

Topic Test

Summer 2022

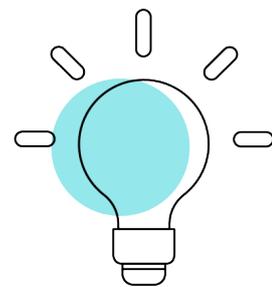
Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 9: Numerical methods

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both [published](#) (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the [A level](#) advance information for summer 2022:

- Topic 9: Numerical methods
 - o Formal proof Maximum point; iteration
 - o The trapezium rule

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Question T9_Q4

11.

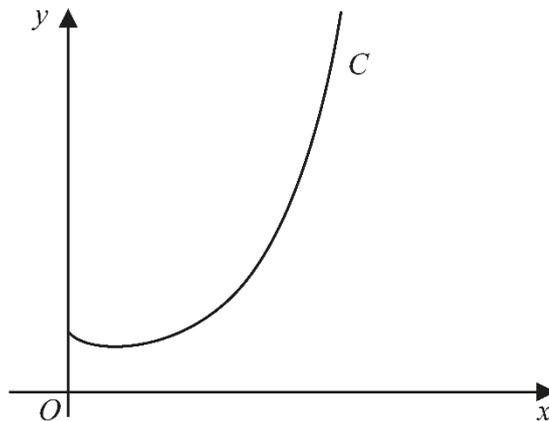


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C .
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

The point $P(\alpha, 2)$ lies on C .

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

Question T9_Q5

1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

Question T9_Q6

7.

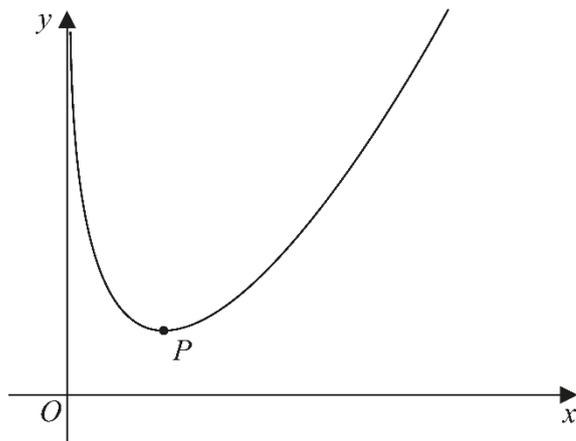


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4 \ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

Question T9_Q7

4. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0 \tag{4}$$

The iterative formula

$$x_{n+1} = \frac{1}{7} (2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α .

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

Question T9_Q8

11.

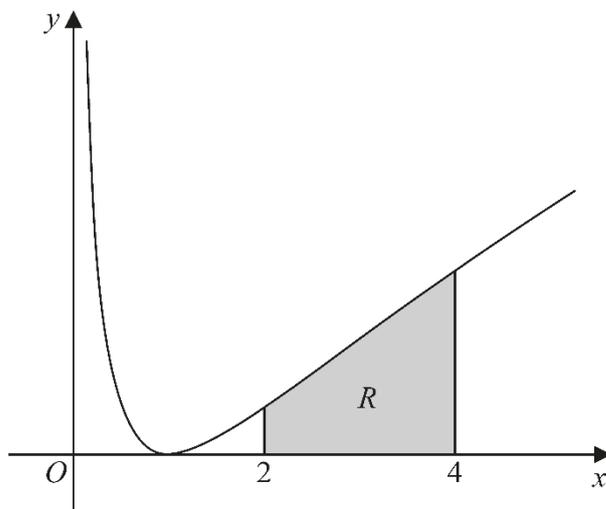


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

The table below shows corresponding values of x and y , with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R , giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R , giving your answer in the form

$$y = a(\ln 2)^2 + b \ln 2 + c$$

where a , b and c are integers to be found.

(5)

Mark Scheme

Question T9_Q1

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$	M1	2.1
	$f(3) = (2\ln(5) - x) = (+)0.22$ and $f(4) = (2\ln(4) - 4) = -1.23$ <u>Change of sign</u> and function <u>continuous</u> in interval $[3, 4] \Rightarrow$ <u>Root</u> *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula can be used to find an approximation for α because the cobweb spirals inwards for the cobweb diagram	A1	2.2a
		(2)	

(4 marks)

Notes:

(a)

M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

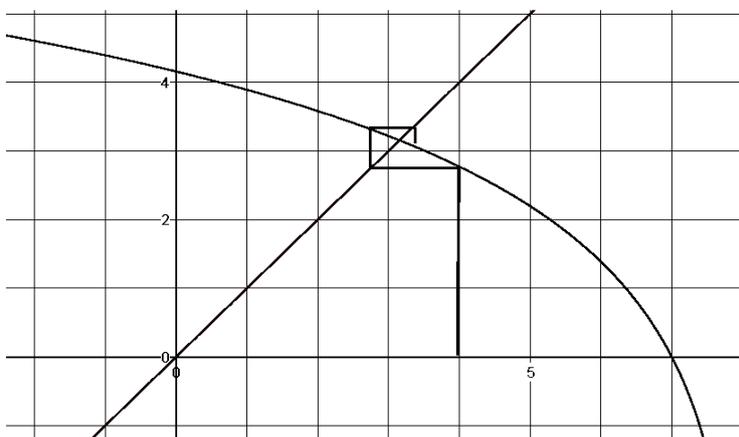
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2\ln 8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must state that it can be used with a suitable reason. Suitable reasons could be "it spirals inwards", "it gets closer to the root", "it converges"



Question T9_Q2

Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\} f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow\right\} \{x_{n+1}\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	M1	1.1b
	$= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} *$	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\} x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \frac{3}{4}, x_3 = \frac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson method cannot be used with $x_1 = 0$ which refer or allude to either the stationary point or the tangent. E.g. <ul style="list-style-type: none"> • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal 	B1	2.3
		(1)	

(6 marks)

Notes for Question 5

(a)	
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^2 + 2x_n$ (Condone $\frac{dy}{dx} = 6x^2 + 2x$)
M1:	Substitutes $f(x_n) = 2x_n^3 + x_n^2 - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \frac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for <ul style="list-style-type: none"> • $x_{n+1} = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x} = \frac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \frac{4x^3 + x^2 + 1}{6x^2 + 2x_n}$
Note	Condone $x = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ for M1
Note	Condone $x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$ or $x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ (i.e. no $x_{n+1} = \dots$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^3 + x_n^2 - 1 - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1 = 1$
A1:	$x_2 = \frac{3}{4}$ and $x_3 = \frac{2}{3}$
Note:	Condone $x_2 = \frac{3}{4}$ and $x_3 = \text{awrt } 0.667$ for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. <ul style="list-style-type: none"> • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x = 0$ • At $x = 0, f'(x_1) = 0$ • x_1 cannot be 0 • $6x^2 + 2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1 = 0, 6x^2 + 2x = 0$

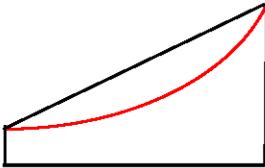
Question T9_Q3

Question	Scheme	Marks	AOs
2	Time (s) 0 5 10 15 20 25		
	Speed (m s ⁻¹) 2 5 10 18 28 42		
(a)	Uses an allowable method to estimate the area under the curve. E.g. Way 1: an attempt at the trapezium rule (see below)		
	Way 2: $\{s = \left(\frac{2+42}{2}\right)(25) \{= 550\}$		
	Way 3: $42 = 2 + 25(a) \Rightarrow a = 1.6 \Rightarrow s = 2(25) + (0.5)(1.6)(25)^2 \{= 550\}$		
	Way 4: $\{d = \}(2)(5) + 5(5) + 10(5) + 18(5) + 28(5) \{= 63(5) = 315\}$	M1	3.1a
	Way 5: $\{d = \}5(5) + 10(5) + 18(5) + 28(5) + 42(5) \{= 103(5) = 515\}$		
	Way 6: $\{d = \} \frac{315 + 515}{2} \{= 415\}$		
	Way 7: $\{d = \} \left(\frac{2+5+10+18+28+42}{6}\right)(25) \{= 437.5\}$		
	$\frac{1}{2} \times (5) \times [2 + 2(5+10+18+28) + 42]$ or $\frac{1}{2} \times ["315" + "515"]$	M1	1.1b
	$= 415 \{m\}$	A1	1.1b
	(3)		
(b) Alt 1	Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a) Overestimate and a relevant explanation e.g. <ul style="list-style-type: none"> • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{d^2y}{dx^2} > 0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5) 	B1ft	2.4
		(1)	
(b) Alt 2	Uses a Way 4 method in (a) Underestimate and a relevant explanation e.g. <ul style="list-style-type: none"> • All the rectangles are below the curve 	B1ft	2.4
		(1)	

(4 marks)

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2 + 2(5 + 10 + 18 + 28) + 42)$; $\lambda > 0$ for 1 st M1
	Way 2: Uses $s = \left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed) × (time)

Notes for Question 2 Continued	
(a)	<i>continued</i>
M1:	Correct trapezium rule method with $h = 5$. Condone a slip on one of the speeds. The '2' and '42' should be in the correct place in the [.....].
A1:	415
Note:	Units do not have to be stated
Note:	Give final A0 for giving a final answer with incorrect units. e.g. give final A0 for 415 km or 415ms ⁻¹
Note:	Only the 1 st M1 can only be scored for Way 2, Way 3, Way 4, Way 5 and Way 7 methods
Note:	Full marks in part (a) can only be scored by using a Way 1 or a Way 6 method.
Note:	Give M0 M0 A0 for $\{d = \} 2(5) + 5(5) + 10(5) + 18(5) + 28(5) + 42(5) \{= 105(5) = 525\}$ (i.e. using too many rectangles)
Note	Condone M1 M0 A0 for $\left[\frac{(2+10)}{2}(10) + \frac{(10+18)}{2}(5) + \frac{(18+28)}{2}(5) + \frac{(28+42)}{2}(5) \right] = 395 \text{ m}$
Note:	Give M1 M1 A1 for $5 \left[\frac{(2+5)}{2} + \frac{(5+10)}{2} + \frac{(10+18)}{2} + \frac{(18+28)}{2} + \frac{(28+42)}{2} \right] = 415 \text{ m}$
Note:	Give M1 M1 A1 for $\frac{5}{2}(2+42) + 5(5+10+18+28) = 415 \text{ m}$
Note:	<u>Bracketing mistake:</u> Unless the final calculated answer implies that the method has been applied correctly
	give M1 M0 A0 for $\frac{5}{2}(2) + 2(5+10+18+28) + 42 \{= 169 \}$
	give M1 M0 A0 for $\frac{5}{2}(2+42) + 2(5+10+18+28) \{= 232 \}$
Note:	Give M0 M0 A0 for a Simpson's Rule Method
(b)	Alt 1
B1ft:	This mark depends on both an answer to part (a) being obtained and the first M in part (a) See scheme
Note:	Allow the explanation "curve concaves upwards"
Note:	Do not allow explanations such as "curve is concave" or "curve concaves downwards"
Note:	Do not allow explanation "gradient of the curve is positive"
Note:	Do not allow explanations which refer to "friction" or "air resistance"
Note:	The diagram opposite is sufficient as an explanation. It must show the top of a trapezium lying above the curve. <div style="text-align: right;">  </div>
(b)	Alt 2
B1ft:	This mark depends on both an answer to part (a) being obtained and the first M in part (a) See scheme
Note:	Do not allow explanations which refer to "friction" or "air-resistance"

Question T9_Q4

Question	Scheme	Marks	AOs	
11 (a) Way 1	$\{y = x^x \Rightarrow\} \ln y = x \ln x$	B1	1.1a	
	$\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1}$ or awrt 0.368	A1	1.1b	
	Note: $k \neq 0$	(5)		
(a) Way 2	$\{y = x^x \Rightarrow\} y = e^{x \ln x}$	B1	1.1a	
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x \right) e^{x \ln x}$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1}$ or awrt 0.368	A1	1.1b	
	Note: $k \neq 0$	(5)		
(b) Way 1	Attempts both $1.5^{1.5} = 1.8\dots$ and $1.6^{1.6} = 2.1\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b	
	$1.8\dots < 2$ and $2.1\dots > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1	
		(2)		
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$	M1	1.1b	
	Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63			
	$\{x_4 = 1.67313\dots \Rightarrow\} x_4 = 1.673$ (3 dp) cao	A1	1.1b	
		(2)		
(d)	Give 1 st B1 for any of <ul style="list-style-type: none"> oscillates periodic non-convergent divergent fluctuates goes up and down 1, 2, 1, 2, 1, 2 alternates (condone) 	Give B1 B1 for any of <ul style="list-style-type: none"> periodic {sequence} with period 2 oscillates between 1 and 2 	B1	2.5
		Condone B1 B1 for any of <ul style="list-style-type: none"> fluctuates between 1 and 2 keep getting 1, 2 alternates between 1 and 2 goes up and down between 1 and 2 1, 2, 1, 2, 1, 2, ... 	B1	2.5
			(2)	
(11 marks)				
Note	<p>A common solution A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution</p> $\log y = x \log x \Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \log x$ $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} 1 + \log x = 0 \Rightarrow x = 10^{-1}$ <ul style="list-style-type: none"> 1st B1 for $\log y = x \log x$ 1st M1 for $\log y \rightarrow \lambda \frac{1}{y} \frac{dy}{dx}; \lambda \neq 0$ or $x \log x \rightarrow 1 + \log x$ or $\frac{x}{x} + \log x$ 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = \dots; k \neq 0$ 			

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16\dots$ and $1.6^{1.6} - 2 = 0.12\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16\dots < 0$ and $0.12\dots > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x \ln x$, attempts both $1.5 \ln 1.5 = 0.608\dots$ and $1.6 \ln 1.6 = 0.752\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608\dots < 0.69\dots$ and $0.752\dots > 0.69\dots$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x \log x$, attempts both $1.5 \log 1.5 = 0.264\dots$ and $1.6 \log 1.6 = 0.326\dots$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264\dots < 0.301\dots$ and $0.326\dots > 0.301\dots$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 11

(a)	Way 1
B1:	$\ln y = x \ln x$. Condone $\log_x y = x \log_x x$ or $\log_x y = x$
M1:	For either $\ln y \rightarrow \frac{1}{y} \frac{dy}{dx}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = \dots$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x \ln x \rightarrow x = 0.368$ with no intermediate working
(a)	Way 2
B1:	$y = e^{x \ln x}$
M1:	For either $y = e^{x \ln x} \Rightarrow \frac{dy}{dx} = f(\ln x) e^{x \ln x}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right) e^{x \ln x}$ or $\frac{dy}{dx} = (1 + \ln x) e^{x \ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = \dots$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\} \ln y = x \ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question 11 Continued	
(b)	Way 1
M1:	Attempts both $1.5^{1.5} = 1.8\dots$ and $1.6^{1.6} = 2.1\dots$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} = \text{awrt } 1.8\dots$ and $1.6^{1.6} = \text{awrt } 2.1\dots$, reason (e.g. $1.8\dots < 2$ and $2.1\dots > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	Way 2
M1:	Attempts both $1.5^{1.5} - 2 = -0.16\dots$ and $1.6^{1.6} - 2 = 0.12\dots$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16\dots$ and $1.6^{1.6} - 2 = 0.12\dots$ correct to awrt 1 dp, reason (e.g. $-0.16\dots < 0$ and $0.12\dots > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	Way 3
M1:	Attempts both $1.5 \ln 1.5 = 0.608\dots$ and $1.6 \ln 1.6 = 0.752\dots$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5 \ln 1.5 = 0.608\dots$ and $1.6 \ln 1.6 = 0.752\dots$ correct to awrt 1 dp, reason (e.g. $0.608\dots < 0.69\dots$ and $0.752\dots > 0.69\dots$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	Way 4
M1:	Attempts both $1.5 \log 1.5 = 0.264\dots$ and $1.6 \log 1.6 = 0.326\dots$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5 \log 1.5 = 0.264\dots$ and $1.6 \log 1.6 = 0.326\dots$ correct to awrt 2 dp, reason (e.g. $0.264\dots < 0.301\dots$ and $0.326\dots > 0.301\dots$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ cao (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 = \text{awrt } 1.673$
Note:	$x_2 = 1.63299\dots$, $x_3 = 1.46626\dots$, $x_4 = 1.67313\dots$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for “Converges in a cob-web pattern” or “Converges up and down to α ”

Question T9_Q5

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2} \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: The integration on a calculator gives 1.511549071 The full accuracy for y values gives 1.504726147 The accuracy from the table gives 1.50475		
		(3)	
(b)	$3 \times$ their (a) If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working. Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5 If (a) is incorrect allow $3 \times$ their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a)) For reference the integration on a calculator gives 4.534647213	B1ft	2.2a
		(1)	
(c)	<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u> Look for a sensible comment. Some examples: <ul style="list-style-type: none"> The answer is accurate to 2 sf or one decimal place Answer to (b) is accurate as $4.535 \approx 4.50$ Very accurate as 4.535 to 2 sf is 4.5 $4.51425 < 4.535$ so my answer is underestimate but not too far off It is an underestimate but quite close It is a very good estimate High accuracy (Quite) accurate It is less than 1% out $4.535 - 4.5 = 0.035$ so not far out <p style="text-align: center;">But not just "it is an underestimate" or</p> <p>Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left \frac{4.535 - 4.50}{4.535} \right \times 100 = 0.77\% \quad \text{or} \quad \left \frac{4.535 - 4.51}{4.535} \right \times 100 = 0.55\% \quad \text{or}$ $\left \frac{4.535 - 4.51425}{4.535} \right \times 100 = 0.46\% \quad \text{or} \quad \left \frac{4.50}{4.535} \right \times 100 = 99\%$ </p> <p>In these cases don't be too concerned about accuracy e.g. allow 1sf. This mark should be withheld if there are any contradictory statements</p>	B1	3.2b
		(1)	
			(5 marks)

B1: States or uses $h = 0.5$. May be implied by $\frac{1}{4} \times \{ \dots \}$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h \times \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$ condoning slips on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h \times (\text{first } y + \text{last } y + 2 \times \text{"sum of the rest"})$

Give M0 for $\frac{1}{2} \times \frac{1}{2} \times 0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85...

May be awarded for separate trapezia e.g.

$$\frac{1}{4}(0.5774 + 0.7071) + \frac{1}{4}(0.7071 + 0.7746) + \frac{1}{4}(0.7746 + 0.8165) + \frac{1}{4}(0.8165 + 0.8452)$$

May be awarded for using the function e.g. $\frac{1}{2}h \times \left\{ \sqrt{\frac{0.5}{1+0.5}} + \sqrt{\frac{2.5}{1+2.5}} + 2 \left(\sqrt{\frac{1}{1+1}} + \sqrt{\frac{1.5}{1+1.5}} + \sqrt{\frac{2}{1+2}} \right) \right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering $3 \times (a)$ and not e.g. attempting trapezium rule again.

(c)

B1: See scheme

Question T9_Q6

7(a)	$\ln x \rightarrow \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx} = 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
	(4)		
(b)	$12x^2 + x - 16\sqrt{x} = 0 \Rightarrow 12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16 = 0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}} = 16 - \sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
	(3)		
(c)	$x_2 = \sqrt[3]{\left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	$x_2 = \text{awrt } 1.13894$	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
(10 marks)			

Notes:

(a)

B1: Differentiates $\ln x \rightarrow \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$:

Look for $\frac{4x^2 + x}{2\sqrt{x}} \rightarrow \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + \dots$ or $\dots + Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2 + x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(Ax+B) - (4x^2+x)Cx^{\frac{1}{2}}}{(2\sqrt{x})^2} (A, B, C > 0)$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2 + x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{dy}{dx}\right) = Ax^{-\frac{1}{2}}(Bx+C) + D(4x^2+x)x^{-\frac{3}{2}} (A, B, C > 0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and $v = 2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2 + x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(8x+1) - (4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2}, \frac{1}{2}x^{-\frac{1}{2}}(8x+1) - \frac{1}{4}(4x^2+x)x^{-\frac{3}{2}}, 2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.

There must be no errors e.g. missing brackets.

(b)

M1: Sets $12x^2 + x - 16\sqrt{x} = 0$ and divides by \sqrt{x} or equivalent e.g. divides by x and multiplies by \sqrt{x}

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}} \Rightarrow x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow 12x^{\frac{3}{2}} = 16 - \sqrt{x} \Rightarrow 12x^2 = 16\sqrt{x} - x \Rightarrow 12x^2 - 16\sqrt{x} + x = 0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by \sqrt{x} . A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: $x_2 =$ awrt 1.13894

A1: Deduces that $x = 1.15650$

Question T9_Q7

Question	Scheme	Marks	AOs
4(a)	$f'(x) = 2x + \frac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \frac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5) + 4x-4 = 0$	dM1	1.1b
	$2x^3 - 4x^2 + 7x - 2 = 0^*$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \frac{1}{7}(2 + 4(0.3)^2 - 2(0.3)^3)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3 - 4x^2 + 7x - 2$ $h(0.3415) = 0.00366... \quad h(0.3405) = -0.00130...$	M1	3.1a
	States: <ul style="list-style-type: none"> • there is a change of sign • $f'(x)$ is continuous • $\alpha = 0.341$ to 3dp 	A1	2.4
		(2)	
(9 marks)			
Notes			

(a)

M1: Differentiates $\ln(2x^2 - 4x + 5)$ to obtain $\frac{g(x)}{2x^2 - 4x + 5}$ where $g(x)$ could be 1

A1: For $f'(x) = 2x + \frac{4x - 4}{2x^2 - 4x + 5}$

dM1: Sets their $f'(x) = ax + \frac{g(x)}{2x^2 - 4x + 5} = 0$ and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax(2x^2 - 4x + 5) \pm g(x) = 0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3 - 4x^2 + 7x - 2 = 0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1 = 0.3$. If no method is shown award for $x_2 = \text{awrt } 0.33$

A1: $x_2 = \text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4 = \text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute $x = 0.3415$ and $x = 0.3405$ into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3 - 4x^2 + 7x - 2$, $x - \frac{1}{7}(4x^2 - 2x^3 + 2)$ and $f'(x)$ as this has been

found in part (a) with $f'(0.3405) = -0.00067\dots$, $f'(0.3415) = (+) 0.0018$

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x) = 2x^3 - 4x^2 + 7x - 2$ or sight of embedded values that imply the function, not just a value or values

even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415) = 2 \times 0.3415^3 - 4 \times 0.3415^2 + 7 \times 0.3415 - 2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. \checkmark , proven, $\alpha = 0.341$, root

Question T9_Q8

Question	Scheme	Marks	AOs
11(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2} \{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 dx = x(\ln x)^2 - \int x \times \frac{2 \ln x}{x} dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2(x \ln x - \int dx)$ $= x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2x \ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 dx = [x(\ln x)^2 - 2x \ln x + 2x]_2^4$ $= 4(\ln 4)^2 - 2 \times 4 \ln 4 + 2 \times 4 - (2(\ln 2)^2 - 2 \times 2 \ln 2 + 2 \times 2)$ $= 4(2 \ln 2)^2 - 16 \ln 2 + 8 - 2(\ln 2)^2 + 4 \ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12 \ln 2 + 4$	A1	1.1b
		(5)	
(8 marks)			
Notes			

(a)

B1: Correct strip width. May be implied by $\frac{1}{2} \times \frac{1}{2} \{ \dots \}$ or $\frac{1}{4} \times \{ \dots \}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2} \times h \{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta \int \ln x dx$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x dx = x \ln x - x$

who may write $\int (\ln x)^2 dx = \int (\ln x)(\ln x) dx = \ln x(x \ln x - x) - \int \frac{x \ln x - x}{x} dx$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x \ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm \alpha x(\ln x)^2 \pm \beta x \ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2 \ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

It is possible to do $\int (\ln x)^2 dx$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 e^u du = u^2 e^u - \int 2u e^u du = u^2 e^u - 2u e^u + 2e^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2 \ln 2$ at least once to an expression of the form $u^2 e^u - \beta u e^u \pm \gamma e^u$ Both M's must have been awarded