Pearson Edexcel

Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 3 – Mechanics

Topic 5: Moments

Contents

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General guidance to Topic Tests

Context

- Topic Tests have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidates.

Purpose

- The purpose of this resource is to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the advance information for the subject as well as general marking guidance for the qualification (available in published mark schemes).

Revise Revision Guide content coverage

The questions in this topic test have been taken from past papers, and have been selected as they cover the topic(s) most closely aligned to the A level advance information for summer 2022:

- Topic 5: Moments and Quantities and units in mechanics
  - Statics, moments, resolving forces, friction

The focus of content in this topic test can be found in the Revise Pearson Edexcel A level Mathematics Revision Guide. Free access to this Revise Guide is available for front of class use, to support your students' revision.

Contents	Revise Guide page reference	Level
Pure Mathematics	1-111	A level
Statistics	112-147	A level
Mechanics	148-181	A level

Content on other pages may also be useful, including for synoptic questions which bring together learning from across the specification.

Questions

Question T5_Q1

7.

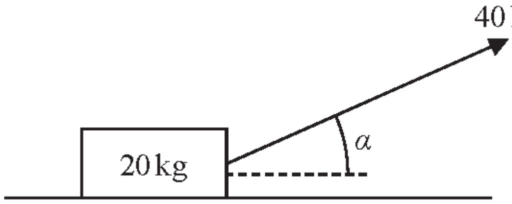


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

Question T5_Q2

9.

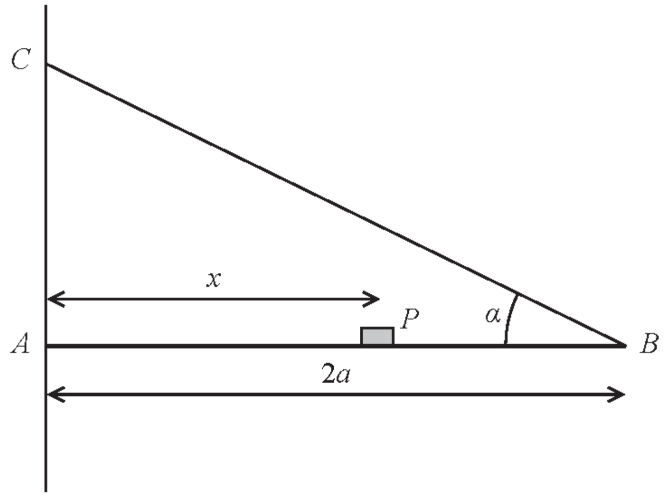


Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

Question 9 continued

Question 9 continued

Question 9 continued

Question T5_Q3

4.

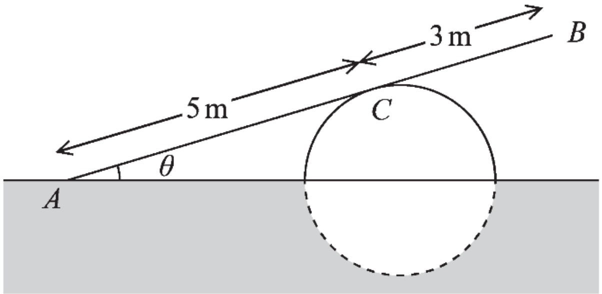


Figure 2

A ramp, *AB*, of length 8 m and mass 20 kg, rests in equilibrium with the end *A* on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as *A*.

The point of contact between the ramp and the drum is *C*, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point *C* acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at *A*.

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to *A* than to *B*,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at *C*.

(1)

Question 4 continued

Question 4 continued

Question 4 continued

Question T5_Q4

4.

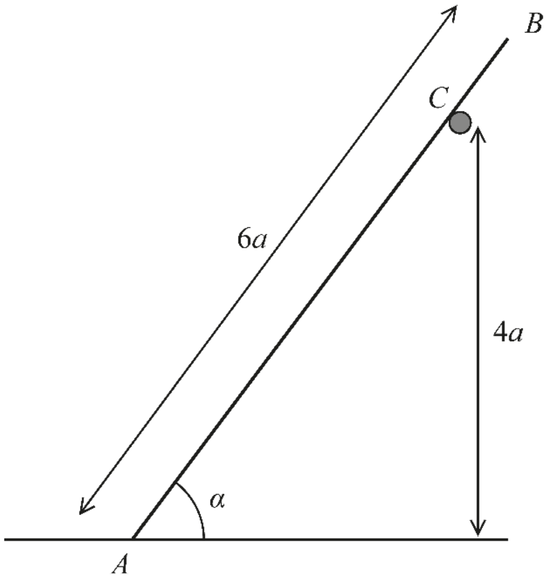


Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

Question 4 continued

Question 4 continued

Question 4 continued

Question T5_Q5

3.

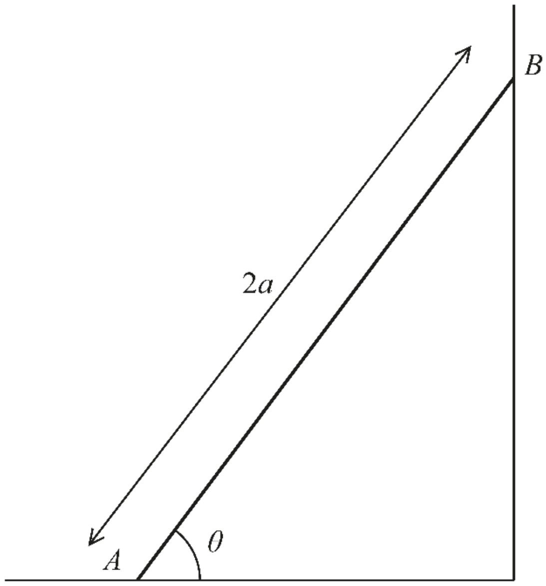


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

Mark Scheme

Question T5_Q1

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation. Neither g nor $\sin\alpha$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos\alpha$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

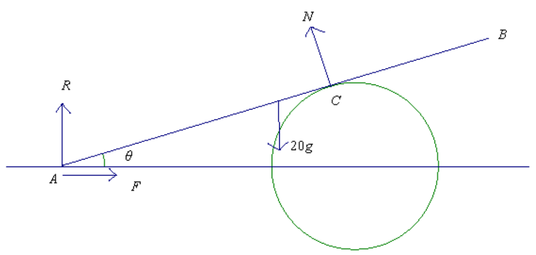
A1: Either answer

Question T5_Q2

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a+3x)}{2a \times \frac{3}{5}} = \frac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \frac{5Mg}{2}$ N.B. May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \frac{Y}{X}$ or $\frac{R\sin\beta}{R\cos\beta}$ $= \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \le \frac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 A1	2.4
		(3)	
	(13 marks)		

Notes:
(a) **M1**: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only. **A1**: Correct equation **A1***: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.
(b) **M1**: Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the given expression for T. **A1**: Accept $0.67a$ or better
(c) **M1**: Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules **A1 ft**: Correct equation with their x substituted in T expression or using $T = \frac{2Mg}{\cos\alpha}$ **A1**: Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$ **M1**: For finding an equation **in tan** β **only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$ This is independent but must have found a Y. **A1**: Accept $\frac{-5}{4}$ if it follows from their working.
(d) **M1**: Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$) **A1**: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better. **B1**: **Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Question T5_Q3

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\text{M}(A):\ 20g \times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\text{M}(A):\ 20g \times 4\cos\theta = 5N$ $\text{M}(B):\ 3N + R \times 8\cos\theta = F \times 8\sin\theta + 20g \times 4\cos\theta$ $\text{M}(C):\ R \times 5\cos\theta = F \times 5\sin\theta + 20g \times \cos\theta$ $\text{M}(G):\ R \times 4\cos\theta = F \times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X = 20g\sin\theta$ $(\nwarrow): Y + N = 20g\cos\theta$ $(\uparrow): X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ $(\rightarrow): X\cos\theta = Y\sin\theta + N\sin\theta$ $\text{M}(A): 20g \times 4\cos\theta = 5N$ $\text{M}(B): 20g \times 4\cos\theta = 8Y + 3N$ $\text{M}(C): 20g \times \cos\theta = 5Y$ $\text{M}(G): 4Y = N \times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (H) and cpt perp to ramp (S)		
	$(\nearrow): H\cos\theta = 20g\sin\theta$	M1	3.3
	$(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$	A1	1.1b
	$(\uparrow): S\cos\theta + N\cos\theta = 20g$	M1	3.4
	$(\rightarrow): H = S\sin\theta + N\sin\theta$		
	$\text{M}(A): 20g \times 4\cos\theta = 5N$	A1	1.1b
	$\text{M}(B): 20g \times 4\cos\theta + H \times 8\sin\theta = 8S + 3N$	M1	3.4
	$\text{M}(C): 20g \times \cos\theta + H \times 5\sin\theta = 5S$		
	$\text{M}(G): 4S = N \times 1 + H \times 4\sin\theta$	A1	1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^\circ - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

<table>
<tr><th colspan="2">Marks</th><th>Notes</th></tr>
<tr><td>4a</td><td>B1</td><td>Ignore any extra incorrect comments.</td></tr>
<tr><td></td><td></td><td>Generally 3 independent equations required so at least one moments equation.: M1A1M1A1M1A1.
More than 3 equations, give marks for the best 3. For each:
M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion.
A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover
<u>Enter marks on ePEN in order in which equations appear</u>.
N.B. If reaction at C is not perpendicular to the ramp, can only score marks for M(C)
Allow use of (μR) for F</td></tr>
<tr><td>4b</td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>M1</td><td>All terms required, dim correct, condone sin/cos confusion</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td colspan="2"></td><td>N.B. They can find F and R using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1</td></tr>
<tr><td>Alt 1</td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td colspan="2"></td><td>N.B. They can find X and Y using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1</td></tr>
<tr><td>Alt 2</td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
<tr><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>M1</td><td>All terms required. Must be dimensionally correct. Condone sin/cos confusion.</td></tr>
</table>

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find H and S using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for F (or X) and a value for R (or Y)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for H and a value for S
	A1	Correct answer only
	B1	Ignore reasons

Question T5_Q4

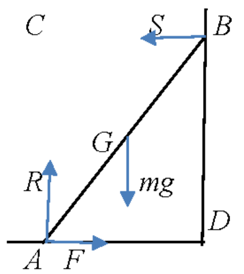
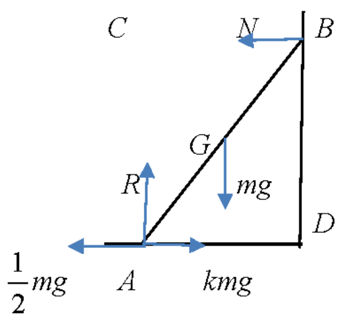
Question	Scheme	Marks	AOs
4(a)	Take moments about A	M1	3.3
	$N\times\dfrac{4a}{\sin\alpha}=Mg\times 3a\cos\alpha$	A1	1.1b
	$\dfrac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F=\dfrac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow) R+\dfrac{9Mg}{25}\cos\alpha=Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R\cos\alpha+\dfrac{9Mg}{25}=Mg\cos\alpha+F\sin\alpha$ $(\nearrow), Mg\sin\alpha=F\cos\alpha+R\sin\alpha$ $\text{M}(C), Mg.2a\cos\alpha+F.5a\sin\alpha=R.5a\cos\alpha$ $\text{M}(G), \dfrac{9Mg}{25}.2a+F.3a\sin\alpha=R.3a\cos\alpha$ $\text{M}(B), Mg.3a\cos\alpha+F.6a\sin\alpha=R.6a\cos\alpha+\dfrac{9Mg}{25}a$ $(F=\dfrac{36Mg}{125}, R=\dfrac{98Mg}{125})$		
	$F=\mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow), Y+\dfrac{9Mg}{25}=Mg\cos\alpha+X\sin\alpha$ $(\nearrow), Mg\sin\alpha=X\cos\alpha$ $(\uparrow), \dfrac{9Mg}{25}\cos\alpha+Y\cos\alpha=Mg$ $(\rightarrow), Y\sin\alpha+\dfrac{9Mg}{25}\sin\alpha=X$		

	$M(C), Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ $M(G), \frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 $M(B), Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1		
	$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

Notes:

4a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of : $\frac{4a}{\sin\alpha}, \frac{4a}{\cos\alpha}, 5a$ but use of any of : $6a, 5a\sin\alpha, 4a\cos\alpha, \ldots$ or anything involving $\tan\alpha$ is M0 Also M0 if no a's in their first equation.
	A1	Correct equation, trig does not need to be substituted
	A1*	Given answer correctly obtained.
4b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)
	M1	Must have 3 equations (<u>and all 3 previous M marks</u>)
	A1	Accept 0.37 or better

Question T5_Q5

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
3(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F \times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations <u>(they must have enough)</u> and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	
3(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b

	S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ **M1A1B1** $1+\frac{5}{4}=\frac{5k}{2}$ **M1** $k = 0.9$ **A1**		
	$N = kmg - F$ **OR** $R = mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k = 0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:

3a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F = \mu R$ anywhere, A0)
3b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao