Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 7: Differentiation

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 7: Differentiation
  - Formal proof Differentiation: stationary points, minima. Radian measure
  - Differentiation; roots of equations
  - Differentiation from first principles
  - Find maximum and minimum points; Newton- Raphson method
  - Differentiation of curves defined parametrically

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T7_Q1

2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Question 2 continued

Question T7_Q2

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

Question 5 continued

Question T7_Q3

6.

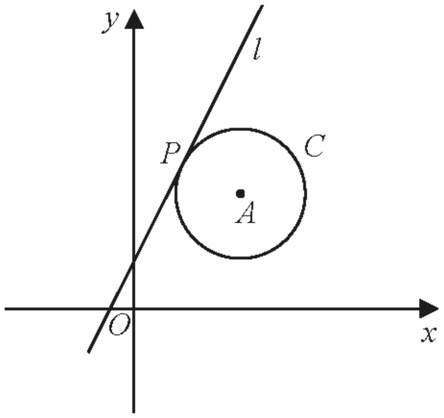


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

Question T7_Q4

9.

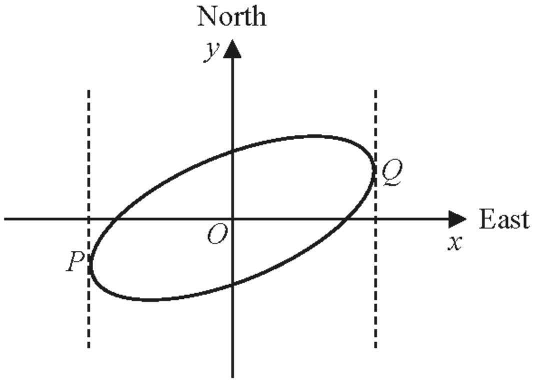


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

Question 9 continued

Question 9 continued

Question 9 continued

Question T7_Q5

9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Question 9 continued

Question T7_Q6

11.

$$\frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \equiv A + \frac{B}{(x - 3)} + \frac{C}{(1 - 2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$\mathrm{f}(x) = \frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \qquad x > 3$$

(b) Prove that $\mathrm{f}(x)$ is a decreasing function.

(3)

Question 11 continued

Question 11 continued

Question 11 continued

Question T7_Q7

14. A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

Question 14 continued

Question 14 continued

Question T7_Q8

3.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$

(1)

Question 3 continued

Question T7_Q9

12.

$$f(x) = 10e^{-0.25x} \sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

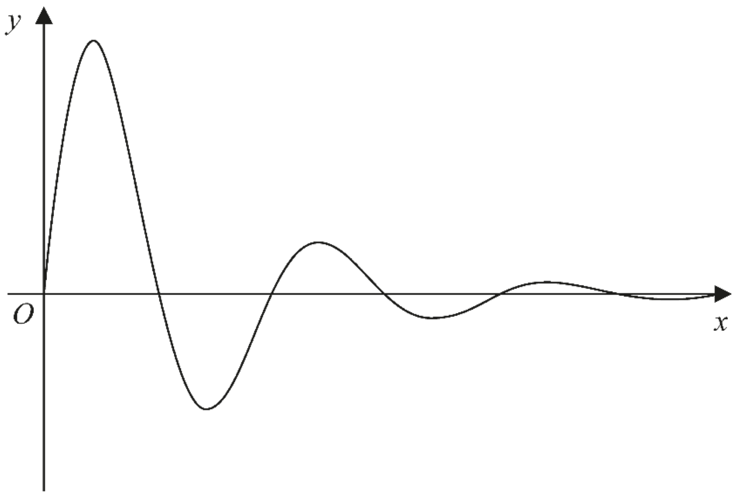


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t} \sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function H(t) is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

Question 12 continued

Question 12 continued

Question 12 continued

Question T7_Q10

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

Question 14 continued

Question 14 continued

Question T7_Q11

13.

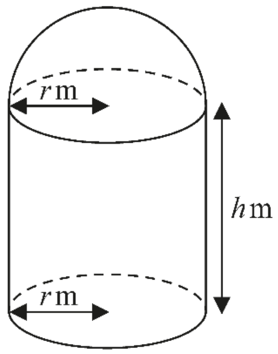


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

Question 13 continued

Question 13 continued

Question 13 continued

Question T7_Q12

9.

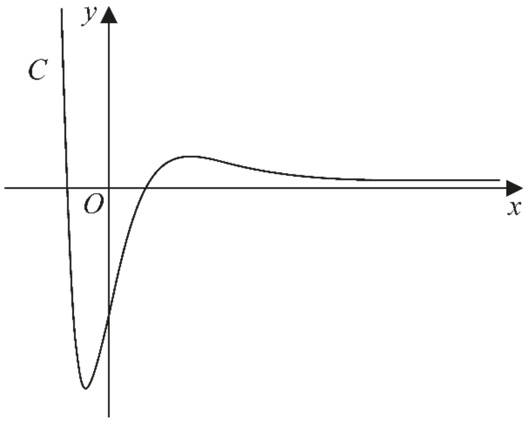


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

Question 9 continued

Question 9 continued

Question 9 continued

Question T7_Q13

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

Question 15 continued

Question 15 continued

Question 15 continued

Question T7_Q14

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

Question 13 continued

Question 13 continued

Question 13 continued

Question T7_Q15

14. Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

Question 14 continued

Question 14 continued

Question 14 continued

Question T7_Q16

5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

Question 5 continued

Question T7_Q17

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

Question 8 continued

Question 8 continued

Question 8 continued

Question T7_Q18

13. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \operatorname{cosec}^3 \theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

Question 13 continued

Mark Scheme

Question T7_Q1

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x = 4$ into their $\frac{dy}{dx} = 2\times4 - 2 - 12\times4^{-\frac{1}{2}} = ...$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x = 4$ into their $\frac{d^2y}{dx^2} = 2 + 6\times4^{-\frac{3}{2}} = (2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2} = 2.75 > 0$ and states "hence minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

(a)(i)

M1: Differentiates to $\frac{dy}{dx} = Ax + B + Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4} = ...$

Alternatively substitutes $x = 4$ into an equation resulting from $\frac{dy}{dx} = 0$ Eg. $\frac{36}{x} = (x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓, QED for a minimal conclusion

Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" oe

Alt Shows that $x = 4$ is a root of the resulting equation and states "hence there is a stationary point"

All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x = 4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x = 4 \Rightarrow \frac{d^2y}{dx^2} > 0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x = 4$ or calculates the value of y either side of $x = 4$.

A1ft: For **a correct calculation, a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x = 4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question T7_Q2

Question	Scheme	Marks	AOs
5	$\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{...}{........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\dfrac{dy}{d\theta}=\dfrac{3}{2+2\sin 2\theta}=\dfrac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\dfrac{dy}{d\theta}$ (condone it being stated as $\dfrac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\dfrac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\dfrac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\dfrac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\dfrac{dy}{d\theta}$ condoning it appearing as $\dfrac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{dy}{d\theta}=\dfrac{...}{........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\dfrac{3}{2}$ stated but allow for example $\dfrac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question T7_Q3

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	

Notes:
(a)
M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark
M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

Question T7_Q4

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q \frac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\frac{dy}{dx}+2y$ unless you see evidence that they have used the incorrect law vu'-uv'

A1: Fully correct derivative $2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

Allow attempts where candidates write $2xdx-2xdy-2ydx+6ydy=0$

but watch for students who write $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

M1: For a valid attempt at making $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. with two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ coming from $3y^2$ and $2xy$

Look for $(\ldots \pm \ldots)\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots\ldots$ It is implied by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y-2x}{6y-2x}$

This cannot be scored from attempts such as $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y + 6y$ which only has one correct term.

A1*: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y-x}{3y-x}$ with no errors or omissions.

The previous line $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y-2x}{6y-2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y - x = 0$ oe

M1: Attempts to find either the x or y coordinates of P and Q by solving their $y = \frac{1}{3}x$ with $x^2 - 2xy + 3y^2 = 50$ simultaneously. Allow for finding a quadratic equation in x or y and solving to find at least one value for x or y.

This may be awarded when candidates make the numerator = 0 ie using $y = x$

A1: $\Rightarrow x = (\pm)5\sqrt{3}$ OR $\Rightarrow y = (\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the y coordinate from their x (or vice versa)

This may also be scored following the numerator being set to 0 ie using $y = x$

A1: Deduces that $P = \left(-5\sqrt{3}, -\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be $x = \ldots$ $y = \ldots$

(c)

B1ft: Explains that this is where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and so you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution (or larger solution).

Allow a follow through for candidates who mix up parts (b) and (c)

Alternatively candidates could complete the square $(x-y)^2 + 2y^2 = 50$ and state that y would reach a maximum value when $x = y$ and choose the positive solution from $2y^2 = 50$

Question T7_Q5

Question	Scheme	Marks	AOs
9	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
	$\frac{\cos(\theta+h)-\cos\theta}{h}$	B1	2.1
	$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1	1.1b
		A1	1.1b
	$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
	As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
	so $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5
		(5)	
			(5 marks)

Notes for Question 9

B1:	Gives the correct fraction such as $\frac{\cos(\theta+h)-\cos\theta}{h}$ or $\frac{\cos(\theta+\delta\theta)-\cos\theta}{\delta\theta}$ Allow $\frac{\cos(\theta+h)-\cos\theta}{(\theta+h)-\theta}$ o.e. **Note:** $\cos(\theta+h)$ or $\cos(\theta+\delta\theta)$ may be expanded
M1:	Uses the compound angle formula for $\cos(\theta+h)$ to give $\cos\theta\cos h \pm \sin\theta\sin h$
A1:	Achieves $\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$ or equivalent
dM1:	**dependent on both the B and M marks being awarded** Complete attempt to apply the given limits to the gradient of their chord
Note:	They must isolate $\frac{\sin h}{h}$ and $\left(\frac{\cos h - 1}{h}\right)$, and replace $\frac{\sin h}{h}$ with 1 and replace $\left(\frac{\cos h - 1}{h}\right)$ with 0
A1*:	cso. Uses correct mathematical language of limiting arguments to prove $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Acceptable responses for the final A mark include: • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = \lim_{h\to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of chord tends to the gradient of the curve, so derivative is $-\sin\theta$ • Gradient of chord $= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$. As $h \to 0$, gradient of ***curve*** is $-\sin\theta$
Note:	Give final A0 for the following example which shows ***no limiting arguments***: when $h = 0$, $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta = -1\sin\theta + 0\cos\theta = -\sin\theta$
Note:	Do not allow the final A1 for stating $\frac{\sin h}{h} = 1$ or $\left(\frac{\cos h - 1}{h}\right) = 0$ and attempting to apply these
Note:	In this question $\delta\theta$ may be used in place of h
Note:	Condone $\mathrm{f}'(\theta)$ where $\mathrm{f}(\theta) = \cos\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ where $y = \cos\theta$ used in place of $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta)$

Notes for Question 9 Continued	
Note:	Condone x used in place of θ if this is done consistently
Note:	Give final A0 for • $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right) = -1\sin\theta + 0\cos\theta = -\sin\theta$ • $\frac{\mathrm{d}}{\mathrm{d}\theta} = ...$ • Defining $\mathrm{f}(x) = \cos\theta$ and applying $\mathrm{f}'(x) = ...$ • $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\theta)$
Note:	Give final A1 for a correct limiting argument in x, followed by $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$ e.g. $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cos x) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin x + \left(\frac{\cos h - 1}{h}\right)\cos x\right) = -1\sin x + 0\cos x = -\sin x$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\theta}(\cos\theta) = -\sin\theta$
Note:	Applying $h \to 0,\ \sin h \to h,\ \cos h \to 1$ to give e.g. $\lim_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \left(\frac{\cos\theta(1) - \sin\theta(h) - \cos\theta}{h}\right) = \frac{-\sin\theta(h)}{h} = -\sin\theta$ is final M0 A0 for incorrect application of limits
Note:	$\lim_{h \to 0}\left(\frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}\right) = \lim_{h \to 0}\left(-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta\right)$ $= \lim_{h \to 0}\left(-(1)\sin\theta + 0\cos\theta\right) = -\sin\theta$. So for not removing $\lim_{h \to 0}$ when the limit was taken is final A0
Note:	<u>**Alternative Method:**</u> Considers $\frac{\cos(\theta + h) - \cos(\theta - h)}{(\theta + h) - (\theta - h)}$ which simplifies to $\frac{-2\sin\theta\sin h}{2h}$

Question T7_Q6

Question	Scheme	Marks	AOs
11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\frac{B}{(x-3)}+\frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\frac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$f(x)=3+\frac{4}{(x-3)}-\frac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$f'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\frac{4}{(x-3)^2}-\frac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $f'(x)$ and as $(x-3)^2>0$ and $(1-2x)^2>0$, then $f'(x)=-(+\text{ve})-(+\text{ve})<0$, so $f(x)$ is a decreasing function	A1	2.4
		(3)	
		(7 marks)	

Notes for Question 11

(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for B and C. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for B and C
B1:	$A=3$
M1:	Attempts to find the value of either B or C from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: -6=-2A$; $x: 11=7A-2B+C$; constant: $1=-3A+B-3C$ **Way 1:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: -10=-2B+C$; constant: $10=B-3C$ **Way 2:** Substituting: $x=3: -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: 5=-\frac{5}{2}C \Rightarrow C=-2$

Note:	$A=3, B=4, C=-2$ from no working scores M1B1M1A1
Note:	The final A1 mark is effectively dependent upon both M marks

Notes for Question 11 Continued	
(a) ctd	
Note:	Writing $1+11x-6x^2 \equiv B(1-2x)+C(x-3) \Rightarrow B=4,\ C=-2$ will get 1st M0, 2nd M1, 1st A0
Note:	**Way 1**: You can imply a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ from seeing $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv \frac{A(1-2x)(x-3)+B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
Note:	**Way 2**: You can imply a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ from seeing $\frac{-10x+10}{(x-3)(1-2x)} \equiv \frac{B(1-2x)+C(x-3)}{(x-3)(1-2x)}$
(b)	
M1:	Differentiates to give $\{\mathrm{f}'(x)=\}\ \pm\lambda(x-3)^{-2} \pm\mu(1-2x)^{-2};\ \lambda,\ \mu \neq 0$
A1ft:	$\mathrm{f}'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$, which can be simplified or un-simplified
Note:	Allow A1ft for $\mathrm{f}'(x) = -(\text{their } B)(x-3)^{-2} + (2)(\text{their } C)(1-2x)^{-2}$; $(\text{their } B), (\text{their } C) \neq 0$
A1:	$\mathrm{f}'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$ or $\mathrm{f}'(x) = -\frac{4}{(x-3)^2} - \frac{4}{(1-2x)^2}$ **and** a correct explanation e.g. $\mathrm{f}'(x) = -(+\text{ ve}) - (+\text{ ve}) < 0$, so $\mathrm{f}(x)$ is a decreasing {function}
Note:	The final A mark can be scored in part (b) from an incorrect $A = ...$ or from $A=0$ or no value of A found in part (a)

<table>
<tr><th colspan="4">Notes for Question 11 Continued - Alternatives</th></tr>
<tr><td>(a)</td><td colspan="3"></td></tr>
<tr><td>Note:</td><td colspan="3">Be aware of the following alternative solutions, by initially dividing by $"(x-3)"$ or $"(1-2x)"$

• $\dfrac{1+11x-6x^2}{"(x-3)"(1-2x)} \equiv \dfrac{-6x-7}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)} \equiv 3 - \dfrac{10}{(1-2x)} - \dfrac{20}{(x-3)(1-2x)}$

$\dfrac{20}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 20 \equiv D(1-2x) + E(x-3) \Rightarrow D=-4,\ E=-8$

$\Rightarrow 3 - \dfrac{10}{(1-2x)} - \left(\dfrac{-4}{(x-3)} + \dfrac{-8}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$

• $\dfrac{1+11x-6x^2}{(x-3)"(1-2x)"} \equiv \dfrac{3x-4}{(x-3)} + \dfrac{5}{(x-3)(1-2x)} \equiv 3 + \dfrac{5}{(x-3)} + \dfrac{5}{(x-3)(1-2x)}$

$\dfrac{5}{(x-3)(1-2x)} \equiv \dfrac{D}{(x-3)} + \dfrac{E}{(1-2x)} \Rightarrow 5 \equiv D(1-2x) + E(x-3) \Rightarrow D=-1,\ E=-2$

$\Rightarrow 3 + \dfrac{5}{(x-3)} + \left(\dfrac{-1}{(x-3)} + \dfrac{-2}{(1-2x)}\right) \equiv 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)};\ A=3,\ B=4,\ C=-2$</td></tr>
<tr><td>(b)</td><td colspan="3"></td></tr>
<tr><td rowspan="4"></td><td colspan="3"><u>Alternative Method 1:</u></td></tr>
<tr><td colspan="3">$\mathrm{f}(x) = \dfrac{1+11x-6x^2}{(x-3)(1-2x)},\ x>3 \Rightarrow \mathrm{f}(x) = \dfrac{1+11x-6x^2}{-2x^2+7x-3};\ \left\{\begin{matrix} u=1+11x-6x^2 & v=-2x^2+7x-3 \\ u'=11-12x & v'=-4x+7 \end{matrix}\right\}$</td></tr>
<tr><td rowspan="2">$\mathrm{f}'(x) = \dfrac{(-2x^2+7x-3)(11-12x)-(1+11x-6x^2)(-4x+7)}{(-2x^2+7x-3)^2}$</td><td>Uses quotient rule to find $\mathrm{f}'(x)$</td><td>M1</td></tr>
<tr><td>Correct differentiation</td><td>A1</td></tr>
<tr><td></td><td colspan="2">$\mathrm{f}'(x) = \dfrac{-20((x-1)^2+1)}{(-2x^2+7x-3)^2}$ and a correct explanation,
e.g. $\mathrm{f}'(x) = -\dfrac{(+\text{ ve})}{(+\text{ ve})} < 0$, so $\mathrm{f}(x)$ is a decreasing {function}</td><td>A1</td></tr>
<tr><td rowspan="2"></td><td colspan="3"><u>Alternative Method 2:</u></td></tr>
<tr><td colspan="3">Allow M1A1A1 for the following solution:
Given $\mathrm{f}(x) = 3 + \dfrac{4}{(x-3)} - \dfrac{2}{(1-2x)} = 3 + \dfrac{4}{(x-3)} + \dfrac{2}{(2x-1)}$
as $\dfrac{4}{(x-3)}$ decreases when $x>3$ and $\dfrac{2}{(2x-1)}$ decreases when $x>3$
then $\mathrm{f}(x)$ is a decreasing {function}</td></tr>
</table>

Question T7_Q7

Question	Scheme	Marks	AOs
14	$N = \dfrac{900}{3+7e^{-0.25t}} = 900(3+7e^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\ \dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt} = -900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{= \dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{dN}{dt} = \dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt} = -900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{= \dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200} = \dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS} = \dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e., $\text{RHS} = \dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2} = \dfrac{1575e^{-0..25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt} = \dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N = 150$ (can be implied)	B1	2.2a
	so $150 = \dfrac{900}{3+7e^{-0.25T}} \Rightarrow e^{-0.25T} = \dfrac{3}{7}$	M1	3.4
	$T = -4\ln\left(\dfrac{3}{7}\right)$ or $T =$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
		(10 marks)	

Notes for Question 14	
14 (b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N} - 3}{7}$ into their $\frac{dN}{dt} = ...$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
14	$N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}} = 900(3+7\mathrm{e}^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\quad \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$		
(b) Way 3	$\displaystyle\int \frac{1}{N(300-N)}\,\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$	M1	2.1
	$\displaystyle\int \frac{1}{300}\left(\frac{1}{N}+\frac{1}{300-N}\right)\mathrm{d}N = \int \frac{1}{1200}\,\mathrm{d}t$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t\ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90 \Rightarrow\}\ c = \dfrac{1}{300}\ln(90) - \dfrac{1}{300}\ln(210) \Rightarrow c = \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\dfrac{1}{300}\ln N - \dfrac{1}{300}\ln(300-N) = \dfrac{1}{1200}t + \dfrac{1}{300}\ln\left(\dfrac{3}{7}\right)$ $\ln N - \ln(300-N) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right)$ $\ln\left(\dfrac{N}{300-N}\right) = \dfrac{1}{4}t + \ln\left(\dfrac{3}{7}\right) \Rightarrow \dfrac{N}{300-N} = \dfrac{3}{7}\mathrm{e}^{\frac{1}{4}t}$	dM1	2.1
	$7N = 3\mathrm{e}^{\frac{1}{4}t}(300-N) \Rightarrow 7N + 3N\mathrm{e}^{\frac{1}{4}t} = 900\mathrm{e}^{\frac{1}{4}t}$ $N(7+3\mathrm{e}^{\frac{1}{4}t}) = 900\mathrm{e}^{\frac{1}{4}t} \Rightarrow N = \dfrac{900\mathrm{e}^{\frac{1}{4}t}}{7+3\mathrm{e}^{\frac{1}{4}t}} \Rightarrow N = \dfrac{900}{3+7\mathrm{e}^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7\mathrm{e}^{-0.25t}) = 900 \Rightarrow \mathrm{e}^{-0.25t} = \dfrac{1}{7}\left(\dfrac{900}{N}-3\right) \Rightarrow \mathrm{e}^{-0.25t} = \dfrac{900-3N}{7N}$	M1	2.1
	$\Rightarrow t = -4\left(\ln(900-3N) - \ln(7N)\right)$ $\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = -4\left(\dfrac{-3}{900-3N} - \dfrac{7}{7N}\right)$	A1	1.1b
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{1}{300-N} + \dfrac{1}{N}\right) \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = 4\left(\dfrac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\dfrac{\mathrm{d}t}{\mathrm{d}N} = \left(\dfrac{1200}{N(300-N)}\right) \Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question 14 Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give $\ln$ terms $= kt\ \{+c\},\ k \neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N - \frac{1}{300}\ln(300-N) = \frac{1}{1200}t\ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0,\ N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t} = f(N);\ \lambda \neq 0$ or $\lambda e^{-\frac{1}{4}t} = f(N);\ \lambda \neq 0$
A1*:	Correct manipulation leading to $N = \frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN} = -4\left(\frac{-3}{900-3N} - \frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ *

Question T7_Q8

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{dy}{dx}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\frac{dy}{dx}=\frac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
		(5 marks)	

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\frac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{((x+1)^2)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$ but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

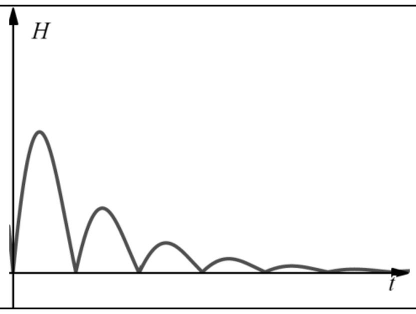
A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{\mathrm{d}y}{\mathrm{d}x}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
		(5 marks)	

Question T7_Q9

Question	Scheme	Marks	AOs
12 (a)	$f(x) = 10e^{-0.25x} \sin x$		
	$\Rightarrow f'(x) = -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x$ oe	M1 A1	1.1b 1.1b
	$f'(x) = 0 \Rightarrow -2.5\cancel{e^{-0.25x}} \sin x + 10\cancel{e^{-0.25x}} \cos x = 0$	M1	2.1
	$\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5} \Rightarrow \tan x = 4$ *	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \left\|10e^{-0.25\times 4.47} \sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...e^{-0.25x} \sin x \pm ...e^{-0.25x} \cos x$ M1

Sight of $vdu - udv$ however is M0

A1: $f'(x) = -2.5e^{-0.25x} \sin x + 10e^{-0.25x} \cos x$ which may be unsimplified

M1: For clear reasoning in setting their $f'(x) = 0$, factorising/ cancelling out the $e^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x = \arctan 4$ into $f'(x)$ to score this mark.

A1*: Shows the steps $\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4$ *. $\dfrac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt $76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change"

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Question T7_Q10

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x = 4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy} = k\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$ or $1 = k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = ...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8\sin y\cos y \Rightarrow \frac{dx}{dy} = \pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy} = 8\cos 2x \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x = 8y$ oe such as $x = 4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x = 4\sin 2y \rightarrow y = \ldots\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx} = \frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

Question T7_Q11

Question	Scheme	Marks	AOs
13 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \ \pi r h = \frac{6}{r} - \frac{2}{3}\pi r^2,\ \ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi r h + 2\pi r^2\ \{\Rightarrow A = 3\pi r^2 + 2\pi r h\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2\ \Rightarrow\ A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\}\ \frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{dA}{dr} = 0 \Rightarrow\right\}\ -\frac{12}{r^2} + \frac{10}{3}\pi r = 0\ \Rightarrow\ -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \ldots \left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736... \Rightarrow r =$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	(4)	
(c)	$A_{\min} = \frac{12}{(1.046...)} + \frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min} = 17.20...\ \Rightarrow\}\ A = 17\ (\text{m}^2)$ or $A =$ awrt 17 (m^2)	A1ft	1.1b
		(2)	
		(10 marks)	

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = \ldots$ or $\pi h = \ldots$ or $\pi r h = \ldots$ or $rh = \ldots$, where $'\ldots' = \text{f}(r)$ into an expression for the surface area which is of the form $A = \lambda \pi r^2 + \mu \pi r h;\ \lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\ 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = \ldots$ or $S = \ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{dA}{dr} =\right\}\ -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{dA}{dr} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \rightarrow r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r =$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Notes for Question 13 Continued	
Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$
(c)	
M1:	Substitutes their $r = 1.046...$, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ in part (b), into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their *r* which has been found from solving $\frac{\mathrm{d}^2A}{\mathrm{d}r^2} = 0$ or from using $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
A1ft:	$\{A =\}$ 17 or $\{A =\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of *r* for $0.6 \le$ their $r \le 1.3$ (and where their *r* has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ in part (b))

r	***A***	***A* (nearest integer)**
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Note:	Give M1 A1 for $A = 17\ (\mathrm{m}^2)$ or $A =$ awrt 17 (m^2) from no working

Question T7_Q12

Question	Scheme	Marks	AOs
9(a)	$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x}$		
	Differentiates to $\mathrm{e}^{-2x} \times 8x + 4(x^2 - 2) \times -2\mathrm{e}^{-2x}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x) = 8\mathrm{e}^{-2x}\{x - (x^2 - 2)\} = 8(2 + x - x^2)\mathrm{e}^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $\mathrm{f}'(x) = 0$ $x = -1, 2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $(-1, -4\mathrm{e}^2)$ and $(2, 8\mathrm{e}^{-4})$	A1	1.1b
		(3)	
(c)	(i) Range $[-8\mathrm{e}^2, \infty)$ o.e. such as $\mathrm{g}(x) \geqslant -8\mathrm{e}^2$	B1ft	2.5
	(ii) For • Either attempting to find $2\mathrm{f}(0) - 3 = 2 \times -8 - 3 = (-19)$ and identifying this as the **lower bound** • Or attempting to find $2 \times "8\mathrm{e}^{-4}" - 3$ and identifying this as the **upper bound**	M1	3.1a
	Range $[-19, 16\mathrm{e}^{-4} - 3]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $\mathrm{e}^{-2x} \to k\mathrm{e}^{-2x}, \quad k \neq 0$

If candidate states $u = 4(x^2 - 2), v = \mathrm{e}^{-2x}$ with $u' = ..., v' = ...\mathrm{e}^{-2x}$ it can be implied by their $vu' + uv'$

If they just write down an answer without working award for $\mathrm{f}'(x) = px\mathrm{e}^{-2x} \pm q(x^2 - 2)\mathrm{e}^{-2x}$

They may multiply out first $\mathrm{f}(x) = 4x^2\mathrm{e}^{-2x} - 8\mathrm{e}^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $\mathrm{f}(x) = \dfrac{u}{v} = \dfrac{4(x^2 - 2)}{\mathrm{e}^{2x}}$ with $v' = k\mathrm{e}^{2x}$ and $\mathrm{f}'(x) = \dfrac{vu' - uv'}{v^2}$

A1: A correct $\mathrm{f}'(x)$ which may be unsimplified.

Via the quotient rule you can award for $\mathrm{f}'(x) = \dfrac{8x\mathrm{e}^{2x} - 8(x^2 - 2)\mathrm{e}^{2x}}{\mathrm{e}^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $\mathrm{f}'(x)$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $\mathrm{f}'(x) = 8x\mathrm{e}^{-2x} - 8(x^2 - 2)\mathrm{e}^{-2x}$

to the given answer in one step.

Do not allow it from an **unsimplified** $f'(x) = 4 \times 2xe^{-2x} + 4(x^2 - 2) \times -2e^{-2x}$

Allow the expression / bracketed expression to be written in a different order.

So, for example, $8(x - x^2 + 2)e^{-2x}$ is OK

(b)

B1: States or implies $x = -1, 2$ (as the roots of $f'(x) = 0$)

M1: Substitutes one x value of their solution to $f'(x) = 0$ in f(x) to find a y value.

Allow decimals here (3sf). FYI, to 3 sf, $-4e^2 = -29.6$ and $8e^{-4} = 0.147$

Some candidates just write down the x coordinates but then go on in part (c) to find the ranges using the y coordinates. Allow this mark to be scored from work in part (c)

A1: Obtains $(-1, -4e^2)$ and $(2, 8e^{-4})$ as the stationary points. This must be scored in (b). Remember to isw after a correct answer. Allow these to be written separately. E.g. $x = -1,\ y = -4e^2$

Extra solutions, e.g. from $x = 0$ will be penalised on this mark.

(c)(i)

B1ft: For a correct range written using correct notation.

Follow through on 2 $\times$their minimum "y" value from part (b), providing it is negative.

Condone a decimal answer if this is consistent with their answer in (b) to 3sf or better.

Examples of correct responses are $[-8e^2, \infty)$, $g \geqslant -8e^2$, $y \geqslant -8e^2$, $\{q \in \mathbb{R}, q \geqslant -8e^2\}$

(c)(ii)

M1: See main scheme. Follow through on $2 \times$ their "$8e^{-4}$"$- 3$ for the upper bound.

A1: Range $[-19, 16e^{-4} - 3]$ o.e. such as $-19 \leqslant y \leqslant 16e^{-4} - 3$ but must be exact

Question T7_Q13

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2 \sec^2 y\dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18\times(x^4+81)-(-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
		(7 marks)	
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y\dfrac{dy}{dx}$ or $\cot y \to -\text{cosec}^2 y\dfrac{dy}{dx}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y\dfrac{dy}{dx} = ...$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x\tan y + x^2 \sec^2 y\dfrac{dy}{dx} = 0$

Allow also $\sec^2 y\dfrac{dy}{dx} = -\dfrac{18}{x^3}$ or $2x = -9\text{cosec}^2 y\dfrac{dy}{dx}$ amongst others

M1: Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$

(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu'-uv'}{v^2}$ with $u=-18x, v=x^4+81, u'=\pm18, v'=...x^3$

If no method is seen or implied award for $\frac{\pm18\times\left(x^4+81\right)\pm18x\left(ax^3\right)}{\left(x^4+81\right)^2}$

Using the product rule award for $\pm18\left(x^4+81\right)^{-1}\pm18x\left(x^4+81\right)^{-2}\times cx^3$

A1: Correct **simplified** $\frac{d^2y}{dx^2}=\frac{54\left(x^4-27\right)}{\left(x^4+81\right)^2}$ o.e. such as $\frac{d^2y}{dx^2}=\frac{54x^4-1458}{\left(x^4+81\right)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{d^2y}{dx^2}=0\Rightarrow x^4=27\Rightarrow x=\sqrt[4]{27}$

Or for substituting $x=\sqrt[4]{27}$ into an unsimplified but correct $\frac{d^2y}{dx^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.
See scheme.
It can be also be argued from $x^4<27$, $x^4=27$ and $x^4>27$ provided the conclusion states that the point of inflection is at $x=\sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{d^2y}{dx^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{d^3y}{dx^3}$ and showing that $\frac{d^3y}{dx^3}\neq0$ and $\frac{d^2y}{dx^2}=0$ when $x=\sqrt[4]{27}$

FYI $\frac{d^3y}{dx^3}=\frac{23328x^3}{\left(x^4+81\right)^3}=0.219$ when $x=\sqrt[4]{27}$

Alternative part (a) using arctan

M1: Sets $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times...$ where ... could be 1

A2: $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times-\frac{18}{x^3}$

A1*: $\frac{dy}{dx}=\frac{1}{1+\frac{81}{x^4}}\times-\frac{18}{x^3}=\frac{-18x}{x^4+1}$ showing correct intermediate step and no errors.

Question T7_Q14

Question	Scheme	Marks	AOs
13(a)	$k = e^2$ **or** $x \neq e^2$	B1	2.2a
		(1)	
(b)	$g'(x) = \dfrac{(\ln x - 2)\times\frac{3}{x} - (3\ln x - 7)\times\frac{1}{x}}{(\ln x-2)^2} = \dfrac{1}{x(\ln x-2)^2}$ or $g'(x) = \dfrac{d}{dx}\left(3-(\ln(x)-2)^{-1}\right) = (\ln x-2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x-2)^2}$ or $g'(x) = (\ln x-2)^{-1}\times\dfrac{3}{x} - (3\ln x-7)(\ln x-2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x-2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7$ … 0 or $\ln x - 2$ … 0 or $3\ln a - 7$ … 0 or $\ln a - 2$ … 0 where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x >$ … or $a >$ …	M1	3.1a
	$0 < a < e^2, \quad a > e^{\frac{7}{3}}$	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(a)

B1: Deduces $k = e^2$ **or** $x \neq e^2$ Condone k = awrt 7.39 or $x \neq$ awrt 7.39

(b)

M1: Attempts to differentiate via the quotient rule and with $\ln x \to \dfrac{1}{x}$ so allow for:

$$\frac{d}{dx}(g(x)) = \frac{(\ln x-2)\times\frac{\alpha}{x} - (3\ln x-7)\times\frac{\beta}{x}}{(\ln x-2)^2}, \quad \beta > 0$$

But a correct rule may be implied by their u, v, u', v' followed by applying $\dfrac{vu' - uv'}{v^2}$ etc.

Alternatively attempts to write $g(x) = \dfrac{3\ln(x)-7}{\ln(x)-2} = 3-(\ln(x)-2)^{-1}$ and attempts the chain rule so allow for:

$$3-(\ln(x)-2)^{-1} \to (\ln(x)-2)^{-2}\times\frac{\alpha}{x}$$

Alternatively writes $g(x) = (3\ln(x)-7)(\ln(x)-2)^{-1}$ and attempts the product rule so allow for:

$$g'(x) = (\ln x-2)^{-1}\times\frac{\alpha}{x} - (3\ln x-7)(\ln x-2)^{-2}\times\frac{\beta}{x}$$

In general condone missing brackets for the M mark. E.g. if they quote $u = 3\ln x - 7$ and $v = \ln x - 2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: $\dfrac{1}{x(\ln x-2)^2}$ Allow $\dfrac{\frac{1}{x}}{(\ln x-2)^2}$ **i.e. we need to see the numerator simplified to $1/x$**

Note that some candidates establish the correct numerator and correct denominator independently and provided they obtain the correct expressions, this mark can be awarded.

But allow a correctly expanded denominator.

A1cso: States that as $x > 0$ AND $\ln x - 2$ is squared so $g'(x) > 0$

(c)

M1: Attempts to solve either 3ln x – 7 = 0 or ln x – 2 = 0 or using inequalities e.g. 3ln x – 7 > 0

A1: $0 < a < e^2, \; a > e^{\frac{7}{3}}$

Question T7_Q15

Question	Scheme	Marks	AOs
14	$y=\dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y=\dfrac{t^2-4}{2+t}\Rightarrow\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}=\dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
14	$y=\dfrac{x-4}{2+\sqrt{x}}\Rightarrow y=\dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}}=\sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x}\Rightarrow y=\dfrac{t^2-4}{t+2}\Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question T7_Q16

Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = \ldots$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = \ldots$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0,$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = \ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = \ldots$ $\left(\frac{dy}{dx}\right)_{x=1.2} = \ldots$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0,$ $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)

M1: $x^n \to x^{n-1}$ for at least one power of x

A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of $(x - 1)$ or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.

A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)

M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "≠ 0" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Question T7_Q17

Question	Scheme	Marks	AOs
8(a)	$\frac{d}{dx}(3y^2)=6y\frac{dy}{dx}$ or $\frac{d}{dx}(qxy)=qx\frac{dy}{dx}+qy$	M1	2.1
	$3px^2+qx\frac{dy}{dx}+qy+6y\frac{dy}{dx}=0$	A1	1.1b
	$(qx+6y)\frac{dy}{dx}=-3px^2-qy \Rightarrow \frac{dy}{dx}=...$	dM1	2.1
	$\frac{dy}{dx}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3+q(-1)(-4)+3(-4)^2=26$	M1	1.1b
	$19x+26y+123=0 \Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2-q(-4)}{q(-1)+6(-4)}=\frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624 \Rightarrow p=...,q=...$	dM1	1.1b
	$p=2,\ q=-5$	A1	1.1b
		(5)	
			(9 marks)

Notes

(a)

M1: For selecting the appropriate method of differentiating:

Allow this mark for either $3y^2 \to \alpha y\frac{dy}{dx}$ or $qxy \to \alpha x\frac{dy}{dx}+\beta y$

A1: Fully correct differentiation. Ignore any spurious $\frac{dy}{dx}=...$

dM1: A valid attempt to make $\frac{dy}{dx}$ the subject with 2 terms only in $\frac{dy}{dx}$ coming from qxy and $3y^2$

Depends on the first method mark.

A1: Fully correct expression

(b)

M1: Uses $x=-1$ and $y=-4$ in the equation of C to obtain an equation in p and q

B1: Deduces the correct gradient of the given normal.

This may be implied by e.g.

$19x+26y+123=0 \Rightarrow y=-\frac{19}{26}x+... \Rightarrow$ Tangent equation is $y=\frac{26}{19}x+...$

M1: Fully correct strategy to establish an equation connecting p and q using $x=-1$ and $y=-4$ in their $\frac{dy}{dx}$ and the gradient of the normal. E.g. $(a)=-1\div \text{their}-\frac{19}{26}$ or $-1\div(a)=\text{their}-\frac{19}{26}$

dM1: Solves simultaneously to obtain values for p and q.

Depends on both previous method marks.

A1: Correct values

Note that in (b), attempts to form the equation of the normal in terms of p and q and then compare coefficients with $19x + 26y + 123 = 0$ score no marks. If there is any doubt use Review.

Question T7_Q18

Question	Scheme	Marks	AOs
13(a)	$y = \text{cosec}^3\theta \Rightarrow \dfrac{dy}{d\theta} = -3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \text{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \dfrac{1}{8} \Rightarrow \sin\theta = \dfrac{1}{2}$	M1	3.1a
	$\theta = \dfrac{\pi}{6} \Rightarrow \dfrac{dy}{dx} = \dfrac{-3\text{cosec}^3\left(\dfrac{\pi}{6}\right)\cot\left(\dfrac{\pi}{6}\right)}{2\cos\left(\dfrac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \dfrac{1}{2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\dfrac{-3}{\sin^3\theta}\times\dfrac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \dfrac{-3\times 8\times\dfrac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\dfrac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: Correct expression for $\dfrac{dy}{d\theta}$ seen or implied in any form e.g. $\dfrac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\dfrac{dx}{d\theta} = k\cos 2\theta$ or $\alpha\cos^2\theta + \beta\sin^2\theta$ (from product rule on $\sin\theta\cos\theta$)

and attempts $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$

A1: Correct expression in any form.

May see e.g. $\dfrac{-3\cos\theta}{2\sin^4\theta\cos 2\theta}$, $-\dfrac{3}{4\sin^4\theta\cos\theta - 2\sin^3\theta\tan\theta}$

(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when $y = 8$ and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta = \dfrac{1}{2}$ or e.g. $\theta = \dfrac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\dfrac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\dfrac{dy}{dx}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\dfrac{dy}{dx}$.

A1: Deduces the correct gradient