Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 5: Trigonometry

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 5: Trigonometry
  - Trigonometric identities and equations
  - Trigonometric functions and identities: area under a curve
  - Use of a trigonometric function

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T5_Q1

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

Question 1 continued

Question T5_Q2

3.

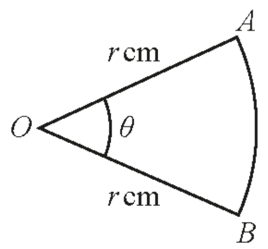


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

Question 3 continued

Question T5_Q3

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 8 continued

Question T5_Q4

7. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question 7 continued

Question 7 continued

Question 7 continued

Question T5_Q5

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

Question 12 continued

Question 12 continued

Question 12 continued

Question T5_Q6

2.

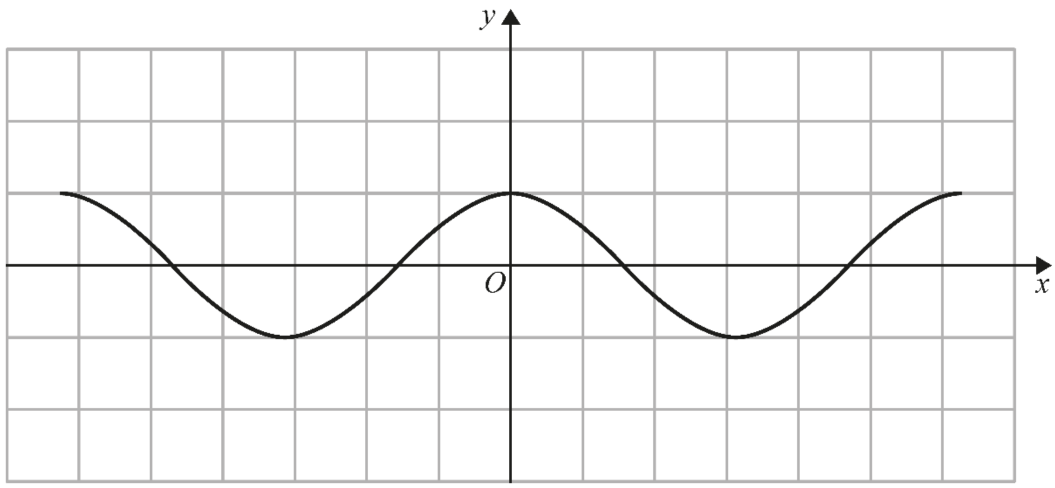


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

Question 2 continued

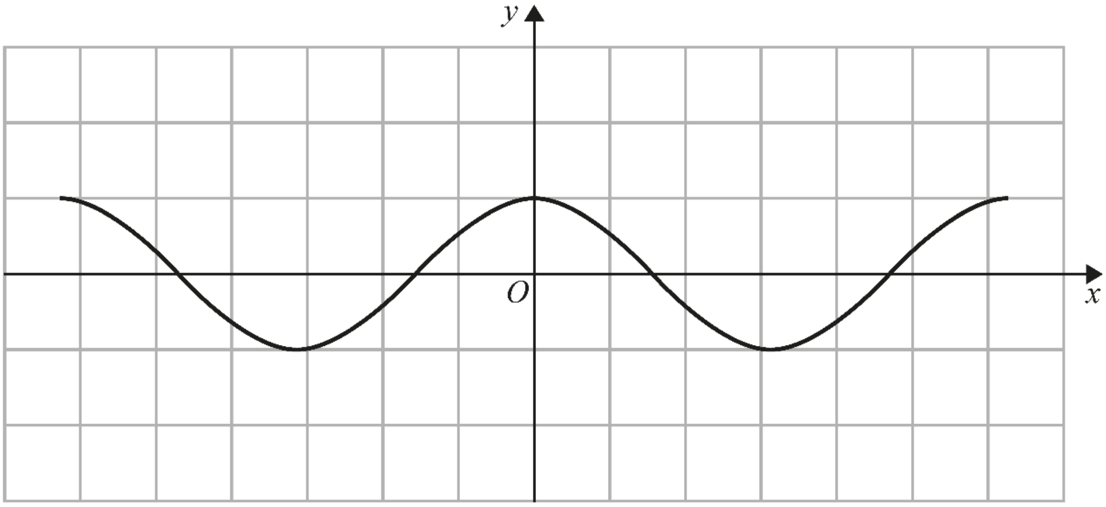


Diagram 1

Question T5_Q7

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

Question 6 continued

Question 6 continued

Question 6 continued

Question T5_Q8

3.

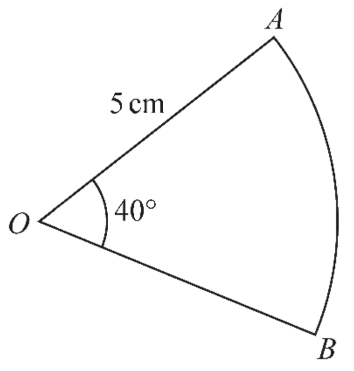


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O*, radius 5 cm and angle *AOB* = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

Question 3 continued

Question T5_Q9

4.

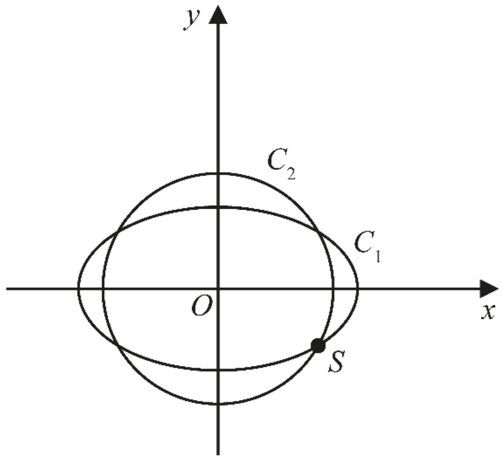


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

Question 4 continued

Question T5_Q10

12. (a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90^\circ < \theta < 180^\circ$, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

Question 12 continued

Question 12 continued

Question 12 continued

Question T5_Q11

6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

Question T5_Q12

12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec}x - \sin x = \cos x\cot(3x - 50^\circ)$$

(5)

Question 12 continued

Question 12 continued

Question 12 continued

Question T5_Q13

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

Question 10 continued

Question 10 continued

Question 10 continued

Question T5_Q14

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

Question T5_Q15

4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4\sin\frac{\theta}{2} + 3\cos^2\theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(3)

Question 4 continued

Question T5_Q16

6.

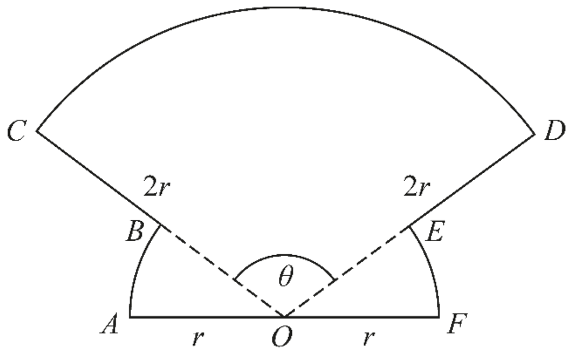


Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius $2r$
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

Question T5_Q17

15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

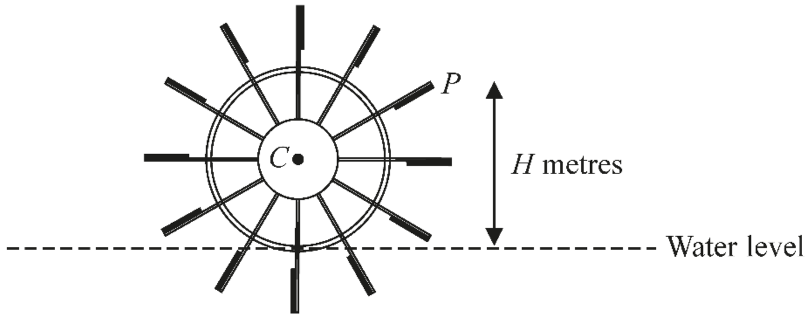


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

Question 15 continued

Question 15 continued

Question 15 continued

Question 15 continued

Mark Scheme

Question T5_Q1

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$\to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

....

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2\times(2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

Question T5_Q2

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $...r = ...r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

....

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

Question T5_Q3

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = \textit{awrt}\ 4.48\text{m}$. The units must be seen somewhere in (a). So allow when $D = 4.482.. = 4.5$ m

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k,\ |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{inv}\sin\ \textit{their} - 0.6$ to give the first value of t where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

Question T5_Q4

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1 - \sin^2 x) = 1$ | $16(1 - \cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}\ \left\{= \frac{2 \pm \sqrt{3}}{4}\right.$ or $\left. 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2 \pm \sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta - \alpha) = k,\ \|k\| < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \|k\| < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme		Marks	AOs
7	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2 + 5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1 - \cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ **or** e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1 - \sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \dfrac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \dfrac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\dfrac{-2 + \sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\dfrac{2 + \sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question 7

(i)	
B1:	For recalling that $\sec x = \dfrac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ and no other values in the range $0 \le x < \dfrac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \dfrac{1}{2} \Rightarrow$ any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°, 75°$, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}, 15°$ or $75°$ with no working

Notes for Question 7 Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy**: Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k \rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \leq \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working
Note:	<u>**Alternative solutions:**</u> (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\operatorname{cosec}\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\operatorname{cosec}\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\operatorname{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only

Question T5_Q5

Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x - 5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x - 5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x = 0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x - 3\tan x - 5)\tan x = 0$ **or** $(1+\tan^2 x - 3\tan x - 5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x - 5 = 3\tan x$	M1	2.1
	$\tan^2 x - 3\tan x - 4 = 0$	A1	1.1b
	$(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4}, 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

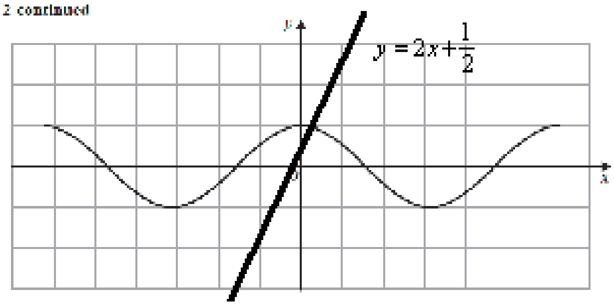
(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question 12 Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x=0$
M1:	For using the key step of $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Note: <u>**Alternative Method (Alt 1)**</u>

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$			
Deduces $x=0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x\cos x$ $1 - 5\left(\frac{1+\cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x+1.03) = -3$	Expresses their answer in the form $R\sin(2x+\alpha) = k;\ k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x+1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1 A1	1.1b 1.1b

Question T5_Q6

Question	Scheme	Marks	AOs
2(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2 + \sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2 + \sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

Question T5_Q7

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\dfrac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\dfrac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \dfrac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

Question T5_Q8

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2 \left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2 \left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
			(3 marks)

Notes for Question 3

(a)	
B1:	Explains that the formula use is only valid when angle AOB is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

Question T5_Q9

Question	Scheme		Marks	AOs
4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1 A1	2.1 1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \dfrac{1}{2}$ $\Rightarrow \sin t = \ldots$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \dfrac{1}{2}$ $\Rightarrow \cos t = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\dfrac{x}{10}\right)^2 + \left(\dfrac{y}{4\sqrt{2}}\right)^2 = 1\ \ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\dfrac{x^2}{100} + \dfrac{66 - x^2}{32} = 1$	$\dfrac{66 - y^2}{100} + \dfrac{y^2}{32} = 1$	M1 A1	2.1 1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = \ldots$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding *x*-coordinate or *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 3	$\{C_2 : x^2 + y^2 = 66 \Rightarrow\}\ \ x = \sqrt{66}\cos\alpha,\ \ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ \ 10\cos t = \sqrt{66}\cos\alpha,\ \ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\dfrac{10\cos t}{\sqrt{66}}\right)^2 + \left(\dfrac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100\left(\dfrac{1+\cos 2t}{2}\right) + 32\left(\dfrac{1-\cos 2t}{2}\right) = 66$		M1 A1	2.1 1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = \ldots$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the *x*-coordinate and value of the *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$			
				(6 marks)

Notes for Question 4

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t+\cos^2 t\equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: Note:	**dependent on both the previous M marks** Rearranges to make $\sin t=...$ where $-1\le\sin t\le 1$ or $\cos t=...$ where $-1\le\cos t\le 1$ Condone 3rd M1 for $\sin^2 t=\frac{1}{2}\Rightarrow\sin t=\frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S=(5\sqrt{2},-4)$ or $S=(\text{awrt } 7.07,-4)$

	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t+\sin^2 t\equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: Note:	**dependent on both the previous M marks** Rearranges to make $x=...$ or $y=...$ their x^2 or their y^2 must be >0 for this mark
M1: Note:	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S=(5\sqrt{2},-4)$ or $S=(\text{awrt } 7.07,-4)$ or $S=(\sqrt{50},-4)$ or $S=\left(\frac{10}{\sqrt{2}},-4\right)$

	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha+\sin^2\alpha\equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1

	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: Note:	Uses the identities $\cos 2t\equiv 2\cos^2 t-1$ and $\cos 2t\equiv 1-2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t\equiv 2\cos^2 t-1$ or $\cos 2t\equiv 1-2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t=...$ where $-1\le\cos 2t\le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S=(5\sqrt{2},-4)$ or $S=(\text{awrt } 7.07,-4)$ or $S=(\sqrt{50},-4)$ or $S=\left(\frac{10}{\sqrt{2}},-4\right)$

4	$C_1 : x = 10\cos t\,,\; y = 4\sqrt{2}\sin t,\;\; 0 \le t < 2\pi;\;\; C_2 : x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\; y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question T5_Q10

Question	Scheme	Marks	AOs
12	$\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS} =\}\ \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} =\}\ \dfrac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\sin\theta} + \dfrac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\cos\theta}$		
	$= \dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos 2\theta(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} =\}\ \dfrac{2\cos 2\theta}{\sin 2\theta} = \dfrac{2\cos(3\theta-\theta)}{\sin 2\theta} = \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$= \dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k;\ k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \dfrac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \dfrac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)} = 4 \Rightarrow 2(1-\tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta + 4\tan\theta - 1 = 0 \Rightarrow \tan\theta = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2 \pm \sqrt{5}\} \Rightarrow \tan\theta = k;\ k \neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2-\sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 12	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\frac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta} = \frac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \frac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=\frac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=\frac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\frac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=\frac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=\frac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\frac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \frac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \frac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\frac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta} = \frac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \frac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $=\frac{\cos 2\theta}{\sin\theta\cos\theta}=\frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\frac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\frac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \dfrac{1}{2}$
Note:	Condone 2nd M1 for applying $\dfrac{1}{2}\arctan\left(\dfrac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta = \text{awrt } 103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta = \text{awrt } 103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

Question T5_Q11

Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2} \Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha =$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24°C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $(5+"R")$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12}\pm 1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t\pm 63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2} \Rightarrow t=13.23=13:14$ scores M1 A1 A1

$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0 \Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2} \Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

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Question T5_Q12

Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec}x - \sin x = \cos x\cot(3x-50°)$ $\Rightarrow \cos x\cot x = \cos x\cot(3x-50°)$		
	$\cot x = \cot(3x-50°) \Rightarrow x = 3x-50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x-50°) \Rightarrow x+180° = 3x-50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in *x* (or other variable) instead of θ's here

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone θ's instead of *x*'s here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

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Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x\cot x = \cos x\cot(3x-50°) \Rightarrow \cot x = \cot(3x-50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta} = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{1}{\sin\theta} - \sin\theta = \text{cosec}\,\theta - \sin\theta$ *	A1*	2.1
		(3)	

Alt 2- Works on both sides

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ or $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$LHS = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\text{cosec}\,\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone = for ≡)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \dfrac{\cos x}{\sin x} = \dfrac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x-50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x-50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

Question T5_Q13

Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$	ddM1	2.1

	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = ...$	dM1	3.1a
	Two of $-90°, 0, 90°$, awrt $139°$	A1	1.1b
	All four of $-90°, 0, 90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of *x* rather than A

M1: Attempts to use the compound angle formula for cos(2*A* + *A*) or cos(*A* + 2*A*)
Condone a slip in sign

dM1: Uses correct double angle identities for cos 2*A* and sin 2*A*
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.

ddM1: Attempts to get all terms in terms of cos *A* using correct and appropriate identities.
Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$

$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in cos *x* using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in cos *x* and making a valid attempt to solve. This could include factorisation or division of a cos *x* term followed by an attempt to solve the 3 term quadratic equation in cos *x* to reach at least one non zero value for cos *x*.
May also be scored for solving the cubic equation in cos *x* to reach at least one non zero value for cos *x*.

A1: Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the <u>first</u> method mark.**

A1: All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1-\cos 3x=\sin^2 x \Rightarrow 1-\cos 3x=\frac{1}{2}\left(1-\cos 2x\right)$$
$$\Rightarrow 2-2\cos 3x=1-\cos 2x$$
$$\Rightarrow 1=2\cos 3x-\cos 2x$$
$$\Rightarrow 1=2\left(4\cos^3 x-3\cos x\right)-\left(2\cos^2 x-1\right)$$
$$\Rightarrow 0=4\cos^3 x-3\cos x-\cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos $2x$

Question T5_Q14

Question	Scheme	Marks	AOs
10(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x(1-3\cos 2x)=0$ $\Rightarrow(\sin 2x=0,)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
		(8 marks)	

Notes

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of $(\sin\theta + \cos\theta)$

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta\,\cancel{(\sin\theta+\cos\theta)}}{2\cos\theta\,\cancel{(\cos\theta+\sin\theta)}}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \dfrac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \dfrac{1}{3}$ or $\cos^2 x = \dfrac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°$, $35°$, $145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

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Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

Question T5_Q15

Question	Scheme	Marks	AOs
4	**Examples:** $4\sin\frac{\theta}{2}\approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta\approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta=3\left(1-\sin^2\theta\right)\approx 3\left(1-\theta^2\right)$ $3\cos^2\theta=3\frac{(\cos 2\theta+1)}{2}\approx\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta\approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta=4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right)\approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta=4\sin\frac{\theta}{2}+3\frac{(\cos 2\theta+1)}{2}\approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$=2\theta+3\left(1-\theta^2+\ldots\right)=3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

Notes

M1: Attempts to use at least one correct approximation **within the given expression**.

Either $\sin\frac{\theta}{2}\approx\frac{\theta}{2}$ or $\cos\theta\approx 1-\frac{\theta^2}{2}$ or e.g. $\sin\theta\approx\theta$ if they write $\cos^2\theta$ as $1-\sin^2\theta$ or e.g. $\cos 2\theta\approx 1-\frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2\theta$ as $\frac{1+\cos 2\theta}{2}$.

Allow sign slips only with any identities used but the appropriate approximations must be applied.

dM1: Attempts to use correct approximations **with the given expression** to obtain an expression in terms of θ only. **Depends on the first method mark.**

A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.

Question T5_Q16

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area $= 2\times\dfrac{1}{2}r^2\left(\dfrac{\pi-\theta}{2}\right)+\dfrac{1}{2}(2r)^2\theta$	M1	2.1
	$=\dfrac{1}{2}r^2\pi-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{3}{2}r^2\theta+\dfrac{1}{2}r^2\pi=\dfrac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter $= 4r+2r\left(\dfrac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
		(5 marks)	

Notes

(a)

B1: Deduces the correct expression for angle AOB

Note that $\dfrac{180-\theta}{2}$ scores B0

(b)

M1: Fully correct strategy for the area using their angle from (a) appropriately.

Need to see $2\times\dfrac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\dfrac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\dfrac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$

The first term must be seen expanded as e.g. $\dfrac{1}{2}r^2\pi-\dfrac{1}{2}r^2\theta$ or equivalent

(c)

M1: Fully correct strategy for the perimeter using their angle from (a) appropriately

Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ

A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\dfrac{180-\dfrac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\dfrac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times\pi r^2+\dfrac{\theta}{360}\times\dfrac{180}{\pi}\times\pi(2r)^2=\dfrac{1}{2}\pi r^2-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\dfrac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times2\pi r+\dfrac{180\theta}{\pi}\times\dfrac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$

Question T5_Q17

Question	Scheme	Marks	AOs
15(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	$t=11.6$	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	$=3.34$	A1	1.1b
		(4)	
(d)	e.g. the “3” would need to vary	B1	3.5c
		(1)	
		(11 marks)	

Notes

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$awrt 0.464

(b)(i)

B1ft: For $\left(3+2\sqrt{5}\right)$ m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow 3 + 2×Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

Alternative for (b):

$$H=3+4\cos(0.5t)-2\sin(0.5t)\Rightarrow\frac{dH}{dt}=-2\sin(0.5t)-\cos(0.5t)=0$$

$$\Rightarrow\tan(0.5t)=-\frac{1}{2}\Rightarrow 0.5t=2.677...,5.819...\Rightarrow t=5.36,\ 11.6$$

$$t=11.6\Rightarrow H=7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{dH}{dt}$ and attempts to solve $\frac{dH}{dt}=0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3+2"\sqrt{5}"\cos(...)=0$ and proceeds to $\cos(...)=k$ where $|k|<1$.

Allow e.g. $3+2"\sqrt{5}"\cos(...)<0$

dM1: Solves $\cos(0.5t\pm"0.464")=k$ where $|k|<1$ to obtain at least one value for t

This requires e.g. $2\left(\pi+\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi-\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when H = 0 and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

Examples:

Second time at water level – first time at water level:

$$2\left(\pi+\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)=7.02685...-3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)\right)=2(5.35589...-3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ which is not immediately obvious but may be seen as an overall method.

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

Special Cases in (c):

Note that if candidates have an incorrect α and have e.g. $3+2\sqrt{5}\cos(0.5t-0.464)$, this has no impact on the final answer. So for candidates using $3+2\sqrt{5}\cos(0.5t\pm\alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

Some values to look for:

$$0.5t\pm"0.464"=\pm2.306,\ \pm3.977,\ \pm8.598,\ \pm10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the "3" then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.