Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 2: Algebra and functions

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 2: Algebra and functions
  - The factor theorem
  - Understand and use graphs of functions
  - Use intersection points of graphs to solve equations
  - Transformations of a curve
  - Use of functions in modelling
  - The modulus of a linear function
  - Understand and use function notation

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T2_Q1

1.

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

Question 1 continued

Question T2_Q2

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

Question 6 continued

Question 6 continued

Question 6 continued

Question T2_Q3

8.

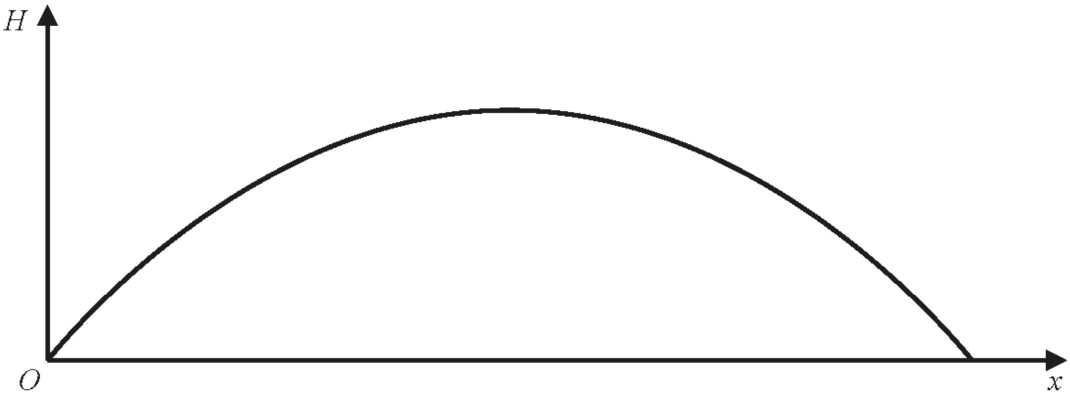


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

Question 8 continued

Question 8 continued

Question 8 continued

Question T2_Q4

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

Question 1 continued

Question T2_Q5

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

Question 5 continued

Question 5 continued

Question 5 continued

Question T2_Q6

Answer ALL questions. Write your answers in the spaces provided.

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

Question 1 continued

Question T2_Q7

6.

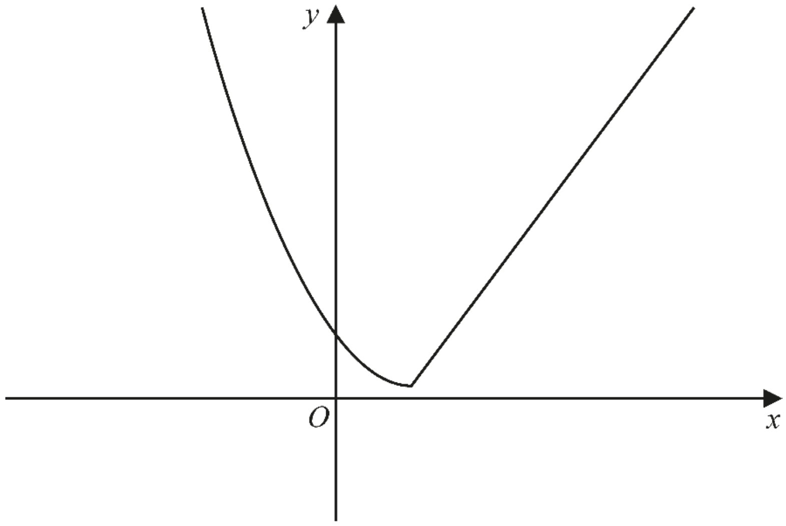


Figure 4

Figure 4 shows a sketch of the graph of $y = \text{g}(x)$, where

$$\text{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$\text{g}(x) > 28$$

(4)

The function h is defined by

$$\text{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\text{h}^{-1}(x) = -\frac{1}{2}$$

(3)

Question 6 continued

Question 6 continued

Question 6 continued

Question T2_Q8

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \frac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

Question 4 continued

Question T2_Q9

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Question 5 continued

Question T2_Q10

11.

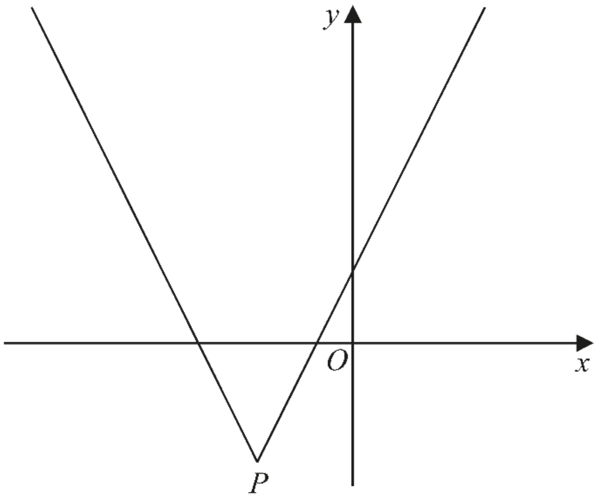


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Question 11 continued

Question 11 continued

Question 11 continued

Question T2_Q11

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

Question 1 continued

Question T2_Q12

12.

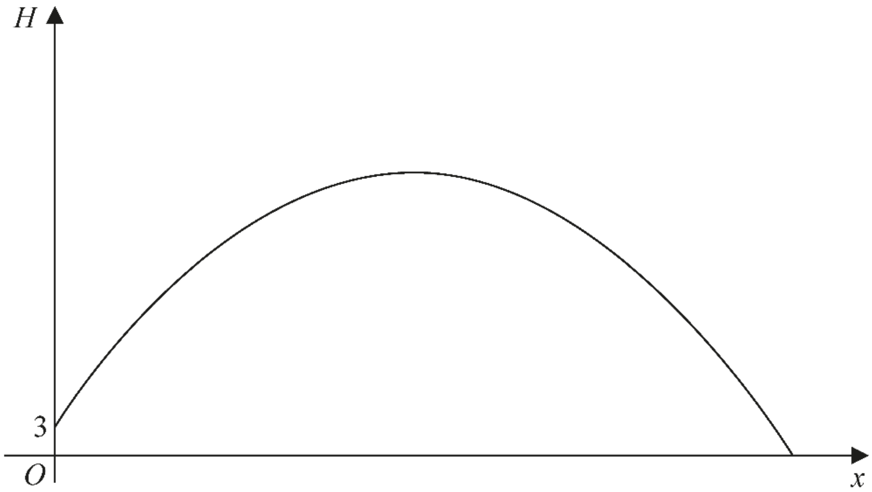


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

Question 12 continued

Question 12 continued

Question 12 continued

Question T2_Q13

2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

Question 2 continued

Question T2_Q14

11.

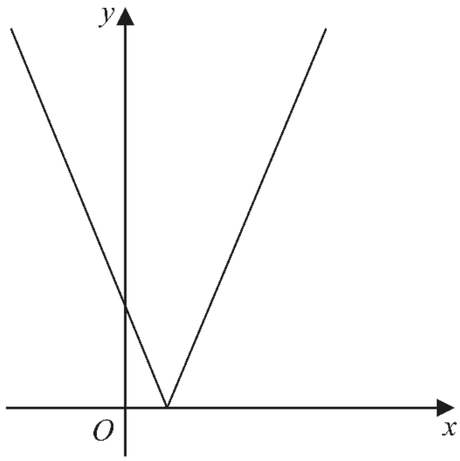


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

Question 11 continued

Question 11 continued

Question 11 continued

Mark Scheme

Question T2_Q1

Question	Scheme	Marks	AOs
1	$\mathrm{g}(x) = \dfrac{2x+5}{x-3}, \ x \geq 5$		
(a) **Way 1**	$\mathrm{g}(5) = \dfrac{2(5)+5}{5-3} = 7.5 \Rightarrow \mathrm{gg}(5) = \dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9} \ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$\mathrm{gg}(x) = \dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow \mathrm{gg}(5) = \dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9} \ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \dfrac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y = \dfrac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \dfrac{3y+5}{y-2} \ \left\{\text{or } y = \dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{3x+5}{x-2}, \quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y = \dfrac{2x-6+11}{x-3} \Rightarrow y = 2 + \dfrac{11}{x-3} \Rightarrow y - 2 = \dfrac{11}{x-3}$	M1	1.1b
	$x - 3 = \dfrac{11}{y-2} \Rightarrow x = \dfrac{11}{y-2} + 3 \ \left\{\text{or } y = \dfrac{11}{x-2} + 3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{11}{x-2} + 3, \quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question 1

(a)	
M1:	Full method of attempting $\mathrm{g}(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x = 5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $\mathrm{gg}(x) = \dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Notes for Question 1 Continued	
(b)	
B1:	States $2 < y \leq \frac{15}{2}$ Accept any of $2 < \mathrm{g} \leq \frac{15}{2}$, $2 < \mathrm{g}(x) \leq \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $\mathrm{g}(x) > 2$ **and** $\mathrm{g}(x) \leq \frac{15}{2}$ o.e.
(c) **Way 1**	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in *x* or terms in a swapped *y*
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1}, where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $\mathrm{g}^{-1} : x \rightarrow$. Condone $\mathrm{g}^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $\mathrm{g}^{-1}(x)$. Allow $2 < x \leq \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < \mathrm{g} \leq \frac{15}{2}$, $2 < \mathrm{g}^{-1}(x) \leq \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) **Way 2**	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \neq 0$ and rearranges to isolate *y* and 2 on one side of their equation. **Note:** Allow the equivalent method with *x* swapped with *y*
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Question T2_Q2

Question	Scheme	Marks	AOs
6	(a) $\mathrm{f}(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \mathrm{f}(2) = 0$	B1	1.1b
	(ii) $\{\mathrm{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 6

(a)(i)	
B1:	$\mathrm{f}(2) = 0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + ...,\ k = \text{value} \neq 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta = \text{value} \neq 0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c),\ k = \text{value} \neq 0,$ c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta),\ \beta = \text{value} \neq 0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2 + 2x - 5),\ (2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5),\ 4 - 60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions • or for the omission of < 0
Note:	< 0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \dfrac{-2 \pm \sqrt{-56}}{-6}$ or $\dfrac{-1 \pm \sqrt{-14}}{-3}$

Notes for Question 6 Continued	
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\frac{2}{3}x+\frac{5}{3}=0 \Rightarrow \left(x-\frac{1}{3}\right)^2-\frac{1}{9}+\frac{5}{3}=0 \Rightarrow x=\frac{1}{3}\pm\sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta =$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

Question T2_Q3

Question	Scheme	Marks	AOs
8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \dfrac{3}{100}$	dM1	3.1b
	$H = \dfrac{3}{100}x(40-x)$ or $H = -\dfrac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \dfrac{3}{100}$	dM1	3.1b
	$H = 12 - \dfrac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\dfrac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}\ 3 = \dfrac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H = 3 \Rightarrow\}\ 3 = 12 - \dfrac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \dfrac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20 + \sqrt{300} \Rightarrow\right\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12 - \lambda(x-20)^2$ or $H = 12 + \lambda(x-20)^2$
	Way 3: Writes $H = ax^2 + bx + c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking H to x E.g. $H = \frac{3}{100}x(40-x)$, $H = 12 - \frac{3}{100}(x-20)^2$ or $H = -0.03x^2 + 1.2x$
Note:	Condone writing y in place of H for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2 + 1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12-(x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2 + 1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied $ax^2 + bx$ or $ax^2 + bx + c$ (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta;\ \alpha, \beta \neq 0$
Note:	E.g. $1.2x - 0.03x^2 = 3$ or $40x - x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for (their A)$x^2 = 3 \Rightarrow x = ...$ or their (their A)x^2 + (their k) $= 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20 + \sqrt{300}\right)$ m or $\left(20 + 10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • H (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Question T2_Q4

Question	Scheme	Marks	AOs
1	Attempts $f(-3) = 3\times(-3)^3 + 2a\times(-3)^2 - 4\times -3 + 5a = 0$	M1	3.1a
	Solves linear equation $23a = 69 \Rightarrow a = ...$	M1	1.1b
	$a = 3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3) = 0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3) = 0$ method

$$\begin{array}{r} 3x^2 + (2a-9)x + 23 - 6a \\ x+3\,\overline{)\,3x^3 + 2ax^2 - 4x + 5a} \\ \underline{3x^3 + 9x^2} \\ (2a-9)x^2 - 4x \\ \underline{(2a-9)x^2 + (6a-27)x} \\ (23-6a)x + 5a \\ (23-6a)x + 69 - 18a \end{array}$$

So accept $5a = 69 - 18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a - 27 + \frac{5a}{3} = -4$$

M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm 3) = 0$. Allow for a linear equation in a leading to $a = ...$. Don't be too concerned with the mechanics of this.

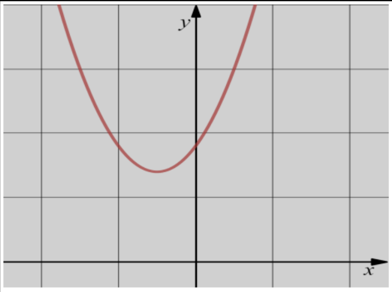
Via division accept $x+3\,\overline{)\,3x^3 + 2ax^2 - 4x + 5a}$ with quotient $3x^2 ...$ followed by a remainder in a set $= 0 \Rightarrow a = ...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a + 12 - 81$ oe

A1: $a = 3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question T2_Q5

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&\, b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix}2\\-4\end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $\mathrm{g}(x)=\mathrm{f}(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $\mathrm{f}(x)\to\mathrm{g}(x)$ is $2x^2+4x+9\to2(x-2)^2+4(x-2)+5$ So $\mathrm{g}(x)=\mathrm{f}(x-2)-4$
- $\mathrm{g}(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=\mathrm{g}(x)$ and compares to the sketch of $y=\mathrm{f}(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix}2\\-4\end{pmatrix}$', Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix}0\\-4\end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix}-2\\4\end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $\mathrm{h}(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=\mathrm{h}(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<\mathrm{h}(x)\leqslant3$ Allow for $0<\mathrm{h}\leqslant3$ $(0,3]$ and $0<y\leqslant3$ but not $0<x\leqslant3$

Follow through on their $a(x+b)^2+c$ so award for $0<\mathrm{h}(x)\leqslant\dfrac{21}{c}$

Question T2_Q6

Question	Scheme	Marks	AOs
1	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\frac{1}{2^x\, 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^{y} = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 1	
	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain y written as a function of x.
A1:	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \frac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where y is written as a function of x
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \frac{1}{2\sqrt{2}} \Rightarrow 4^y = \frac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\frac{1}{2^x\, 2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

Question T2_Q7

Question	Scheme	Marks	AOs
6 (a)	$gg(0) = g((0-2)^2+1) = g(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{h^{-1}(x)=-\frac{1}{2} \Rightarrow\right\}$ $x=h\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $h^{-1}(x)$ } $=\pm 2\pm\sqrt{x\pm 1}$	M1	1.1b
	Attempts to solve $\pm 2\pm\sqrt{x\pm 1}=-\frac{1}{2} \Rightarrow \pm\sqrt{x\pm 1}=...$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
			(10 marks)

Notes for Question 6

(a)	
M1:	Uses a complete method to find $gg(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $g(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$gg(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x\in\mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Notes for Question 6 Continued	
(b)	***continued***
Note:	Give final A0 for $\{ x \in \mathbb{R} : x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \mathrm{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \mathrm{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\mathrm{h}^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

Question T2_Q8

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*\_*}{*\_*}\right)-a}{\left(\frac{*\_*}{*\_*}\right)-b}$ where a and b are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Question T2_Q9

Question	Scheme	Marks	AOs
5	$15-2^{x+1}=3\times 2^x$	B1	1.1b
	$\Rightarrow 15-2\times 2^x=3\times 2^x \Rightarrow 2^x=3$ or e.g. $\Rightarrow \dfrac{15}{2^x}-2=3 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^x \Rightarrow 2^x=\dfrac{y}{3} \Rightarrow y=15-2\times\dfrac{y}{3}$	B1	1.1b
	$3y+2y=45 \Rightarrow y=9 \Rightarrow 3\times 2^x=9 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $15-2^{x+1}=3\times 2^x$ or equivalent e.g. $15-2^{x+1}-3\times 2^x=0$

M1: Uses $2^{x+1}=2\times 2^x$ oe e.g. $\dfrac{2^{x+1}}{2^x}=2$ to obtain an equation in 2^x and attempts to make 2^x the subject.

See scheme but e.g. $y=2^x \Rightarrow 3\times 2^x=15-2^{x+1} \Rightarrow 3y=15-2y \Rightarrow y=...$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x=3$ so $x=1.584$.. but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Alternative

B1: Correct equation in y

M1: Solves their equation in y and attempts to make 2^x the subject.

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x=3$ so $x=1.584$.. but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Question T2_Q10

Question	Scheme	Marks	AOs
11(a)	$x = -4$ **or** $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x+4) - 5 \Rightarrow x = \ldots$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
			(7 marks)

Notes:

(a)

B1: One correct coordinate. Either $x = -4$ or $y = -5$ or $(-4, \ldots)$ or $(\ldots, -5)$ seen.

B1: Deduces that $P(-4, -5)$ Accept written separately e.g. $x = -4,\ y = -5$

(b)

M1: Attempts to solve $3x + 40 = -2(x+4) - 5 \Rightarrow x = \ldots$ Must reach a value for x.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x = -10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x = -37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find y.

Alternative by squaring:

$$3x + 40 = 2|x+4| - 5 \Rightarrow 3x + 45 = 2|x+4| \Rightarrow 9x^2 + 270x + 2025 = 4\left(x^2 + 8x + 16\right)$$

$$\Rightarrow 5x^2 + 238x + 1961 = 0 \Rightarrow x = -37, -\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(c)

B1: Deduces that $a > 2$

M1: Attempts to find a value for a using their $P(-4, -5)$

Alternatively attempts to solve $ax = 2(x+4) - 5$ and $ax = 2(x+4) - 5$ to obtain a value for a.

A1: Correct range in acceptable set notation.

Examples:

$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$

$\{a : a \leqslant 1.25\},\ \{a : a > 2\}$

$\{a : a \leqslant 1.25 \text{ or } a > 2\}$

$\{a : a \leqslant 1.25,\ a > 2\}$

$(-\infty, 1.25] \cup (2, \infty)$

$(-\infty, 1.25],\ (2, \infty)$

Question T2_Q11

Question	Scheme	Marks	AOs
1	$f(1)=a(1)^3+10(1)^2-3a(1)-4=0$	M1	3.1a
	$6-2a=0\Rightarrow a=...$	M1	1.1b
	$a=3$	A1	1.1b
		(3)	
			(3 marks)
Notes			

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a+10-3a-4=0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a\pm 10\pm 3a\pm 4=0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

………

Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a=3$, $f(x)=3x^3+10x^2-9x-4$ and shows that $(x-1)$ is a factor of this $f(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3+10x^2-9x-4=(x-1)(3x^2+13x+4)$ Hence $a=3$

E.g. 2: $f(x)=3x^3+10x^2-9x-4$, $f(1)=3+10-9-4=0$ Hence $a=3$

The solutions via this method must end with the value for a to score the A1

....

Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

Alt (1) by inspection which may be seen in a table/;

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$ and sets terms in x equal

$-3a=-(10+a)+4 \Rightarrow 2a=6 \Rightarrow a=3$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

....

Alt (2) By division:

$$\begin{array}{r|l} & ax^2+(\pm 10\pm a)x+(10-2a) \\ x-1 & ax^3+10x^2 \quad -3ax-4 \\ & ax^3-ax^2 \\ \hline & (10+a)x^2-3ax \\ & (10+a)x^2-(10+a)x \\ \hline & (-2a+10)x \end{array}$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm 10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

Question T2_Q12

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\text{(m)}$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • H is not likely to be a quadratic function in x	B1	3.5b
		(1)	
		(9 marks)	

Notes

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model
Either uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)
B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x=90$

So award M1 for $H=\pm a(x-90)^2+c$ or $H=\pm a(90-x)^2+c$. Condone for this mark an equation with $a=1 \Rightarrow H=(x-90)^2+c$ or $c=3 \Rightarrow H=a(x-90)^2+3$ but will score little else

Alt (a)	$H=a(x+b)^2+c$ and $x=90$ at $H_{max} \Rightarrow H=a(x-90)^2+c$	M1	3.3
	$H=3$ when $x=0 \Rightarrow 3=8100a+c$ **or** $H=27$ when $x=120 \Rightarrow 27=900a+c$	M1 A1	3.1b 1.1b
	$H=3$ when $x=0 \Rightarrow 3=8100a+c$ **and** $H=27$ when $x=120 \Rightarrow 27=900a+c$ $\Rightarrow a=...,c=...$	dM1	3.1b
	$H=-\frac{1}{300}(x-90)^2+30$ o.e	A1	1.1b
		(5)	
(b)	$x=90 \Rightarrow H=0^2+30=30\text{m}$	B1	3.4
		(1)	
	$H=0 \Rightarrow 0=-\frac{1}{300}(x-90)^2+30 \Rightarrow x=...$	M1	3.4
	$\Rightarrow x=185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H=-\frac{1}{300}(x-90)^2+30$ is equivalent to $H=-\frac{1}{300}(90-x)^2+30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H=a(x-60)(x-120)+b$ leads to $H=-\frac{1}{300}(x-60)(x-120)+27$

Question T2_Q13

Question	Scheme	Marks	AOs
2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\mathrm{f}(1.8) = 7 - 2\times1.8^2 = 0.52 \Rightarrow \mathrm{gf}(1.8) = \mathrm{g}(0.52) = \dfrac{3\times0.52}{5\times0.52-1} = ...$	M1	1.1b
	$\mathrm{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\mathrm{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
			(5 marks)

Notes

(a)

B1: Correct range. Allow f(x) or f for y. Allow e.g. $\{y \in \mathbb{R} : y \leqslant 7\}$, $-\infty < y \leqslant 7$, $(-\infty, 7]$

(b)

M1: Full method to find f(1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute x = 1.8 into an attempt at gf(x).

E.g. $\mathrm{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3(7-2(1.8)^2)}{5(7-2\times(1.8)^2)-1} = ...$

A1: Correct value

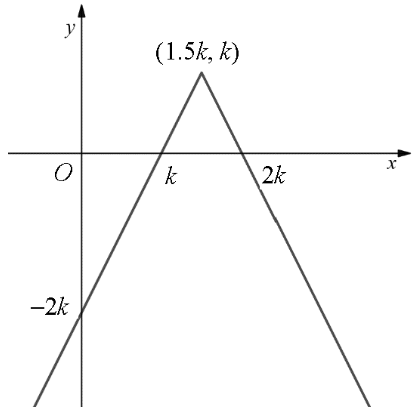
(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}$, $\dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

Question T2_Q14

Question	Scheme	Marks	AOs
11(a)			
	∧ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a ∧ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x = ...$	M1	3.1a
	$x=\dfrac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x < \dfrac{5k}{3}\right\} \cap \{x : x > k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes

(a) **Note that the sketch may be seen on Figure 4**
B1: See scheme
B1: Correct x-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places.
B1: Correct y-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$.
B1: Correct coordinates as shown
Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0$, $x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.
(b)
B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$
M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = \ldots$ or $x = \ldots$ as long as they are solving the required equation.
A1: Correct value
A1: Correct answer using the correct set notation.

Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x :\ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "|" for " :"

But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\quad\left\{x :\ k < x,\ x < \frac{5k}{3}\right\}$ scores A0

(c)

B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..,\ y = \ldots$

B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..,\ y = \ldots$

If coordinates are given the wrong way round and not seen correctly as $x = \ldots,\ y = \ldots$

e.g. $(3 - 5k, 3k)$ this is B0B0

Alternative to part (b) by squaring:

$$k - |2x - 3k| = x - k \Rightarrow |2x - 3k| = 2k - x$$

$$4x^2 - 12kx + 9k^2 = 4k^2 - 4kx + x^2 \Rightarrow 3x^2 - 8kx + 5k^2 = 0$$

$$(3x - 5k)(x - k) = 0 \Rightarrow x = \frac{5k}{3},\ k$$

Score M1 for isolating the $|2x - 3k|$, squaring both sides to obtain 3 appropriate terms for each side, collects terms to obtain $Ax^2 + Bkx + Ck^2 = 0$ and solves for x

A1 for $x = \frac{5k}{3}$ and B1 for $x = k$

Then A1 as in the scheme.