

General Certificate of Education
January 2009
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Thursday 15 January 2009 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A curve passes through the point $(0, 1)$ and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{1 + x^2}$$

Starting at the point $(0, 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. (5 marks)

- 2 The complex number $2 + 3i$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

- (a) Write down the other root of this equation. (1 mark)
- (b) Find the values of b and c . (4 marks)

- 3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3} \quad (5 \text{ marks})$$

- 4 It is given that

$$S_n = \sum_{r=1}^n (3r^2 - 3r + 1)$$

- (a) Use the formulae for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$ to show that $S_n = n^3$. (5 marks)

- (b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k . (2 marks)

5 The matrices \mathbf{A} and \mathbf{B} are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k :

(i) $\mathbf{A} + \mathbf{B}$; (1 mark)

(ii) \mathbf{A}^2 . (2 marks)

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. (4 marks)

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix \mathbf{A}^2 . (2 marks)

(ii) The matrix \mathbf{A} represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. (3 marks)

6 A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. (3 marks)

(ii) State the coordinates of the points at which the curve intersects the x -axis. (1 mark)

(iii) Sketch the curve.

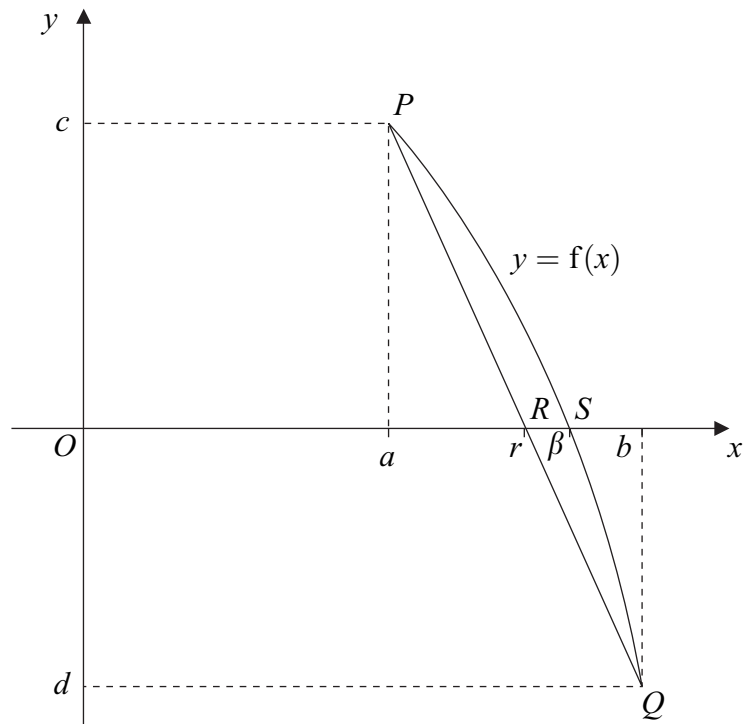
(You are given that the curve has no stationary points.) (4 marks)

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0 \quad (2 \text{ marks})$$

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- 7 The points $P(a, c)$ and $Q(b, d)$ lie on the curve with equation $y = f(x)$. The straight line PQ intersects the x -axis at the point $R(r, 0)$. The curve $y = f(x)$ intersects the x -axis at the point $S(\beta, 0)$.



- (a) Show that

$$r = a + c \left(\frac{b - a}{c - d} \right) \quad (4 \text{ marks})$$

- (b) Given that

$$a = 2, b = 3 \text{ and } f(x) = 20x - x^4$$

- (i) find the value of r ; (3 marks)
- (ii) show that $\beta - r \approx 0.18$. (3 marks)

- 8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\int_1^{\infty} x^{-\frac{3}{4}} dx;$ (3 marks)

(b) $\int_1^{\infty} x^{-\frac{5}{4}} dx;$ (3 marks)

(c) $\int_1^{\infty} (x^{-\frac{3}{4}} - x^{-\frac{5}{4}}) dx.$ (1 mark)

- 9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

- (a) Find the equations of the two asymptotes of H , giving each answer in the form $y = mx$. (2 marks)

- (b) Draw a sketch of the two asymptotes of H , using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H . (3 marks)

- (c) (i) Show that, if the line $y = x + c$ intersects H , the x -coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$
 (4 marks)

- (ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c . (2 marks)

- (iii) Find, in terms of c , the y -coordinates of these two points. (3 marks)

END OF QUESTIONS

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