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General Certificate of Education (A-level)
January 2012

Mathematics MFP1

(Specification 6360)

Further Pure 1

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

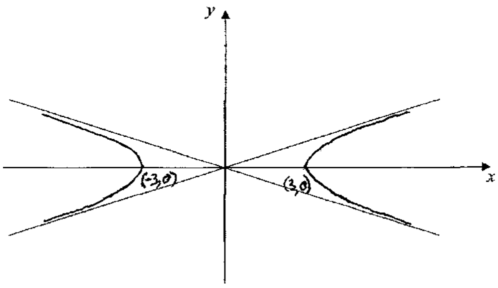
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

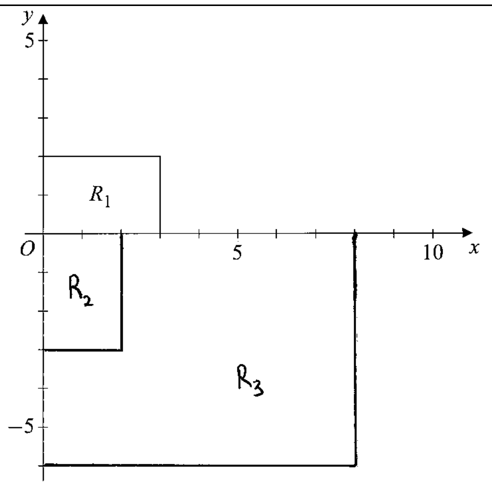
Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\alpha + \beta = -\frac{7}{2}$	B1		
	$\alpha\beta = 4$	B1	2	
(b)	$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{7}{2}\right)^2 - 2(4)$	M1		Using correct identity with ft or correct substitution
	$= \frac{49}{4} - 8 = \frac{17}{4}$	A1	2	CSO AG. A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{17/4}{16} \left(= \frac{17}{64}\right)$	M1		Writing $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in a correct suitable form with ft or correct substitution
	$= \frac{17}{64}$	A1F		ft wrong value for $\alpha\beta$
	(Product =) $\frac{1}{(\alpha\beta)^2} = \frac{1}{16} \left(= \frac{4}{64}\right)$	B1F		ft wrong value for $\alpha\beta$
	$x^2 - Sx + P \ (= 0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values. PI
	Eqn is $64x^2 - 17x + 4 = 0$	A1	5	CSO Integer coefficients and '= 0' needed
	Total		**9**	
2(a)	$\int x^{-2/3}\, dx = 3x^{1/3} (+c)$	B1		$kx^{\frac{1}{3}}, k \neq 0$ ie condone incorrect non-zero coefficient here
	$(3)x^{1/3} \to \infty$ as $x \to \infty$, so no finite value	E1		
(b)	$\int x^{-4/3}\, dx = -3x^{-1/3} (+c)$	M1		$\lambda x^{-1/3}, \ \lambda \neq 0$
		A1		$-3x^{-1/3}$ OE
	$\int_8^{\infty} x^{-4/3}\, dx = -3(0 - \frac{1}{2}) = \frac{3}{2}$	A1	5	CSO
	Total		**5**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$x = \pm 3\mathrm{i}$	B1	1	$\pm 3\mathrm{i}$ $(a = 0, b = \pm 3)$
(ii)	$x = -2 \pm 3\mathrm{i}$	B1F	1	If not correct, ft wrong answer(s) to (i) provided (i) has a non-zero b value
(b)(i)	$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$	B1	1	Terms simplified in any order.
(ii)	$(1 + 2\mathrm{i})^3 = 1 + 3(2\mathrm{i}) + 3(2\mathrm{i})^2 + (2\mathrm{i})^3$ $= 1+3(2\mathrm{i}) + 3(4\mathrm{i}^2) + (8\mathrm{i}^3)$	B1F		Replacing x in (b)(i) by 2i, squaring and cubing correctly, only ft on c's wrong non-zero coefficients from (b)(i).
	$= 1 + 3(2\mathrm{i}) +3(4)(-1)+(8)(-\mathrm{i})$ $= -11 - 2\mathrm{i}$	M1 A1	 3	Use of $\mathrm{i}^2 = -1$ at least once. $-11 - 2\mathrm{i}$ $(a = -11,\ b = -2)$
(iii)	$z^* - z^3 = 1 - 2\mathrm{i} - (-11 - 2\mathrm{i})$ $= 12$	M1 A1F	 2	Use of $z^* = 1 - 2\mathrm{i}$ If not correct, only ft on $1 - 2\mathrm{i} -$ c's (b)(ii) if c's (b)(ii) answer is of the form $a + b\mathrm{i}$ with $a \neq 0$ and $b \neq 0$
	Total		**8**	
4(a)	$\Sigma r^2(4r-3) = 4\Sigma r^3 - 3\Sigma r^2$ ...	M1		Splitting up the sum into two separate sums. PI by next line.
	$= 4\left(\frac{1}{4}\right)n^2(n+1)^2 - 3\left(\frac{1}{6}\right)n(n+1)(2n+1)$	m1		Substitution of the two summations from FB
	$= n(n+1)\left[n(n+1) - \frac{1}{2}(2n+1)\right]$	m1 A1		Taking out common factors n and $n + 1$. Remaining expression eg our [...] in ACF not just simplified to AG
	Sum $= \frac{1}{2}n(n+1)(2n^2-1)$	A1	5	Be convinced as form of answer is given, penalise any jumps or backward steps
(b)	$\sum_{r=20}^{40} r^2(4r-3)$ $= \sum_{r=1}^{40} r^2(4r-3) - \sum_{r=1}^{19} r^2(4r-3)$	M1		Attempt to take S(19) from S(40) using part (a)
	$= 20(41)(3199) - 9.5(20)(721)$ $= 2623180 - 136990$ $\sum_{r=20}^{40} r^2(4r-3) = 2486190$	A1	2	2486190 ; Since 'Hence' NMS 0/2. **SC** $\sum_{r=1}^{40}$....... $-\sum_{r=1}^{20}$...... clearly attempted and evaluated to 2455390 scores B1
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	Line joining points A and B	B1	1	Must not be linked to Q
(ii)	$x_P = 2 + w$, $\frac{w}{10} = \frac{5-2}{22-(-10)}$	M1		OE eg correct equation for AB with y replaced by 0
	$x_P = 2 + 10 \times \frac{3}{32}$	A1		$2 + 10 \times \frac{3}{32}$ OE
	$x_P = 2.9375 = 2.9$ (to 1dp)	A1	3	CAO Must be 2.9
(b)(i)	Tangent at A drawn	B1	1	At least as far as meeting the x-axis. Accept reasonable attempt. Must not be linked to P.
(ii)	$x_Q = 2 - \frac{-10}{8}$	M1		PI by 3.25 or 26/8 OE
	$\ldots = 3.25$	A1	2	CAO Must be 3.25
	Total		**7**	
6(a)	$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	B1		OE (PI) Stated or used. A correct angle in 1st or 3rd quadrant for $\tan^{-1}(1/\sqrt{3})$. Condone degrees / decimal equivs.
	$\left(\frac{x}{2} - \frac{\pi}{4}\right) = n\pi + \frac{\pi}{6}$;	M1		Correct use of either $n\pi$ or $2n\pi$. Eg either $n\pi + \alpha$ or **both** $2n\pi + \alpha$ and $2n\pi + \pi + \alpha$ OE where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right)$ $\left(= 2n\pi + \frac{5\pi}{6}\right)$	m1		Either correct rearrangement of $\frac{x}{2} - \frac{\pi}{4} = n\pi + \alpha$ to $x = \ldots$, or correct rearrangements of both the equivalents above in the M1 line involving $2n\pi$, where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
		A1	4	ACF, but must now be exact and in terms of π.
(b)	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \pm\sqrt{\frac{1}{3}}$	M1		PI. Taking square roots, must include the $\pm$ or evidence of its use
	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = -\sqrt{\frac{1}{3}}$ $\Rightarrow \frac{x}{2} - \frac{\pi}{4} = n\pi - \frac{\pi}{6}$;	m1		OE If not correct, ft on c's working in (a) with c's α replaced by $-\alpha$. Condones as in m1 above.
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right)$, $x = 2\left(n\pi - \frac{\pi}{6} + \frac{\pi}{4}\right)$ $\{ x = 2n\pi + \frac{5\pi}{6}$, $x = 2n\pi + \frac{\pi}{6} \}$	A1F	3	Any valid form, but only ft on c's exact value for $\tan^{-1}(1/\sqrt{3})$ in terms of π.
	Total		**7**	

Q	Solution	Marks	Total	Comments
7(a)	$y = \pm\frac{1}{3}x$	B1	1	ACF Need both
(b)		B1 B1 B1	 3	2-branch curve with branches in correct regions above and below x-axis Curve approaching asymptotes Coords (± 3, 0), as **only** points of intersection with coordinate axes, indicated. Condone -3 and $+3$ marked on x-axis at points of intersection as (± 3, 0) indicated.
(c)(i)	$\frac{(x+3)^2}{9} - y^2 = 1$	M1 A1	 2	Replacing x by either $x + 3$ or $x - 3$ ACF
(ii)	$\frac{(x+3)^2}{9} - x^2 = 1$	M1		Substitution into c's (c)(i) eqn of $y = x$ to eliminate y or of $x = y$ to eliminate x
	$x^2 + 6x + 9 = 9(x^2 + 1)$	A1F		Correct expansion of $(x \pm 3)^2$ equated to $9(x^2 + 1)$ OE ft; [OE in y]
	$8x^2 - 6x = 0$ $\quad (8x^2 = 6x)$	A1F		Ft on error $(x - 3)$ for $(x + 3)$ in (c)(i) which gives $8x^2 + 6x = 0$ $\quad (8x^2 = -6x)$ [OE in y]
	Points are (0, 0), $\left(\frac{3}{4}, \frac{3}{4}\right)$	A1	4	Both. ACF
(d)	 Points are (3, 0), $\left(3\frac{3}{4}, \frac{3}{4}\right)$	M1 A1F	 2	Adding 3 to c's (c)(ii) **two** x-coords keeping y-coordinates unchanged. Ft on c's (c)(ii) coordinates for the **two** points If not deduced then M0A0
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)(i)		B1	1	Rectangle with vertices (0, 0), (0, −3), (2, −3), (2, 0)
(ii)		M1		Rectangle with vertices **either** whose x-coords are c's (a)(i) x-coord vertices multiplied by 4 **or** whose y-coords are c's (a)(i) y-coord vertices multiplied by 2
		A2,1	3	A2 if rectangle with vertices (0, 0), (0, −6), (8, −6), (8, 0) (A1 if either the four x-coords are correct or the four y-coords are correct)
(b)(i)	Matrix is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} =$	M1		Attempt to multiply $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ with c's (b)(i) matrix in **either** order.
		m1		Multiplication in **correct** order with at least two of the four ft multiplications carried out correctly.
	$\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$	A1	3	For $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ NMS $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ scores B3 $\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}$ scores B1
	Total		**8**	

Q	Solution	Marks	Total	Comments
9(a)	Asymptotes $x = 1$ $y = 1$	B1 B1	 2	$x = 1$ OE $y = 1$ OE
(b)	$-4x + c = \dfrac{x}{x-1}$	M1		Elimination of y PI by next line
	$(-4x + c)(x - 1) = x$ $-4x^2 + cx + 4x - c = x$ $-4x^2 + cx + 3x - c = 0$	A1		OE (denominators cleared)
	$4x^2 - (c+3)x + c = 0$	A1	3	CSO AG No incorrect algebraic expressions etc
(c)(i)	Discriminant is $(c + 3)^2 - 4(4c)$	B1		OE
	For tangency $c^2 - 10c + 9 = 0$	M1		Forming a quadratic eqn in c after equating discriminant to zero
	$(c - 9)(c - 1) = 0 \Rightarrow c = 1,\ c = 9$	A1	3	Correct values 1, 9 for c.
(ii)	$\underline{c = 1}$: $4x^2 - 4x + 1 = 0$ $\underline{c = 9}$: $4x^2 - 12x + 9 = 0$	M1		Substitutes at least one of c's values for c from (c)(i) either into the given quadratic in (b) OE or into $\dfrac{c+3}{8}$
	$4x^2 - 4x + 1 = 0 \quad \Rightarrow \quad x = 1/2 \quad (= 0.5)$	A1		No other root from quadratic
	$4x^2 - 12x + 9 = 0 \quad \Rightarrow \quad x = 3/2 \quad (= 1.5)$	A1		No other root from quadratic
	When $x = 1/2,\ y = -1$; when $x = 3/2,\ y = 3$ $\left(\dfrac{1}{2}, -1\right)\ \left(\dfrac{3}{2}, 3\right)$	A1	4	Accept in either format
	Total		**12**	
	TOTAL		**75**	