General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MFP1

Unit Further Pure 1

Friday 18 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The quadratic equation

$$5x^2 - 7x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{39}{5}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{1}{\alpha} \quad \text{and} \quad \beta + \frac{1}{\beta}$$ *(5 marks)*

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2 A curve has equation $y = x^4 + x$.

(a) Find the gradient of the line passing through the point $(-2, 14)$ and the point on the curve for which $x = -2 + h$. Give your answer in the form

$$p + qh + rh^2 + h^3$$

where p, q and r are integers. *(5 marks)*

(b) Show how the answer to part **(a)** can be used to find the gradient of the curve at the point $(-2, 14)$. State the value of this gradient. *(2 marks)*

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3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i})$$ *(3 marks)*

(b) Hence find the complex number z such that

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i}) = 0$$ *(3 marks)*

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4 Find the general solution, in degrees, of the equation

$$\sin\left(70° - \tfrac{2}{3}x\right) = \cos 20°$$ *(6 marks)*

5 The curve C has equation $y = \dfrac{x}{(x+1)(x-2)}$.

The line L has equation $y = -\frac{1}{2}$.

(a) Write down the equations of the asymptotes of C. *(3 marks)*

(b) The line L intersects the curve C at two points. Find the x-coordinates of these two points. *(2 marks)*

(c) Sketch C and L on the same axes.

(You are given that the curve C has no stationary points.) *(3 marks)*

(d) Solve the inequality

$$\frac{x}{(x+1)(x-2)} \leqslant -\frac{1}{2}$$ *(3 marks)*

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6 (a) Using surd forms, find the matrix of a rotation about the origin through 135° anticlockwise. *(2 marks)*

(b) The matrix $\mathbf{M}$ is defined by $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$.

(i) Given that $\mathbf{M}$ represents an enlargement followed by a rotation, find the scale factor of the enlargement and the angle of the rotation. *(3 marks)*

(ii) The matrix $\mathbf{M}^2$ also represents an enlargement followed by a rotation. State the scale factor of the enlargement and the angle of the rotation. *(2 marks)*

(iii) Show that $\mathbf{M}^4 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(iv) Deduce that $\mathbf{M}^{2012} = -2^n\mathbf{I}$ for some positive integer n. *(2 marks)*

Turn over ▶

7 The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root, α.

(a) Show that α lies in the interval $0.1 < x < 0.2$. *(2 marks)*

(b) Starting from the interval $0.1 < x < 0.2$, use interval bisection **twice** to obtain an interval of width 0.025 within which α must lie. *(3 marks)*

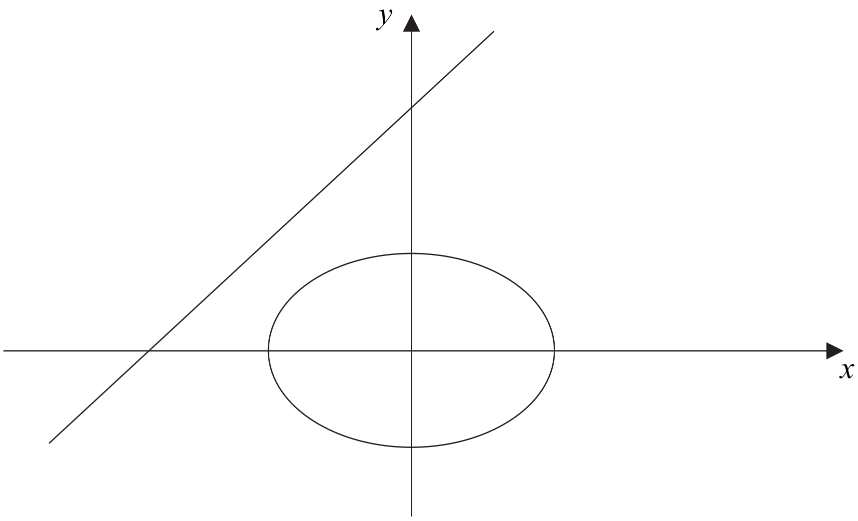
(c) Taking $x_1 = 0.2$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four decimal places. *(4 marks)*

8 The diagram shows the ellipse E with equation

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

and the straight line L with equation

$$y = x + 4$$



(a) Write down the coordinates of the points where the ellipse E intersects the coordinate axes. *(2 marks)*

(b) The ellipse E is translated by the vector $\begin{bmatrix} p \\ 0 \end{bmatrix}$, where p is a constant. Write down the equation of the translated ellipse. *(2 marks)*

(c) Show that, if the translated ellipse intersects the line L, the x-coordinates of the points of intersection must satisfy the equation

$$9x^2 - (8p - 40)x + (4p^2 + 60) = 0$$ *(3 marks)*

(d) Given that the line L is a tangent to the translated ellipse, find the coordinates of the two possible points of contact.

(No credit will be given for solutions based on differentiation.) *(8 marks)*