Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 3: Coordinate geometry in the (x,y) plane

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 3: Coordinate geometry in the (x,y) plane
  - The coordinate geometry of the circle

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T3_Q1

6.

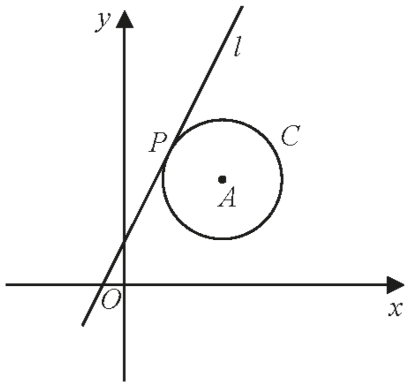


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

Question T3_Q2

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

Question 14 continued

Question 14 continued

Question T3_Q3

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

Question T3_Q4

11.

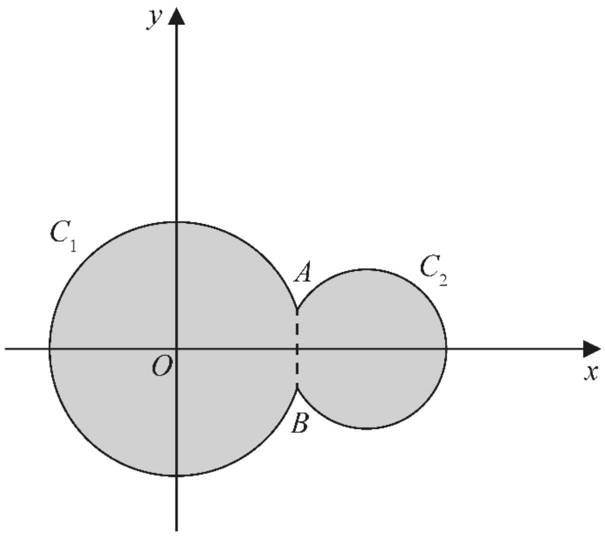


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

Question 11 continued

Question 11 continued

Question 11 continued

Question T3_Q5

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

Question 14 continued

Question 14 continued

Question 14 continued

Question T3_Q6

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

Question 7 continued

Question 7 continued

Question 7 continued

Mark Scheme

Question T3_Q1

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=\ldots$	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=\ldots$ to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=(\sqrt{20})^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or, **in the alternative**, for solving their $(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

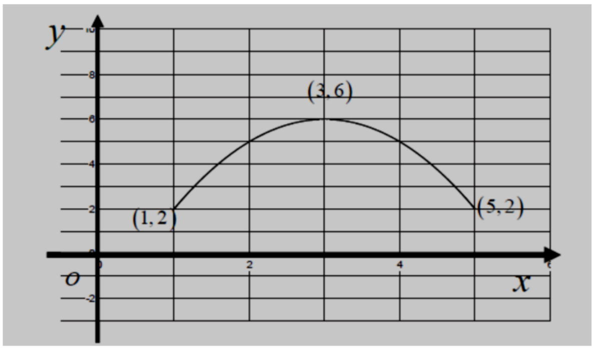
M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

....

Other method are possible using trigonometry.

Question T3_Q2

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)		M1	1.1b
	$\frown$ shaped parabola		
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \; 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)
M1: Uses $\cos 2t = 1 - 2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why C does not include all points of $y=6-(x-3)^2, \quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on x or y**.

For example

'As $-1\le \sin t \le 1$ then $1\leqslant x \leqslant 5$' Condone in words 'x lies between 1 and 5' and strict inequalities

'As $\sin t \le 1$ then $x\leqslant 5$' Condone in words 'x is less than 5'

'As $-1\le \cos(2t) \le 1$ then $2\leqslant y\leqslant 6$ ' Condone in words 'y lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant 5$"

Do not allow a statement on the top limit of y as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$
  $x+y=k \Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0 \Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$
- or deduces that $k<\dfrac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2 ... 4ac$ where ... is any inequality leading to a critical value for k. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\dfrac{37}{4}$

A1: Range of values for $k=\left\{k: 7\leqslant k<\dfrac{37}{4}\right\}$ Accept $k\in\left[7,\dfrac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1\Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of k. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for $k=\{k:7\leqslant k<9.25\}$	A1	2.5

Question T3_Q3

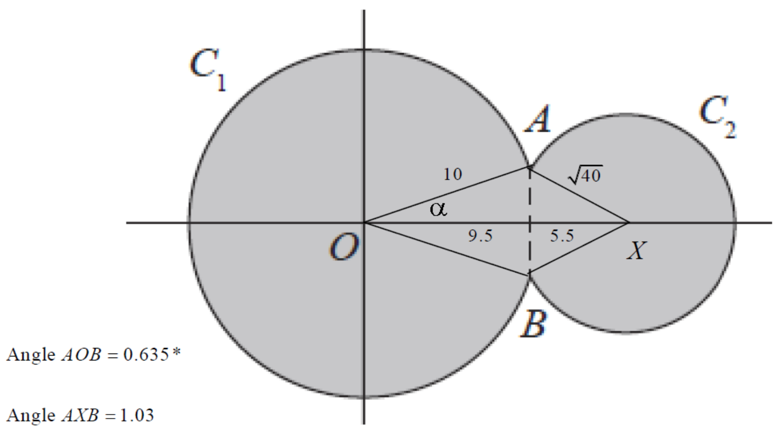
Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) **Way 1**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = \dots$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 2**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = \dots$ and $c = \dots$ e.g. $k = \dfrac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = \dots$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 3**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} (Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) **Way 1**	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = \dots$ (Condone $2x = 0.84x + 428 \Rightarrow x = \dots$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) **Way 2**	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss = 1.12} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit = 0.04} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points (800, their 1100) and (300, their 680)
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c$, $80 = 300k + c \Rightarrow k = \frac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	**<u>Special Case:</u>** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question T3_Q4

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2+y^2=100$ and $(x-15)^2+y^2=40$ simultaneously to find x or y E.g. $(x-15)^2+100-x^2=40 \Rightarrow x=...$	M1	3.1a
	Either $\Rightarrow -30x+325=40 \Rightarrow x=9.5$ Or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos\alpha=\dfrac{"9.5"}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2\times\arccos\left(\dfrac{9.5}{10}\right)=0.635$ rads (3sf) *	A1*	2.1
		(4)	
(b)	Attempts $10\times(2\pi-0.635) = 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos\beta=\dfrac{15-"9.5"}{\sqrt{40}} \Rightarrow \beta=...$ (Note $AXB=1.03$ rads)	M1	3.1a
	Attempts $10\times(2\pi-0.635)+\sqrt{40}\times(2\pi-2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
			(8 marks)
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.
Look for an attempt to set up an equation in a single variable leading to a value for x or y.

A1: $x=9.5$ (or $y=\dfrac{\sqrt{39}}{2}=$ awrt ± 3.12)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos\alpha = \frac{"9.5"}{10}$ to find α then $\times 2$

Alternatives include $\tan\alpha = \frac{\sqrt{100 - "9.5"^2}}{"9.5"} = (0.3286...)$ to find α then $\times 2$

And $\cos AOB = \frac{10^2 + 10^2 - \left(\sqrt{39}\right)^2}{2\times 10\times 10} = \frac{161}{200}$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin\alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt $36.4°$ (without the need to shown any calculation.)

E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{\underline{2\pi 10}} + 2\pi\sqrt{40}\underline{\underline{-10\times 0.635}}$... which is fine for this M1

M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

Amongst many other methods are $\tan\beta = \frac{"3.12"}{15 - 9.5}$ and $\cos AXB = \frac{40 + 40 - \left(\sqrt{39}\right)^2}{2\times\sqrt{40}\times\sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.
It is dependent upon having scored both M's.

A1: awrt 89.7

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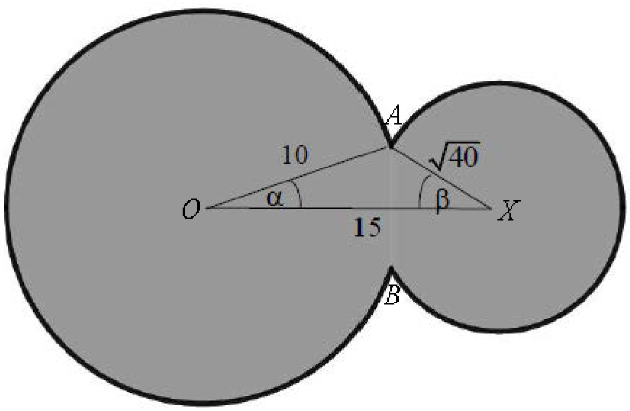

(a)

M1: For the key step in attempting to find all lengths in triangle OAX, condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

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Question T3_Q5

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or	M1	1.1b
	$y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$		
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
		(7 marks)	

Notes:

(a)

B1: Deduces the correct equation of the circle

M1: Attempts to form an equation with terms of the form x^2, x, r^2, and xr **only** using $y=12\pm2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2, y^2, r^2 such as $x^2+y^2=r^2$

If their circle equation starts off as e.g. $(x\pm a)^2+(y\pm b)^2=r^2$ then the B mark and the M mark can be awarded when the "a" and "b" are replaced by r or $-r$ as appropriate for their circle equation.

A1*: Uses correct and accurate algebra leading to the given solution.

(b)

M1: Attempts to use $b^2-4ac...0$ o.e. with $a=5, b=2r-48, c=r^2-24r+144$ and where ... is "=" or any inequality

Allow minor slips when copying the a, b and c provided it does not make the work easier and allow **their** a, b and c if they are similar expressions.

FYI $(2r-48)^2-4\times5\times(r^2-24r+144)=4r^2-192r+2304-20r^2+480r-2880=-16r^2+288r-576$

A1: Correct quadratic **equation** in r (or inequality). Terms need not be all one side but must be collected.

E.g. allow $r^2-18r=-36$ and allow any multiple of this equation (or inequality).

dM1: Correct attempt to solve their 3TQ in r. Dependent upon previous M

A1: Careful and accurate work leading to both answers in the required form (must be simplified surds)

Question T3_Q6

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=\ldots$	M1	1.1b
	$(5,\ -2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k\Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0\Rightarrow(6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0\Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2, (y\pm2)^2=\ldots$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2 . For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0\Rightarrow(x-a)^2+(y-b)^2=k\Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,\ -2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k. See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac \ldots 0$

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2+y^2-10x+4y+11=0\Rightarrow2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3\ \Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=\ldots$ or $y=\ldots$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x\Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms
A1: Correct differentiation.
M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.
M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's
A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k, x+3y=1$ $\Rightarrow x=\ldots$ and $y=\ldots$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10},\ y=\frac{k-3}{10},\ x^2+y^2-10x+4y+11=0 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3
M1 A1: Correct differentiation (See alt 1)
M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k
M1: Substitutes for x and y into C and solves resulting 3TQ in k
A1: Correct simplified values

(b) Alt 3	$y=3x+k \Rightarrow m=3 \Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2=18 \Rightarrow x=\ldots$ or $\Rightarrow 10(y+2)^2=18 \Rightarrow y=\ldots$	M1
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=\ldots$	M1
	$k=-17\pm6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius
A1: Correct equation of radial line passing through the centre of C
M1: Solves simultaneously to find x or y

Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.

M1: Applies $k = y - 3x$ with at least one pair of values to find k
A1: Correct simplified values