Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 10: Vectors

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 10: Vectors
  - Use vectors to solve a problem in pure mathematics

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T10_Q1

2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

Question 2 continued

Question T10_Q2

10.

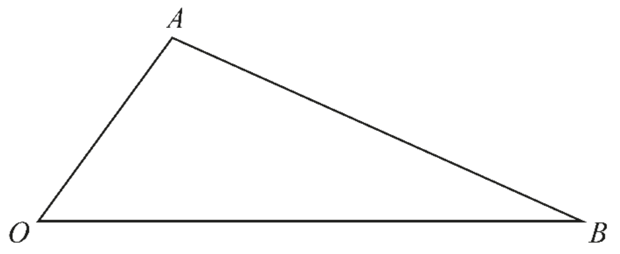


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)

Question 10 continued

Question 10 continued

Question 10 continued

Question T10_Q3

3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

Question 3 continued

Question T10_Q4

2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)

Question 2 continued

Question T10_Q5

6.

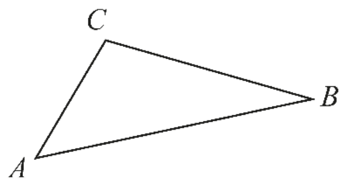


Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

Question 6 continued

Question 6 continued

Question 6 continued

Mark Scheme

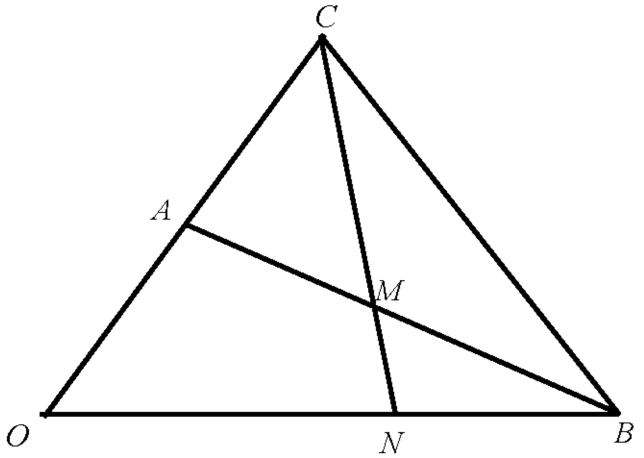
Question T10_Q1

Question	Scheme	Marks	AOs
2	$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k},\ \overrightarrow{OB} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k},\ \overrightarrow{OC} = a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k},\ a < 0$ $\overrightarrow{AB} = \overrightarrow{BD},\ \lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} \left\{ = \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$ **or** $= \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix}\right) \left\{ = \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2 + (5-3)^2 + (-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2 + (5-3)^2 + (-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = \ldots$ **or** $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = \ldots$	dM1	2.1
	(as $a < 0 \Rightarrow$) $a = 2 - 2\sqrt{2}$ (or $a = 2 - \sqrt{8}$)	A1	1.1b
		(3)	
		(5 marks)	

Notes for Question 2

(a)	
M1:	Complete ***applied*** strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) - (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}) = (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB} - \overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components Note: Ignore labelling
dM1:	Complete method of ***correctly*** applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = \ldots$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2 - 2\sqrt{2}$
Note:	Writing $a = 2 \pm 2\sqrt{2}$, without evidence of rejecting $a = 2 + 2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2 - \sqrt{8}$ or $\frac{4 - \sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828\ldots$, without reference to a correct exact value is A0

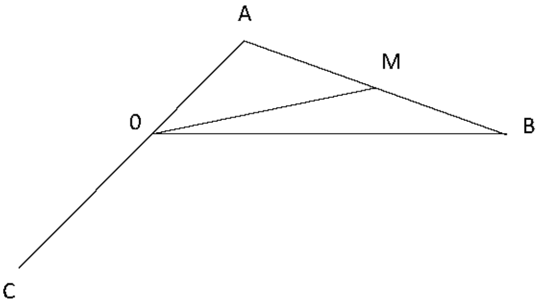
Question T10_Q2

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ ***(needs to be simplified and seen in (a) only)***	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) **Way 1**	$\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \ldots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
(c) **Way 2**	$\overrightarrow{ON} = \mu\mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} = \mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \ldots\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda = \mu\ \&\ \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB}=\overrightarrow{ON}+\overrightarrow{NB} \Rightarrow \mathbf{b}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}+K\mathbf{b}$		
	$\mathbf{a}:\ \left(2-\frac{3}{2}\lambda\right)=0 \Rightarrow \lambda=...\ \left\{\mathbf{b}:\ 1=\frac{1}{2}\lambda+K\ \ \&\ \lambda=\frac{4}{3}\ \Rightarrow K=\frac{1}{3}\right\}$	M1	2.2a
	$\lambda=\frac{4}{3}$ or $K=\frac{1}{3} \Rightarrow \overrightarrow{ON}=\frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB}=\frac{1}{3}\mathbf{b} \Rightarrow ON:NB=2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON}=\mu\mathbf{b}\ \&\ \overrightarrow{CN}=k\overrightarrow{CM} \Rightarrow \overrightarrow{CO}+\overrightarrow{ON}=k\overrightarrow{CM}$		
	$-2\mathbf{a}+\mu\mathbf{b}=k\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}:-2=-\frac{3}{2}k \Rightarrow k=\frac{4}{3},\ \ \mathbf{b}:\ \mu=\frac{1}{2}k \Rightarrow \mu=\frac{1}{2}\left(\frac{4}{3}\right)=...$	M1	2.2a
	$\mu=\frac{2}{3} \Rightarrow \overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\} \Rightarrow ON:NB=2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM}=-\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$ or $\overrightarrow{CM}=(-2\mathbf{a}+\mathbf{b})+\frac{1}{2}(\mathbf{a}-\mathbf{b})$ or for $\left\{\overrightarrow{CM}=\overrightarrow{OM}-\overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM}=\frac{1}{2}(\mathbf{a}+\mathbf{b})-2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON}=\overrightarrow{OC}+\lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**Special Case**
	Give SC M1 A0 for the solution $\overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AM}+\overrightarrow{MN} \Rightarrow \overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AM}+\lambda\overrightarrow{CM}$
	$\overrightarrow{ON}=\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})+\lambda\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)\ \left\{=\left(\frac{1}{2}-\frac{3}{2}\lambda\right)\mathbf{a}+\left(\frac{1}{2}+\frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	**Alternative 1:** Give M1 A1 for the following alternative solution: $\overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AM}+\overrightarrow{MN} \Rightarrow \overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AM}+\mu\overrightarrow{CM}$ $\overrightarrow{ON}=\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})+\mu\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)=\left(\frac{1}{2}-\frac{3}{2}\mu\right)\mathbf{a}+\left(\frac{1}{2}+\frac{1}{2}\mu\right)\mathbf{b}$ $\mu=\lambda-1 \Rightarrow\ \overrightarrow{ON}=\left(\frac{1}{2}-\frac{3}{2}(\lambda-1)\right)\mathbf{a}+\left(\frac{1}{2}+\frac{1}{2}(\lambda-1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2-\frac{3}{2}\lambda\right)=0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	**Special Case where the point *C* is believed to be below the origin *O*** 
	Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$

Question T10_Q3

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB, so $OABC$ is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if AB was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why $OABC$ is a trapezium.

Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore OC is parallel to AB so $OABC$ is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.

Example 3

As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, OC and AB are parallel, so proven

Example 4

Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides OA and CB in this question may be ignored, even if incorrect.

Question T10_Q4

Question Number	Scheme	Marks	AO's
2	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})^{*}$	A1*	2.1
		(3)	
			(3 marks)

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.

Question T10_Q5

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$*	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow \cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$*	A1*	2.1
			(5 marks)
	Notes		

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\dfrac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result