Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MFP1

Unit Further Pure 1

Thursday 27 May 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A curve passes through the point (1, 3) and satisfies the differential equation

$$\frac{dy}{dx} = 1 + x^3$$

Starting at the point (1, 3), use a step-by-step method with a step length of 0.1 to estimate the y-coordinate of the point on the curve for which $x = 1.3$. Give your answer to three decimal places.

(No credit will be given for methods involving integration.) *(6 marks)*

QUESTION PART REFERENCE

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(1 - 2\mathrm{i})z - z^*$$

(4 marks)

(b) Hence find the complex number z such that

$$(1 - 2\mathrm{i})z - z^* = 10(2 + \mathrm{i})$$

(2 marks)

QUESTION PART REFERENCE

3 Find the general solution, in degrees, of the equation

$$\cos(5x - 20°) = \cos 40°$$

(5 marks)

QUESTION PART REFERENCE

4 The variables x and y are related by an equation of the form

$$y = ax^2 + b$$

where a and b are constants.

The following approximate values of x and y have been found.

x	2	4	6	8
y	6.0	10.5	18.0	28.2

(a) Complete the table below, showing values of X, where $X = x^2$. *(1 mark)*

(b) On the diagram below, draw a linear graph relating X and y. *(2 marks)*

(c) Use your graph to find estimates, to two significant figures, for:

(i) the value of x when $y = 15$; *(2 marks)*

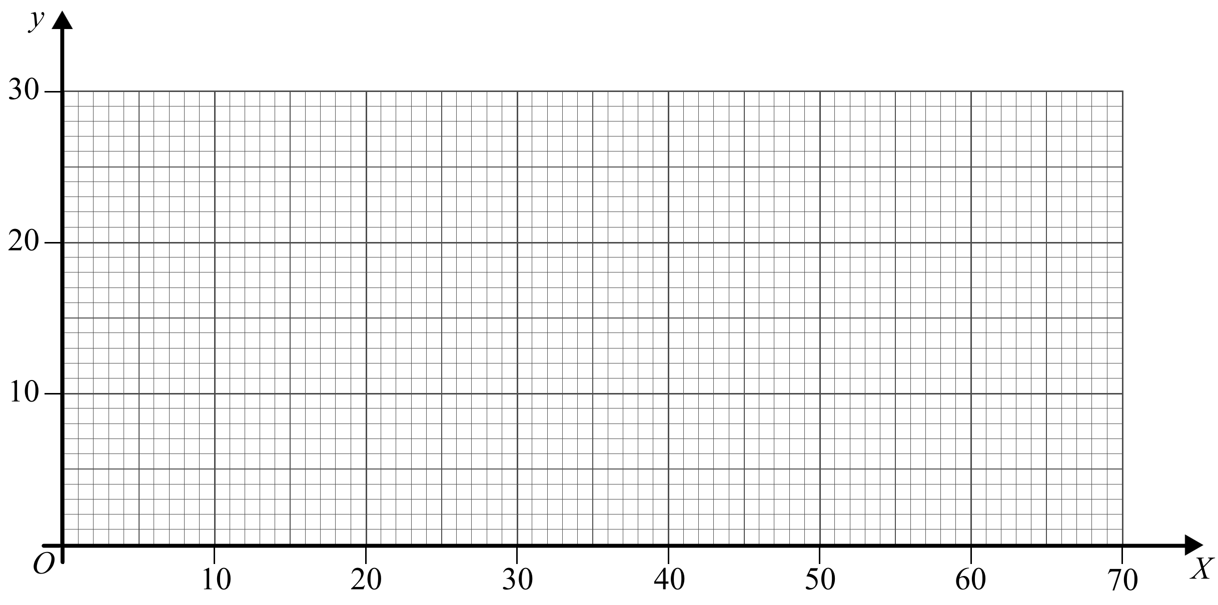
(ii) the values of a and b. *(3 marks)*

QUESTION PART REFERENCE

(a)

x	2	4	6	8
X				
y	6.0	10.5	18.0	28.2

(b)



5 A curve has equation $y = x^3 - 12x$.

The point A on the curve has coordinates $(2, -16)$.

The point B on the curve has x-coordinate $2 + h$.

(a) Show that the gradient of the line AB is $6h + h^2$. *(4 marks)*

(b) Explain how the result of part **(a)** can be used to show that A is a stationary point on the curve. *(2 marks)*

QUESTION PART REFERENCE

6 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

(a) $\mathbf{A}$; *(2 marks)*

(b) $\mathbf{B}$; *(2 marks)*

(c) $\mathbf{A}^2$; *(2 marks)*

(d) $\mathbf{B}^2$; *(2 marks)*

(e) $\mathbf{AB}$. *(3 marks)*

QUESTION PART REFERENCE

7 (a) (i) Write down the equations of the two asymptotes of the curve $y = \dfrac{1}{x-3}$. *(2 marks)*

(ii) Sketch the curve $y = \dfrac{1}{x-3}$, showing the coordinates of any points of intersection with the coordinate axes. *(2 marks)*

(iii) On the same axes, again showing the coordinates of any points of intersection with the coordinate axes, sketch the line $y = 2x - 5$. *(1 mark)*

(b) (i) Solve the equation

$$\frac{1}{x-3} = 2x - 5$$ *(3 marks)*

(ii) Find the solution of the inequality

$$\frac{1}{x-3} < 2x - 5$$ *(2 marks)*

QUESTION PART REFERENCE

8 The quadratic equation

$$x^2 - 4x + 10 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{5}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\alpha + \frac{2}{\beta}$ and $\beta + \frac{2}{\alpha}$. *(6 marks)*

QUESTION PART REFERENCE

9 A parabola P has equation $y^2 = x - 2$.

(a) (i) Sketch the parabola P. *(2 marks)*

(ii) On your sketch, draw the two tangents to P which pass through the point $(-2, 0)$. *(2 marks)*

(b) (i) Show that, if the line $y = m(x + 2)$ intersects P, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$ *(3 marks)*

(ii) Show that, if this equation has equal roots, then

$$16m^2 = 1$$ *(3 marks)*

(iii) Hence find the coordinates of the points at which the tangents to P from the point $(-2, 0)$ touch the parabola P. *(3 marks)*

QUESTION PART REFERENCE