JUNE 2010

4 Two people, Roger and Corrie, play a zero-sum game.

The game is represented by the following pay-off matrix for Roger.

Corrie

Roger

Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{R_1}$	7	3	-5
$\mathbf{R_2}$	-2	-1	4

(a) (i) Find the optimal mixed strategy for Roger. *(7 marks)*

(ii) Show that the value of the game is $\frac{7}{13}$. *(1 mark)*

(b) Given that the value of the game is $\frac{7}{13}$, find the optimal mixed strategy for Corrie. *(5 marks)*

JAN 2011

3 Two people, Rhona and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rhona.

Colleen

Rhona

Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
$\mathbf{R_1}$	2	6	4
$\mathbf{R_2}$	3	-3	-1
$\mathbf{R_3}$	x	$x+3$	3

It is given that $x < 2$.

(a) (i) Write down the three row minima. *(1 mark)*

(ii) Show that there is no stable solution. *(3 marks)*

(b) Explain why Rhona should never play strategy R_3. *(1 mark)*

(c) (i) Find the optimal mixed strategy for Rhona. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

3 (a) Two people, Tom and Jerry, play a zero-sum game. The game is represented by the following pay-off matrix for Tom.

Jerry

	Strategy	**A**	**B**	**C**
	I	−4	5	−3
Tom	**II**	−3	−2	8
	III	−7	6	−2

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Rohan and Carla play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Rohan.

Carla

	Strategy	C_1	C_2	C_3
Rohan	R_1	3	5	−1
	R_2	1	−2	4

(i) Find the optimal mixed strategy for Rohan and show that the value of the game is $\frac{3}{2}$. *(7 marks)*

(ii) Carla plays strategy C_1 with probability p, and strategy C_2 with probability q.

Find the values of p and q and hence find the optimal mixed strategy for Carla. *(4 marks)*

JAN 2012

3 Two people, Roz and Colum, play a zero-sum game. The game is represented by the following pay-off matrix for Roz.

Colum

	Strategy	C_1	C_2	C_3
	R_1	−2	−6	−1
Roz	R_2	−5	2	−6
	R_3	−3	3	−4

(a) Explain what is meant by the term 'zero-sum game'. *(2 marks)*

(b) Determine the play-safe strategy for Colum, giving a reason for your answer. *(2 marks)*

(c) (i) Show that the matrix can be reduced to a 2 by 3 matrix, giving the reason for deleting one of the rows. *(2 marks)*

(ii) Hence find the optimal mixed strategy for Roz. *(7 marks)*

JUNE 2012

4 (a) Two people, Adam and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Adam.

Bill

Adam

Strategy	**B_1**	**B_2**	**B_3**
A_1	−6	−1	−5
A_2	5	2	−3
A_3	−5	4	−4
A_4	2	1	−4

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player. *(1 mark)*

(iii) State the value of the game for **Bill**. *(1 mark)*

4 (b) Roza plays a different zero-sum game against a computer. The game is represented by the following pay-off matrix for Roza.

Computer

Roza

Strategy	**C_1**	**C_2**	**C_3**
R_1	3	4	−3
R_2	−2	−1	5

(i) State which strategy the computer should never play, giving a reason for your answer. *(1 mark)*

(ii) Roza chooses strategy R_1 with probability p. Find expressions for the expected gains for Roza when the computer chooses each of its two remaining strategies. *(2 marks)*

(iii) Hence find the value of p for which Roza will maximise her expected gains. *(2 marks)*

(iv) Find the value of the game for Roza. *(1 mark)*

JAN 2013

2 Harry and Will play a zero-sum game. The game is represented by the following pay-off matrix for Harry.

Will

Harry

Strategy	***D***	***E***	***F***	***G***
A	4	−1	2	3
B	4	6	3	7
C	1	3	−2	4

(a) Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) List any saddle points. *(1 mark)*

6 Kate and Pippa play a zero-sum game. The game is represented by the following pay-off matrix for Kate.

Pippa

Kate

Strategy	***D***	***E***	***F***
A	−2	0	3
B	3	−2	−2
C	4	1	−1

(a) Explain why Kate should not adopt strategy *B*. *(1 mark)*

(b) Find the optimal mixed strategy for Kate and find the value of the game. *(7 marks)*

(c) Find the optimal mixed strategy for Pippa. *(4 marks)*

JUNE 2013

5 Romeo and Juliet play a zero-sum game. The game is represented by the following pay-off matrix for Romeo.

Juliet

Romeo

Strategy	**D**	**E**	**F**
A	4	−4	0
B	−2	−5	3
C	2	1	−2

(a) Find the play-safe strategy for each player. *(3 marks)*

(b) Show that there is no stable solution. *(1 mark)*

(c) Explain why Juliet should never play strategy D. *(1 mark)*

(d) (i) Explain why the following is a suitable pay-off matrix **for Juliet**.

4	5	−1
0	−3	2

(2 marks)

(ii) Hence find the optimal strategy for Juliet. *(7 marks)*

(iii) Find the value of the game for Juliet. *(1 mark)*

JUNE 2014

2 Alex and Roberto play a zero-sum game. The game is represented by the following pay-off matrix for Alex.

Roberto

Alex

Strategy	**D**	**E**	**F**	**G**
A	5	−4	−1	1
B	4	3	0	1
C	−3	0	−5	−2

(a) Show that this game has a stable solution and state the play-safe strategy for each player.

[4 marks]

(b) List any saddle points.

[1 mark]

5 Mark and Owen play a zero-sum game. The game is represented by the following pay-off matrix for Mark.

Owen

Mark

Strategy	**D**	**E**	**F**
A	4	1	−1
B	3	−2	−2
C	−2	0	3

(a) Explain why Mark should never play strategy B.

[1 mark]

(b) It is given that the value of the game is 0.6. Find the optimal strategy for **Owen**.

(You are **not** required to find the optimal mixed strategy for Mark.)

[7 marks]