8x + 1 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

(b) (i) Find the value of $\alpha^2 + \beta^2$.

[2 marks]

(ii) Hence, or otherwise, show that $\alpha^4 + \beta^4 = \frac{449}{2}$.

[2 marks]

(c) Find a quadratic equation, with integer coefficients, which has roots

$$2\alpha^4 + \frac{1}{\beta^2} \text{ and } 2\beta^4 + \frac{1}{\alpha^2}$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

3 Use the formulae for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to find the value of

$$\sum_{r=3}^{60} r^2(r-6)$$

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 Find the complex number z such that

$$5\mathrm{i}z + 3z^* + 16 = 8\mathrm{i}$$

Give your answer in the form $a + b\mathrm{i}$, where a and b are real.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

5 A curve C has equation $y = x(x + 3)$.

(a) Find the gradient of the line passing through the point $(-5, 10)$ and the point on C with x-coordinate $-5 + h$. Give your answer in its simplest form.

[3 marks]

(b) Show how the answer to part **(a)** can be used to find the gradient of the curve C at the point $(-5, 10)$. State the value of this gradient.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

6 A curve C has equation $y = \dfrac{1}{x(x+2)}$.

(a) Write down the equations of all the asymptotes of C.

[2 marks]

(b) The curve C has exactly one stationary point. The x-coordinate of the stationary point is -1.

(i) Find the y-coordinate of the stationary point.

[1 mark]

(ii) Sketch the curve C.

[2 marks]

(c) Solve the inequality

$$\frac{1}{x(x+2)} \leqslant \frac{1}{8}$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

7 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a reflection in the line $y = -x$;

[1 mark]

(ii) a stretch parallel to the y-axis of scale factor 7.

[1 mark]

(b) Hence find the matrix corresponding to the combined transformation of a reflection in the line $y = -x$ followed by a stretch parallel to the y-axis of scale factor 7.

[2 marks]

(c) The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} -3 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix}$.

(i) Show that $\mathbf{A}^2 = k\mathbf{I}$, where k is a constant and $\mathbf{I}$ is the 2×2 identity matrix.

[1 mark]

(ii) Show that the matrix $\mathbf{A}$ corresponds to a combination of an enlargement and a reflection. State the scale factor of the enlargement and state the equation of the line of reflection in the form $y = (\tan\theta)x$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

8 (a) Find the general solution of the equation

$$\cos\left(\frac{5}{4}x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

giving your answer for x in terms of π.

[5 marks]

(b) Use your general solution to find the **sum** of all the solutions of the equation $\cos\left(\frac{5}{4}x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$ that lie in the interval $0 \leqslant x \leqslant 20\pi$. Give your answer in the form $k\pi$, stating the exact value of k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

9 An ellipse E has equation

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

(a) Sketch the ellipse E, showing the values of the intercepts on the coordinate axes.

[2 marks]

(b) Given that the line with equation $y=x+k$ intersects the ellipse E at two distinct points, show that $-5<k<5$.

[5 marks]

(c) The ellipse E is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ to form another ellipse whose equation is $9x^2+16y^2+18x-64y=c$. Find the values of the constants a, b and c.

[5 marks]

(d) **Hence** find an equation for each of the two tangents to the ellipse $9x^2+16y^2+18x-64y=c$ that are parallel to the line $y=x$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 9