A-LEVEL
MATHEMATICS
Further Pure 1 – MFP1
Mark scheme

6360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

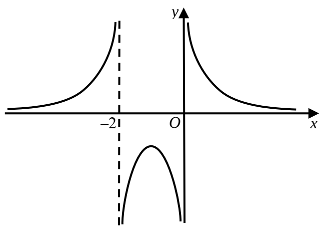
Q	Solution	Mark	Total	Comment
1	$h\,y'(9) = 0.25\times\dfrac{1}{2+\sqrt{9}}$ (=0.05)	M1		Attempt to find $h\,y'(9)$.
	$\{y(9.25)\} \approx 6 + 0.05 = 6.05$	A1		6.05 OE
	$\{y(9.5)\} \approx y(9.25) + 0.25\times y'(9.25)$ $\approx 6.05 + 0.25\times\dfrac{1}{2+\sqrt{9.25}}$	m1		Attempt to find $y(9.25)+0.25\times y'(9.25)$, must see evidence of numerical expression if correct ft [0.049(5..)+c's $y(9.25)$] value is not obtained.
	$\approx 6.05 + 0.25 \times 0.1983(5\ldots)$			
	$\approx 6.05 + 0.0495(8\ldots..)$	A1F		PI; ft on c's value for $y(9.25)$; 4dp value (rounded or truncated) or better.
	$y(9.5) = 6.0996$ (to 4 d.p.)	A1	5	$y(9.5) = 6.0996$
	Total		**5**	
	In this Q1, misreads lose all those A marks that are affected.			

Q	Solution	Mark	Total	Comment
2(a)	$\alpha+\beta=-4;\quad \alpha\beta=\dfrac{1}{2}$	B1; B1	2	Answers −4 & ½ with LHS missing, look for later evidence before awarding B1B1
(b)(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1		PI
	$= 16 - 1 = 15$	A1	2	CSO
(b)(ii)	$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$	M1		OE identity enabling direct substitution.
	$= 225 - 2\times\dfrac{1}{4} = 225-\dfrac{1}{2} = \dfrac{449}{2}$	A1	2	CSO AG Must see evaluations (eg as indicated by either of these two alternatives) before the printed answer.
(c)	$S = 2(\alpha^4+\beta^4)+\dfrac{\alpha^2+\beta^2}{\alpha^2\beta^2}$	M1		OE identity enabling direct substitution, seen or used.
	$P = 4\alpha^4\beta^4+2(\alpha^2+\beta^2)+\dfrac{1}{\alpha^2\beta^2}$	M1		OE identity enabling direct substitution, seen or used.
	$S = 509,\quad P=\dfrac{137}{4}$ (= 34.25)	A1F		Both values correct; ft only on $\alpha+\beta=4$
	Quadratic is $x^2-509x+34.25\,(=0)$	M1		x^2-Sx+P ft c's vals for S and P. M0 if either $S=\alpha+\beta$ or $P=\alpha\beta$ values
	$4x^2-2036x+137=0$	A1F	5	ACF of the equation, but must have integer coefficients; ft only on $\alpha+\beta=4$
	Total		**11**	
Alt (b)(ii)	$\alpha^4+\beta^4=(\alpha+\beta)^4-4\alpha\beta(\alpha^2+\beta^2)-6\alpha^2\beta^2$ (M1) $=256-4\times\dfrac{15}{2}-6\times\dfrac{1}{4}=256-30-\dfrac{3}{2}=\dfrac{449}{2}$ (A1) AG Cand whose only error is $\alpha+\beta=4$ in (**a**) can score B0B1; M1A0; M1A0; 5			

Q	Solution	Mark	Total	Comment
3	$\sum_{r=3}^{60} r^2(r-6) = \sum_{r=3}^{60} r^3 - 6\sum_{r=3}^{60} r^2$	M1		$\sum r^2(r-6) = \sum r^3 - 6\sum r^2$ seen or used
	$= \sum_{r=1}^{60} r^3 - 6\sum_{r=1}^{60} r^2 - \left[\sum_{r=1}^{2} r^3 - 6\sum_{r=1}^{2} r^2\right]$			
	$= \sum_{r=1}^{60} r^3 - 6\sum_{r=1}^{60} r^2 - [9-30]$	B1		B1 for $\left[\sum_{r=1}^{2} r^3 - 6\sum_{r=1}^{2} r^2\right] = 9-30$ OE PI
	$= \frac{1}{4}(60)^2(61)^2 - 6\frac{1}{6}(60)(61)(2\times60+1)+21$	M1		Substitution of n=60 into either (i) the correct formula $\sum_{r=1}^{n} r^3$ or (ii) the correct formula for $\sum_{r=1}^{n} r^2$ or (iii) the c's rearrangement of $\frac{1}{4}n^2(n+1)^2 - 6\frac{n}{6}(n+1)(2n+1)$
	$= 3348900 - 442860 + 21 = 2906061$	A1	4	2906061 NMS Answer only of 2906061 scores 0/4
	Total		**4**	
	Cand who works with Q as $\sum_{r=1}^{60} r^2(r-6)$ can score max of M1B0M1A0 Condone notation $\sum_{1}^{60} r^3$ for $\sum_{r=1}^{60} r^3$ etc **SC** : Let s=r–2; $\sum_{r=3}^{60} r^2(r-6) = \sum_{s=1}^{58}(s+2)^2(s-4) = \sum_{s=1}^{58} s^3 - 12\sum_{s=1}^{58} s - 16\sum_{s=1}^{58} 1$ (**M1** relevant split following expn of $(s+2)^2(s-4)$ into the form $as^3 + (bs^2+)\, cs + d$, ft wrong coeffs provided at least 3 non-zero coefficients.) $= \frac{1}{4}(58)^2(59)^2 - 12\frac{1}{2}(58)(59) - 16(58)$ (**M1** Substitution of n=58 into correct formula for either $\sum_{s=1}^{n} s^3$ or $\sum_{s=1}^{n} s$) (**B1** for $16\sum_{s=1}^{58} 1 = 16(58)$ (=928)) $= 2927521 - 20532 - 928 = 2906061$ (**A1**)			

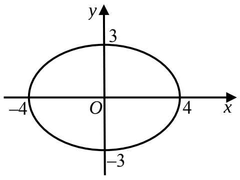
Q	Solution	Mark	Total	Comment
4	5i(a+bi) + 3(a–bi) + 16 = 8i	M1		Use of $z^* = a - b\mathrm{i}$ for $z = a+b\mathrm{i}$ OE
	5ai –5b + 3(a–bi) + 16 = 8i	M1		Use of $\mathrm{i}^2 = -1$
	5ai–5b+3a–3bi+16=8i	A1		5ai–5b+3a–3bi+16=8i OE PI
	3a –5b + 16 = 0, 5a –3b = 8	M1		Equating both the real parts and the imag. parts for the c's eqn.
	16b = 104 (or 16a = 88 etc)	A1		Correct elimination of either a or b from two correct equations involving a and b. OE PI
	$(z=)\frac{11}{2}+\frac{13}{2}\mathrm{i}$	A1	6	ACF isolated, not embedded.
	Total		**6**	

Q	Solution	Mark	Total	Comment
5 (a)	$\{y(-5+h)=\}$ $(-5+h)(-5+h+3)$	M1		Attempt to find *y* when $x=-5+h$ PI
	Gradient $=\dfrac{(-5+h)(-2+h)-10}{-5+h-(-5)}$	M1		Use of gradient = $\dfrac{y_2-y_1}{x_2-x_1}$ OE to obtain an expression in terms of *h*.
	$=\dfrac{-7h+h^2}{h}$ $=-7+h$	A1	**3**	CSO $-7+h$ or $h-7$
(b)	As $h\to 0$, {grad of line in **(a)** → grad of curve at point (−5, 10)}	E1		$\lim_{h\to 0}$ [c's(a+bh)] OE NB 'h=0' instead of '$h\to 0$' gets E0
	{Gradient of curve at point (−5, 10) =} −7	A1F	**2**	ft on c's *a* value only if both Ms have been scored in part **(a)** and *a*+*bh* has been obtained convincingly. Final answer must be −7 not ' → −7 OE'
	Total		**5**	
(b) **(b)**	Note: E0, A1F is possible. OE wording for '→' eg 'tends to', 'approaches', 'goes towards'. Do NOT accept '='.			

Q	Solution	Mark	Total	Comment
6 (a)	$x=0,\quad x=-2,\quad y=0$	B2,1,0	**2**	OE (eg x+2=0) B1 for two correct.
(b)(i)	$(y=)-1$	B1	**1**	
(b)(ii)		M1		Three branches shown on sketch of *C* with either middle branch or outer two branches correct in shape.
		A1	**2**	All three branches, correct shape and positions and approaching correct asymptotes in a correct manner.
(c)	Critical values: (x+4)(x−2) = 0	M1		PI Valid method to find critical values. Condone corresponding inequality. Alternatives must reach an equivalent stage where critical values can be stated.
	Critical values are $x=-4,\ x=2$	A1		Both correct with no extras remaining. Seen or used.
	$x\le -4,\quad x\ge 2$	B1		Both inequalities
	$-2<x<0$	B2,1,0	**5**	B1 if either or both '<' replaced by '≤'
	Total		**10**	
(a)	Must be equations. If more than 3 equations deduct 1 mark for each extra to a minimum of B0			

Q	Solution	Mark	Total	Comment
7(a)(i)	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(a)(ii)	$\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}$	B1	1	
(b)	$\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -7 & 0 \end{bmatrix}$	M1 A1	2	Multiplication of c's matrices from **(a)(i) and (a)(ii)** in correct order. CAO
(c)(i)	$\mathbf{A}^2 = \begin{bmatrix} 9+3 & 3\sqrt{3}-3\sqrt{3} \\ 3\sqrt{3}-3\sqrt{3} & 3+9 \end{bmatrix} = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$ $= 12\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 12\mathbf{I}$	B1	1	Accept either of these two final forms.
(c)(ii)	$\mathbf{A} = \sqrt{12}\begin{bmatrix} -\frac{3}{\sqrt{12}} & -\frac{\sqrt{3}}{\sqrt{12}} \\ -\frac{\sqrt{3}}{\sqrt{12}} & \frac{3}{\sqrt{12}} \end{bmatrix}$	M1		OE eg $-2\sqrt{3}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$
	$= \begin{bmatrix} \sqrt{12} & 0 \\ 0 & \sqrt{12} \end{bmatrix}\begin{bmatrix} \cos 210^\circ & \sin 210^\circ \\ \sin 210^\circ & -\cos 210^\circ \end{bmatrix}$	A1		Either order. OE
	Scale factor of enlargement $= \sqrt{12}$ $(= 2\sqrt{3})$ (line of reflection) $y = \tan 105^\circ\ x$	B1 B1		OE. If not $\sqrt{12}$ OE, ft on $\sqrt{k}$ from (c)(i). OE in form $y = (\tan\theta)x$ ACF
	Combination of enlargement sf $\sqrt{12}$ and reflection in line $y = \tan 105^\circ\ x$	A1	5	OE CSO Need correct combination of sf and eqn and also convincingly shown that the matrix corresponds to a combination of an enlargement and reflection
	Altn for M1A1 in (c)(ii) $\begin{bmatrix} -3 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} =$	(M1)		Attempting to find the image of vertices of a square under **A** with at least two non-origin images obtained and correct.
	$= \begin{bmatrix} 0 & -3 & -\sqrt{3} & -3-\sqrt{3} \\ 0 & -\sqrt{3} & 3 & -\sqrt{3}+3 \end{bmatrix}$	(A1)		Correct image of square under **A** (seen or used) with evidence of either correct length of side of the square or correct angle between a side and an axis.
	Total		**10**	
(c)(ii)	Other correct alternatives' include eg Enlargement sf $-\sqrt{12}$, reflection in $y = \tan 15^\circ\ x$			
(c)(ii)	Other acceptable answers for final B mark above include $y = (\tan\frac{7\pi}{12})\,x$; Condone eg $y = -\tan 75^\circ\ x$, $y = -(\tan\frac{5\pi}{12})\,x$; Apply ISW after a correct form is given			

Q	Solution	Mark	Total	Comment
8(a)	$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	B1		$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$ OE stated or used. B0 if any incorrect angle also used. Condone degrees or decs (3sf or better)
	$\frac{5}{4}x-\frac{\pi}{3}=2n\pi+"\frac{\pi}{4}"$; $\frac{5}{4}x-\frac{\pi}{3}=2n\pi-"\frac{\pi}{4}"$	M1		OE; Either one, showing a correct use of $2n\pi$ in forming a general soln. ft c's $\cos^{-1}(\sqrt{2}/2)$. Condone $360n$ in place of $2n\pi$
	$x=\frac{4}{5}\left(2n\pi+\frac{\pi}{4}+\frac{\pi}{3}\right)$, $x=\frac{4}{5}\left(2n\pi-\frac{\pi}{4}+\frac{\pi}{3}\right)$	m1		Correct rearrangement of $\frac{5}{4}x-\frac{\pi}{3}=2n\pi+\alpha$ OE to $x=........$, where an α is from c's $\cos\alpha=\sqrt{2}/2$. Condone $360n$ in place of $2n\pi$
	$x=\frac{24n\pi+7\pi}{15}$, $x=\frac{24n\pi+\pi}{15}$	A2,1,0	5	OE full set of correct solutions in radians in terms of π written in a simplified form. (A1 if correct but left unsimplified). Accept the simplification retrospectively if it appears in (b)
(b)	For both $\frac{24n\pi+7\pi}{15}$ and $\frac{24n\pi+\pi}{15}$, solns. in $0\le x\le 20\pi$ come from n=0 to n=12 inclusive.	B1F		Values for n, stated or used, ft on c's general solution
	Sum $=\sum_{n=0}^{12}\left[\frac{24n\pi+7\pi}{15}\right]+\sum_{n=0}^{12}\left[\frac{24n\pi+\pi}{15}\right]$ $=\frac{24\pi}{15}\frac{12}{2}(13)+\frac{7\pi}{15}(13)+\frac{24\pi}{15}\frac{12}{2}(13)+\frac{13\pi}{15}$	M1,A1		Method for summing; must be using <u>correct</u> general solution. PI by correct value of k .
	$\{=\frac{\pi}{15}(1872+91+1872+13)\}$ $=\frac{3848}{15}\pi$ (ie $k=\frac{3848}{15}$)	A1	4	OE exact value eg $256\frac{8}{15}\pi$
	Total		9	
(a)	Form of the answer in m1 line of soln above would score A1. If it had been simplified to $x=\frac{4}{5}\left(2n\pi+\frac{7\pi}{12}\right)$, $x=\frac{4}{5}\left(2n\pi+\frac{\pi}{12}\right)$ it would have scored A2			
(a)	Simplification requires terms of the form $a\pi+b\pi$, where a and b are numerical fractions to be combined.			
(a)(b)	Full correct answer might eg be written as $x=\frac{24n\pi+7\pi}{15}$, $x=\frac{24n\pi+25\pi}{15}$ in which case for $\frac{24n\pi+25\pi}{15}$ solns in $0\le x\le 20\pi$ would come from $n=-1$ to n=11 inclusive.			
(b)	Identifying and listing all relevant solns.: (B1F as above); At least 24 of the 26 correct solns (M1 PI) $\frac{3848}{15}\pi$ (OE A2). If not A2 award A1 for **both** $\frac{1963}{15}\pi$ and $\frac{377}{3}\pi$ seen.			

Q	Solution	Mark	Total	Comment
9(a)		B1 B1	 2	Ellipse, 'centre' origin with correct values for at least two intercepts. Correct values shown for the four intercepts
(b)	$\frac{x^2}{16}+\frac{(x+k)^2}{9}=1$; $9x^2+16(x+k)^2=16(9)$ $25x^2+32kx+16k^2-144=0$	M1 A1		Replacing y by (x+k) or by (x−k) OE A correct quadratic equation in the form $Ax^2+Bx+C=0$, PI by later work.
	$B^2-4AC=(32k)^2-4(25)(16k^2-144)$	M1		B^2-4AC in terms of k; ft on c's quadratic provided B and C are both in terms of k
	Roots real and different $\Rightarrow B^2-4AC>0$ $\Rightarrow (32k)^2-4(25)(16k^2-144)>0$	A1		A correct strict inequality where k is the only unknown
	$16k^2-25k^2+25(9)>0$; $9k^2<25(9)$ $k^2<25$; $-5<k<5$	A1	5	CSO AG
(c)	$\frac{(x-a)^2}{16}+\frac{(y-b)^2}{9}=1$	M1		$x\to x\pm a$ and $y\to y\pm b$
	$9(x^2-2ax+a^2)+16(y^2-2by+b^2)=144$	A1		
	$-18a=18$; $-32b=-64$; $144-9a^2-16b^2=c$	m1		Comparing non-zero coeffs to form three equations. PI
	$a=-1$, $b=2$, $c=144-9-64=71$	B2,1,0	5	B1 for two correct values.
	Altn: $9x^2+16y^2+18x-64y=c$ $9(x^2+2x)+16(y^2-4y)=c$ $9(x+1)^2+16(y-2)^2=c+9+64$	 (M1) (A1)		 (Completing the square)
	$\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=\frac{c+9+64}{144}$ $a=-1$, $b=2$, $c=144-9-64=71$	(m1) (B2,1,0)	 (5)	$\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=\frac{c+\lambda}{144}$ (B1 for two correct values.)
(d)	Equations of tangents to E that are parallel to y=x are $y=x+5$ and $y=x-5$	B1		Need both equations. PI by M1 line
	Tangents to translated ellipse that are parallel to y=x are $y-b=x-a+5$ and $y-b=x-a-5$ $y=x+8$ and $y=x-2$	 M1 A1	 3	 Since '**Hence**', NMS scores 0/3
	Total		**15**	
	TOTAL		**75**	
	Condone correct coordinates in place of 'intercepts'.			