Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 3 – Statistics

Topic 5: Normal distribution AND Hypothesis testing

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 5: Normal distribution AND Hypothesis testing
  - Normal distribution
  - Hypothesis testing

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T5_Q1

5. The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

(a) Find the probability that a randomly selected battery will last for longer than 16 hours.

(1)

At the start of her exams Alice put 4 new batteries in her calculator.
She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

(b) Find the probability that her calculator will not stop working for Alice's remaining exams.

(5)

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

(c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures.

(3)

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

(d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

Question T5_Q2

5. A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

(c) Test Hannah's belief at the 5% level of significance.
You should state your hypotheses clearly.

(5)

Question 5 continued

Question 5 continued

Question T5_Q3

5. A health centre claims that the time a doctor spends with a patient can be modelled by a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes.

(a) Using this model, find the probability that the time spent with a randomly selected patient is more than 15 minutes.

(1)

Some patients complain that the mean time the doctor spends with a patient is more than 10 minutes.

The receptionist takes a random sample of 20 patients and finds that the mean time the doctor spends with a patient is 11.5 minutes.

(b) Stating your hypotheses clearly and using a 5% significance level, test whether or not there is evidence to support the patients' complaint.

(4)

The health centre also claims that the time a dentist spends with a patient during a routine appointment, T minutes, can be modelled by the normal distribution where $T \sim N(5, 3.5^2)$

(c) Using this model,

(i) find the probability that a routine appointment with the dentist takes less than 2 minutes

(1)

(ii) find $P(T < 2 \mid T > 0)$

(3)

(iii) hence explain why this normal distribution may not be a good model for T.

(1)

The dentist believes that she cannot complete a routine appointment in less than 2 minutes.

She suggests that the health centre should use a refined model only including values of $T > 2$

(d) Find the median time for a routine appointment using this new model, giving your answer correct to one decimal place.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

Question T5_Q4

5. The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1% of females from this country are shorter than k cm,

(a) find the value of k

(2)

(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

(1)

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm

(c) Find the probability that her height is more than 160 cm

(4)

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm

Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm

(d) Carry out a suitable test to assess Mia's belief.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

Question 5 continued.

Question 5 continued.

Question 5 continued.

Mark Scheme

Question T5_Q1

Qu 5	Scheme	Marks	AO
(a)	$P(L > 16) = 0.69146\ldots$ **awrt 0.691**	B1 (1)	1.1b
(b)	$P(L > 20 \mid L > 16) = \dfrac{P(L > 20)}{P(L > 16)}$	M1	3.1b
	$= \dfrac{0.308537...}{(a)}$ <u>or</u> $\dfrac{1-(a)}{(a)}, = 0.44621\ldots$	A1ft, A1	1.1b 1.1b
	For calc to work require $(0.44621...)^4 = 0.03964\ldots$ **awrt <u>0.0396</u>**	dM1 A1 (5)	2.1 1.1b
(c)	Require: $[P(L > 4)]^2 \times [P(L > 20 \mid L > 16)]^2$	M1	1.1a
	$= (0.99976...)^2 \times ("0.44621...")^2$	A1ft	1.1b
	$= 0.19901\ldots$ **awrt <u>0.199</u> (*)**	A1cso* (3)	1.1b
(d)	$H_0 : \mu = 18 \quad H_1 : \mu > 18$	B1	2.5
	$\bar{L} \sim N\left(18, \left(\dfrac{4}{\sqrt{20}}\right)^2\right)$	M1	3.3
	$P(\bar{L} > 19.2) = P(Z > 1.3416\ldots) = 0.089856\ldots$	A1	3.4
	$(0.0899 > 5\%)$ <u>or</u> $(19.2 < 19.5)$ <u>or</u> $1.34 < 1.6449$ so not significant	A1	1.1b
	Insufficient evidence to support Alice's claim (or belief)	A1 (5)	3.5a
		(14 marks)	

	Notes
(a)	B1 for evaluating probability using their calculator (awrt 0.691) Accept 0.6915
(b)	1st M1 for a first step of identifying a suitable conditional probability (either form) 1st A1ft for a ratio of probabilities with numerator = awrt 0.309 or 1 – (a) and denom = their (a) 2nd A1 for awrt 0.446 (o.e.) Accept 0.4465 (from $\frac{0.3085}{0.691} = 0.44645...$) NB $\frac{P(16<L<20)}{P(L>16)} = 0.5538\ldots$ scores M1A1A1 when they do $1 - 0.5538 = 0.4462\ldots$ 2nd M1 (dep on 1st M1) for 2nd correct step i.e. (their $0.446\ldots)^4$ <u>or</u> $X \sim B(4, \text{“}0.446\text{”})$ and $P(X = 4)$ 3rd A1 for awrt 0.0396
(c)	1st M1 for a correct approach to solving the problem (May be implied by A1ft) 1st A1ft for $P(L > 4)$ = awrt 0.9998 used <u>and</u> ft their 0.44621 in correct expression If use $P(L > 20) = 0.3085..$ as 0.446.. in (b) then M1 for $(0.3085..)^2 \times [P(L > 4)]^2$; A1ft as above
*	2nd A1cso for 0.199 or better with clear evidence of M1 [NB $(0.4662..)^2 = 0.199\ldots$ is M0A0A0] **Must see M1 scored by correct expression in symbols or values (M1A1ft)**
(d)	B1 for both hypotheses in terms of μ. M1 for selecting a suitable model. Sight of <u>normal</u>, <u>mean</u> 18, <u>sd</u> $\frac{4}{\sqrt{20}}$ (o.e.) or <u>variance</u> = 0.8 1st A1 for using the model correctly. Allow awrt 0.0899 <u>or</u> 0.09 from correct prob. statement
ALT	**CR** $(\bar{L}) > 19.471\ldots$ (accept awrt 19.5) <u>or</u> **CV** of 1.6449 (or better: calc 1.6448536..)
	2nd A1 for correct non-contextual conclusion. Wrong comparison or contradictions A0 Error giving 2nd A0 implies 3rd A0 but just a correct contextual conclusion can score A1A1 3rd A1 dep on M1 and 1st A1 for a correct contextual conclusion mentioning <u>Alice's claim</u> /<u>belief</u> <u>or</u> there is insufficient evidence that the mean <u>lifetime</u> is more than 18 hours

Question T5_Q2

Question	Scheme	Marks	AOs
5(a)	$\dfrac{24.63-25}{'\sigma'}=-1.0364$	M1	3.1b
	$[\sigma=]0.357$ (must come from compatible signs)	A1	1.1b
	$P(D>k)=0.4$ or $P(D<k)=0.6$	B1	1.1b
	$\dfrac{k-25}{'0.357'}=0.2533$	M1	3.4
	$k=$ awrt **<u>25.09</u>**	A1	1.1b
		(5)	
(b)	$[Y\sim B(200, 0.45)\rightarrow]\ W\sim N(90, 49.5)$	B1	3.3
	$P(Y<100)\approx P(W<99.5)\left[=P\left(Z<\dfrac{99.5-90}{\sqrt{49.5}}\right)\right]$	M1	3.4
	$=0.9115\ldots$ awrt **<u>0.912</u>**	A1	1.1b
		(3)	
(c)	$H_0: \mu=25$ $H_1: \mu<25$	B1	2.5
	$[\bar{D}\sim]N\left(25, \dfrac{0.16^2}{20}\right)$	M1	3.3
	$P(\bar{D}<24.94)[=P(Z<-1.677...)]=0.046766...$	A1	3.4
	$p=0.047<0.05$ <u>or</u> $z=-1.677\ldots<-1.6449$ <u>or</u> $24.94<24.94115\ldots$ <u>or</u> reject H_0/in the critical region/significant	M1	1.1b
	There is sufficient evidence to support <u>Hannah's</u> <u>belief</u>.	A1	2.2b
		(5)	
			(13 marks)

Notes

(a)	**M1:** for standardising 24.63, 25 and 'σ' (ignore label) and setting = to z where $1<\lvert z\rvert<2$ **A1:** $[\sigma=]$ awrt 0.36. Do **not** award this mark if signs are not compatible. **B1:** for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram **M1:** for a correct expression with z = awrt 0.253 (may be implied by correct answer) **A1:** awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)
(b)	**B1:** setting up normal distribution approximation of binomial N(90, 49.5) (may be implied by a correct answer) Look out for e.g. $\sigma=\frac{3\sqrt{22}}{2}$ or $\sigma=$ awrt 7.04 **M1:** attempting a probability using a continuity correction i.e. $P(W<100.5)$, $P(W<99.5)$ or $P(W<98.5)$ condone $\leq$ (The continuity correction may be seen in a standardisation). **A1:** awrt 0.912 [Note: 0.911299… from binomial scores 0 out of 3]
(c)	**B1:** for both hypotheses in terms of μ **M1:** selecting suitable model must see N(ormal), mean 25, sd = $\frac{0.16}{\sqrt{20}}$ (o.e.) or var = $\frac{4}{3125}$ (o.e.) Condone N(25, $\frac{0.16}{\sqrt{20}}$) if $\frac{0.16}{\sqrt{20}}$ then used as s.d. **A1:** p value = awrt 0.047 <u>or</u> **test statistic** awrt −1.68 <u>or</u> **CV** awrt 24.941 (any of these values imply the M1 provided they do not come from Normal mean = 24.94) **M1:** a correct comparison (including compatible signs) **or** correct non-contextual conclusion (f.t. their p value, test statistic or critical value in the comparison) M1 may be implied by a correct contextual statement **NB** Any contradictory non contextual statements/comparisons score M0A0 e.g. '$p<0.05$, not significant' **A1:** correct conclusion in context mentioning <u>Hannah's belief</u> <u>or</u> the mean <u>amount</u>/<u>liquid</u> in each bottle is now <u>less than 25</u>ml (**dep on M1A1M1**)

Question T5_Q3

Qu 5	Scheme	Marks	AO
(a)	{Let X = time spent, $P(X > 15) =$ } 0.105649… awrt **0.106**	B1 **(1)**	1.1b
(b)	$H_0: \mu = 10 \quad H_1: \mu > 10$	B1	2.5
	$\bar{X} \sim N\left(10, \left(\frac{4}{\sqrt{20}}\right)^2\right)$; $P\left(\bar{X} > 11.5\right) = 0.046766\ldots$ [Condone 0.9532…]	M1;A1	3.3;3.4
	[This is significant (< 5%) so] there is evidence to support the complaint	A1 **(4)**	2.2b
(c)(i)	[$P(T < 2) =$] 0.1956… awrt **0.196**	B1 **(1)**	1.1b
(ii)	Require $\frac{P(0 < T < 2)}{P(T > 0)} = \frac{0.119119...}{0.923436...}$; $= 0.1289955\ldots$ awrt **0.129**	M1 A1;A1 **(3)**	3.4 1.1bx2
(iii)	The current model suggests **non-negligible** probability of T values < 0 which is impossible	B1 **(1)**	3.5b
(d)	Require t such that $P(T > t \mid T > 2) = 0.5$ or $P(T < t \mid T > 2) = 0.5$	M1	3.1b
	e.g. $\frac{P(T > t)}{P(T > 2)} = 0.5$; so $P(T > t) = 0.5 \times [1 - \text{(c)(i)}]$ or $P(T > t) = 0.5 \times 0.8043..$	M1; A1ft	1.1b 3.4
	[i.e. $P(T > t) = 0.40\ldots$ implies] $\frac{t - 5}{3.5} = 0.2533$ or $P(T < t) =$ "0.5978.."	M1	1.1b
	$t = 5.886\ldots$ or from calculator 5.867… so awrt **5.9**	A1 **(5)**	1.1b
		(15 marks)	

	Notes
(a)	B1 for awrt 0.106 (from calculator) [Allow 10.6%]
(b)	B1 for both hypotheses correct in terms of μ. M1 for selection of a correct model (sight or use of correct normal- may not have label $\bar{X}$) 1st A1 for use of this model to get probability allow 0.046~0.047 [Condone awrt 0.953]
ALT	**OR** test statistic $z = 1.677\ldots$ (awrt 1.68) and cv of 1.64 (or better) **or** CR $\bar{X} > 11.47..$ 2nd A1 (dep on 1st A1 or at least $P\left(\bar{X} > 11.5\right) < 0.05$ (o.e.)) for a correct conclusion in context -must mention **complaint**/claim or **time**/mins is > 10
SC	**(M0 for $\bar{X}$ ~N(11.5, …)** for correct probability **and** conclusion (score M0A0A1 on epen)
(c)(i)	B1 for awrt 0.196 (from calculator) [Allow 19.6%]
(ii)	M1 for a correct probability ratio expression (may be implied by 1st A1 scored) 1st A1 for a correct ratio of probabilities (both correct or truncated to 2 dp) 2nd A1 for awrt 0.129
(iii)	B1 for a suitable explanation of why model is not suitable based on negative T values Must say that a **significant** proportion of values < 0 (o.e.) e.g. $P(T > 0)$ should be **closer** to 1 or Difference between $P(T < 2 \mid T > 0)$ and $P(T < 2)$ is **too big** (o.e.)
(d)	1st M1 for a correct conditional probability statement to start the problem or $0.5 \times P(T > 2)$ 2nd M1 for correct ratio of probability expressions [Must have $P(T > t)$ or $P(2 < T < t)$] 1st A1ft for a correct equation for $P(T > t)$ (o.e.) ft their answer to part (c)[May be in a diagram] 3rd M1 for attempt to find t (standardising and sight of 0.2533) or prepare to use calc (ft) Arriving at $P(T <$ median$) = 1 - 0.5 \times$ "their 0.8043" will score 1st 4 marks 2nd A1 for awrt 5.9 Sight of awrt 5.9 and at least one M mark scores 5/5 [Answer only send to review]

Question T5_Q4

Qu 5	Scheme	Marks	AO
(a)	[Let $F \sim N(166.5, 6.1^2)$] $P(F < k) = 0.01 \Rightarrow \frac{k-166.5}{6.1} = -2.3263$	M1	3.4
	$k = 152.309\ldots$ **152** or awrt **152.3**	A1	1.1b
		(2)	
(b)	[$P(150 < F < 175) =$] $0.914840\ldots$ awrt **0.915**	B1	1.1b
		(1)	
(c)	$P(F > 160 \mid 150 < F < 175)$	M1	3.1b
	$= \frac{P(160 < F < 175)}{P(150 < F < 175)}$ or $\frac{P(160 < F < 175)}{\text{"(b)"}}$	M1	1.1b
	$= \frac{0.7749487...}{\text{"}0.91484...\text{"}}$	A1ft	1.1b
	$= 0.84708\ldots$ awrt **0.847**	A1	1.1b
		(4)	
(d)	$H_0 : \mu = 166.5 \quad H_1 : \mu < 166.5$	B1	2.5
	[Let X = height of female from 2nd country] $\bar{X} \sim N\left(166.5, \left(\frac{7.4}{\sqrt{50}}\right)^2\right)$	M1	3.3
	$P(\bar{X} < 164.6) = 0.03472...$	A1	3.4
	[$0.0347\ldots < 0.05$ so significant or reject H_0] There is evidence to support Mia's belief	dA1	2.2b
		(4)	
		(11 marks)	

	Notes
(a)	M1 for standardising (allow ±) with k, 166.5 and 6.1 and set equal to a z value $2.3 < \lvert z \rvert < 2.4$ A1 for 152 or awrt 152.3 **Ans only** 2/2 [Condone poor use of notation e.g. $P(\frac{k-166.5}{6.1}) = -2.3263$]
	Allow percentages instead of probabilities throughout.
(b)	B1 for awrt 0.915
(c)	1st M1 for interpreting demand as an appropriate conditional probability ($\Rightarrow$ by 2nd M1) 2nd M1 for correct ratio of expressions (can ft their (b) on denominator) ($\Rightarrow$ by 1st A1ft) 1st A1ft for a correct ratio of probs (can ft their "0.9148…" to 3sf from (b) if > 0.775) 2nd A1 for awrt 0.847
(d)	B1 for both correct hypotheses in terms of μ 1st M1 for selecting the correct model (needn't use $\bar{X}$ $\Rightarrow$ by standardisation or 1st A1) 1st A1 for correct use of the correct model i.e. awrt 0.035 (allow 0.04 if $P(\text{"}\bar{X}\text{"} < 164.6)$ seen) Condone $P(\text{"}\bar{X}\text{"} > 164.6) = 0.9652$ or awrt 0.97 only if comparison with 0.95 is made
ALT	**Use of z value:** Need to see $Z = -1.8(15\ldots)$ **and** cv of ± 1.6449 (allow 1.64 or better) for 1st A1
ALT	**Use of CR or CV for $\bar{X}$:** Need to see "$\bar{X}$"< 164.7786… or CV = … (awrt 164.8) for 1st A1 Condone truncation i.e 164.7 or better 2nd dA1 (**dep on M1A1** only) for a correct inference in context. Must mention Mia's belief **or** mean height of females/women Do NOT award if contradictory statements about hypotheses made e.g. "not sig"
SC	**M0 for** $\bar{X} \sim N(\mathbf{164.6}, \ldots)$ If they achieve p = awrt 0.035 (o.e. with z-value or CV of 166.3) **and** a correct conclusion in context is given score M0A0A1 [and SC for awrt 0.97 > 0.95 case]