JUNE 2010

3 (a) Given that k is a constant, display the following linear programming problem in a Simplex tableau.

$$\begin{aligned} &\text{Maximise} && P = 6x + 5y + 3z \\ &\text{subject to} && x + 2y + kz \leqslant 8 \\ & && 2x + 10y + z \leqslant 17 \\ & && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0 \end{aligned}$$

(3 marks)

(b) (i) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the x-column as pivot. *(4 marks)*

(ii) Given that the maximum value of P has not been achieved after this first iteration, find the range of possible values of k. *(2 marks)*

(c) In the case where $k = -1$, perform one further iteration and interpret your final tableau. *(6 marks)*

JAN 2011

4 The Simplex method is to be used to maximise $P = 3x + 2y + z$ subject to the constraints

$$-x + y + z \leqslant 4$$

$$2x + y + 4z \leqslant 10$$

$$4x + 2y + 3z \leqslant 21$$

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−3	−2	−1	0	0	0	0
0	−1	1	1	1	0	0	4
0	2	1	4	0	1	0	10
0	4	2	3	0	0	1	21

(a) (i) The first pivot is to be chosen from the x-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method and explain how you know that the optimal value has not been reached. *(5 marks)*

(b) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and write down the initial inequality that still has slack. *(4 marks)*

JUNE 2011

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 6y + kz$, where k is a constant.

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−2	−6	$-k$	0	0	0	0
0	5	3	10	1	0	0	15
0	7	6	4	0	1	0	28
0	4	3	6	0	0	1	12

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot **from the *y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value has **not** been reached, find the possible values of k. *(2 marks)*

(c) In the case when $k = 20$:

(i) perform one further iteration; *(4 marks)*

(ii) interpret the final tableau and state the values of the slack variables. *(3 marks)*

JAN 2012

4 A linear programming problem consists of maximising an objective function P involving three variables, x, y and z, subject to constraints given by three inequalities other than $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$. Slack variables s, t and u are introduced and the Simplex method is used to solve the problem. One iteration of the method leads to the following tableau.

P	x	y	z	s	t	u	*value*
1	−2	11	0	3	0	0	6
0	2	3	1	1	0	0	2
0	6	−30	0	−6	1	0	3
0	−1	−9	0	−3	0	1	4

(a) (i) State the column from which the pivot for the **next** iteration should be chosen. Identify this pivot and explain the reason for your choice. *(3 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) (i) Explain why you know that the maximum value of P has been achieved. *(1 mark)*

(ii) State how many of the three original inequalities still have slack. *(1 mark)*

(c) (i) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(ii) The objective function for this problem is $P = kx - 2y + 3z$, where k is a constant. Find the value of k. *(2 marks)*

JUNE 2012

3 (a) Given that k is a constant, complete the Simplex tableau below for the following linear programming problem.

Maximise $P = kx + 6y + 5z$

subject to $2x + y + 4z \leqslant 11$

$x + 3y + 6z \leqslant 18$

$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$ *(2 marks)*

(b) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the ***y*-column** as pivot. *(4 marks)*

(c) (i) In the case when $k = 1$, explain why the maximum value of P has now been reached and write down this maximum value of P. *(2 marks)*

(ii) In the case when $k = 3$, perform one further iteration and interpret your new tableau. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 3

(a)

P	*x*	*y*	*z*	*s*	*t*	*value*
1	$-k$	−6	−5	0	0	0
0						
0						

.....

.....

.....

(b)

P	*x*	*y*	*z*	*s*	*t*	*value*

(c)(ii)

P	*x*	*y*	*z*	*s*	*t*	*value*

JAN 2013

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = x - 2y + 3z$

subject to
$$x + y + z \leqslant 16$$
$$x - 2y + 2z \leqslant 17$$
$$2x - y + 2z \leqslant 19$$

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$. *(2 marks)*

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret the tableau that you obtained in part **(c)(i)** and state the values of your slack variables. *(3 marks)*

JUNE 2013

6 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x + 3y + z$

subject to
$$2x + y + z \leqslant 25$$
$$x + 2y + z \leqslant 40$$
$$x + y + 2z \leqslant 30$$

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$. *(2 marks)*

(b) The first pivot to be chosen is from the x-column.

Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret your final tableau and state the values of your slack variables. *(3 marks)*

JUNE 2014

4 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 6y + 2z$

subject to $x + 3y + 2z \leqslant 11$

$3x + 4y + 2z \leqslant 21$

and $x \geqslant 0\ ,\ y \geqslant 0\ ,\ z \geqslant 0\ .$

[2 marks]

(b) The first pivot to be chosen is from the y-column.

Perform one iteration of the Simplex method.

[3 marks]

(c) Perform one further iteration.

[3 marks]

(d) Interpret the tableau obtained in part **(c)** and state the values of your slack variables.

[3 marks]