General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MFP1

Unit Further Pure 1

Friday 20 May 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A curve passes through the point (2, 3) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{\sqrt{2+x}}$$

Starting at the point (2, 3), use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 3$. Give your answer to four decimal places. *(5 marks)*

---

2 The equation

$$4x^2 + 6x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{3}{4}$. *(2 marks)*

(c) Find an equation, with integer coefficients, which has roots

$$3\alpha - \beta \quad \text{and} \quad 3\beta - \alpha$$ *(5 marks)*

---

3 It is given that $z = x + iy$, where x and y are real.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(z - i)(z^* - i)$$ *(3 marks)*

(b) Given that

$$(z - i)(z^* - i) = 24 - 8i$$

find the two possible values of z. *(4 marks)*

4 The variables x and Y, where $Y = \log_{10} y$, are related by the equation

$$Y = mx + c$$

where m and c are constants.

(a) Given that $y = ab^x$, express a in terms of c, and b in terms of m. *(3 marks)*

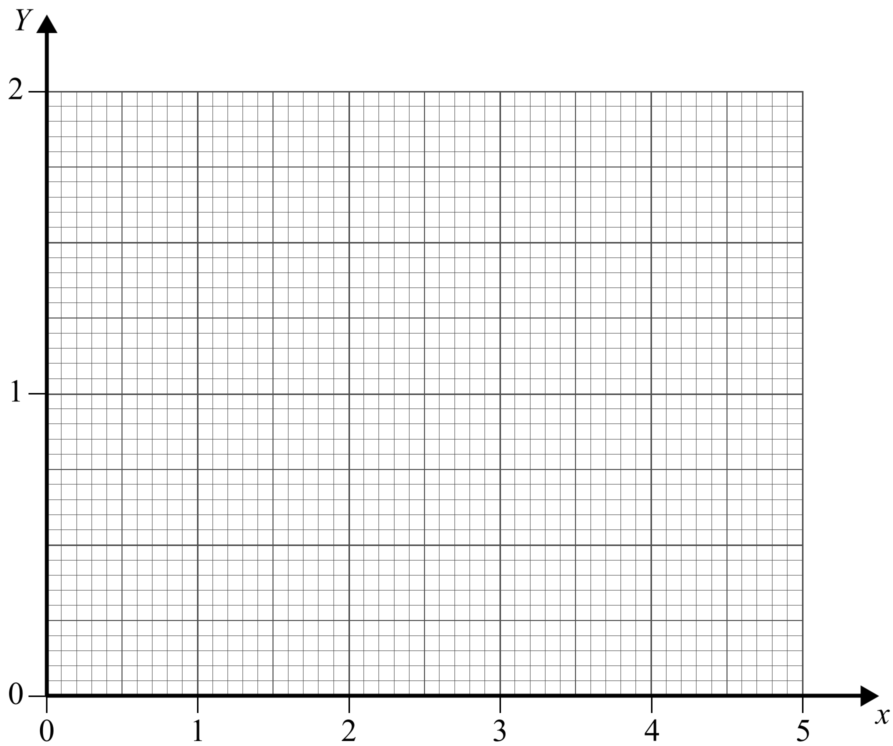
(b) It is given that $y = 12$ when $x = 1$ and that $y = 27$ when $x = 5$.

On the diagram below, draw a linear graph relating x and Y. *(3 marks)*

(c) Use your graph to estimate, to two significant figures:

(i) the value of y when $x = 3$; *(2 marks)*

(ii) the value of a. *(2 marks)*



---

5 (a) Find the general solution of the equation

$$\cos\left(3x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(5 marks)*

(b) Use your general solution to find the smallest solution of this equation which is greater than 5π. *(2 marks)*

Turn over ▶

6 (a) Expand $(5+h)^3$. *(1 mark)*

(b) A curve has equation $y = x^3 - x^2$.

(i) Find the gradient of the line passing through the point $(5, 100)$ and the point on the curve for which $x = 5 + h$. Give your answer in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(4 marks)*

(ii) Show how the answer to part **(b)(i)** can be used to find the gradient of the curve at the point $(5, 100)$. State the value of this gradient. *(2 marks)*

---

7 The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

(a) (i) Calculate the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) Show that $\mathbf{A}^3 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(b) Describe the single geometrical transformation, or combination of two geometrical transformations, corresponding to each of the matrices:

(i) $\mathbf{A}^3$; *(2 marks)*

(ii) $\mathbf{A}$. *(3 marks)*

---

8 A curve has equation $y = \dfrac{1}{x^2 - 4}$.

(a) (i) Write down the equations of the three asymptotes of the curve. *(3 marks)*

(ii) Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes. *(4 marks)*

(b) Hence, or otherwise, solve the inequality

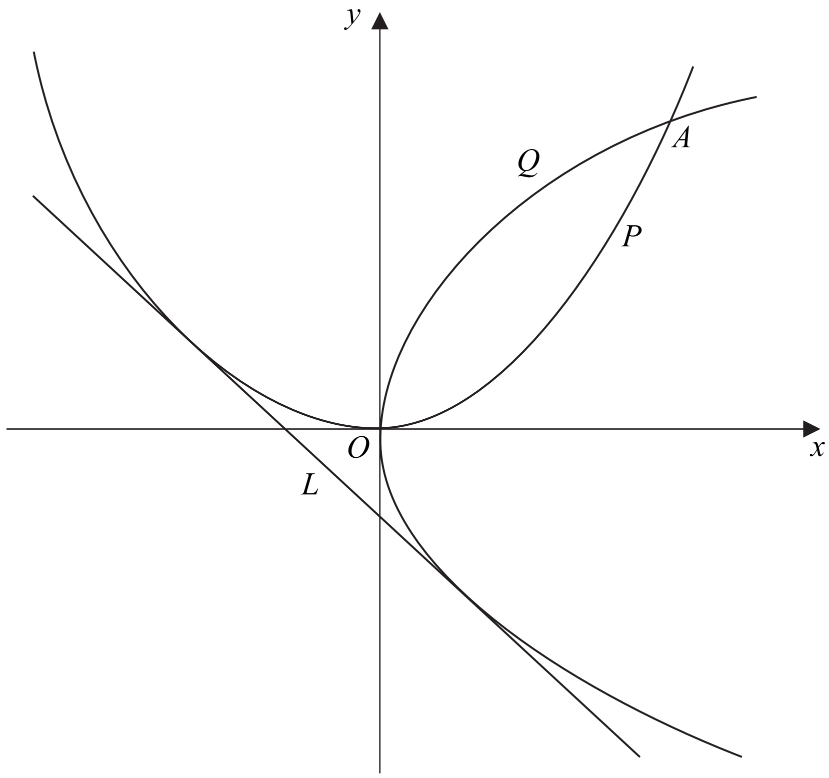
$$\frac{1}{x^2 - 4} < -2$$

(3 marks)

9 The diagram shows a parabola P which has equation $y = \frac{1}{8}x^2$, and another parabola Q which is the image of P under a reflection in the line $y = x$.

The parabolas P and Q intersect at the origin and again at a point A.

The line L is a tangent to both P and Q.



(a) (i) Find the coordinates of the point A. *(2 marks)*

(ii) Write down an equation for Q. *(1 mark)*

(iii) Give a reason why the gradient of L must be -1. *(1 mark)*

(b) (i) Given that the line $y = -x + c$ intersects the parabola P at two distinct points, show that

$$c > -2$$ *(3 marks)*

(ii) Find the coordinates of the points at which the line L touches the parabolas P and Q.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

END OF QUESTIONS