D2 Paper E – Marking Guide

1. *(a)*

P	x	y	z	r	s	
1	$^-3$	$^-3$	$^-4$	0	0	0
0	1	2	1	1	0	30
0	5	1	3	0	1	60

M1 A1

(b) θ values are 30 and 20 so pivot row is 3rd row

P	x	y	z	r	s	
1	$\frac{11}{3}$	$-\frac{5}{3}$	0	0	$\frac{4}{3}$	80
0	$-\frac{2}{3}$	$\frac{5}{3}$	0	1	$-\frac{1}{3}$	10
0	$\frac{5}{3}$	$\frac{1}{3}$	1	0	$\frac{1}{3}$	20

M2 A2

(c) $x = 0, y = 0, z = 20, P = 80$ A1
solution not optimal as there are values < 0 on the objective row B1 **(8)**

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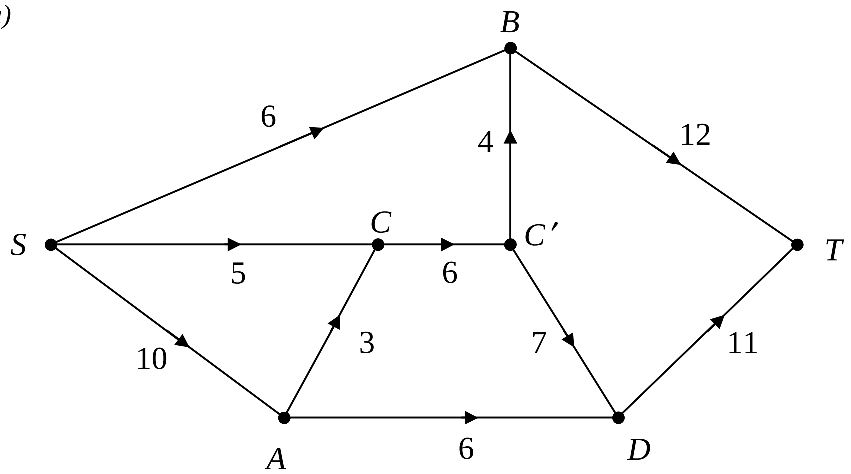
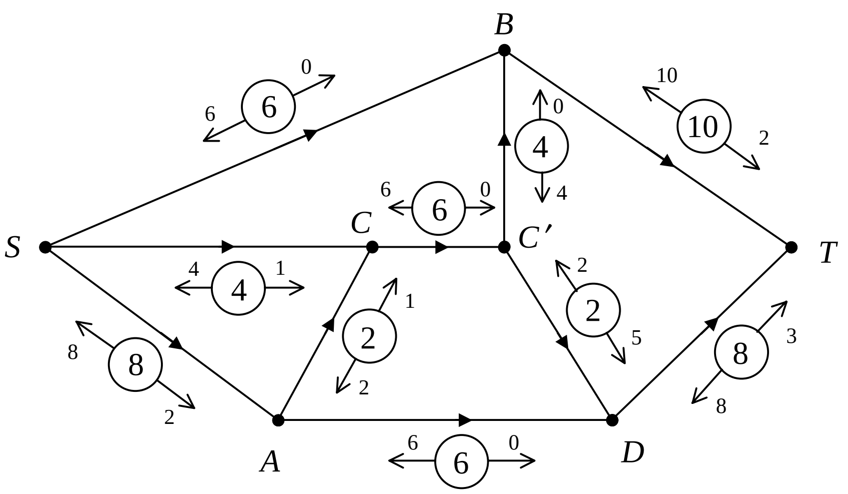
2. *(a)*



M1 A1

(b) e.g. augment $SCC'BT$ by 4 and $SACC'DT$ by 2 giving max. flow below



max. flow = 18 M2 A4 **(8)**

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3. need to maximise so subtract all values from 9 giving — M1

				row min.
2	1	4	3	1
3	0	3	4	0
0	1	4	2	0
2	2	3	3	2

reducing rows gives:

	1	0	3	2
	3	0	3	4
	0	1	4	2
	0	0	1	1
col min.	0	0	1	1

M1 A1

reducing columns gives:

1	0	2	1
3	0	2	3
0	1	3	1
0	0	0	0

(N.B. a different choice of lines will lead to the same final assignment) — A1

3 lines required to cover all zeros, apply algorithm — B1

1	0	1	0*
3	0*	1	2
0*	1	2	0
1	1	0*	0

M1 A1

4 lines are required to cover all zeros so allocation is possible — B1

stage 1 – *C*
stage 2 – *B*
stage 3 – *D* — A1
stage 4 – *A*

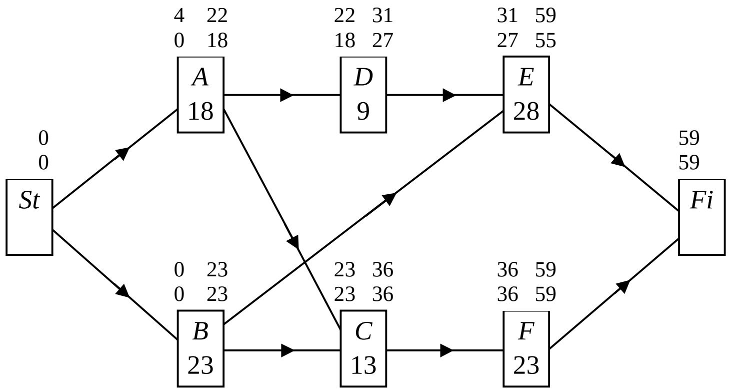
total number of days = 9 + 9 + 6 + 6 = 30 days — A1 **(10)**

4. e.g. using stage, state approach:

Stage	State	Action	Destination	Value	
1	*I*	*IL*	*L*	5*	
	J	*JL*	*L*	6*	M1
	K	*KL*	*L*	10*	
2	*F*	*FI* *FJ* *FK*	*I* *J* *K*	5 + 5 = 10 2 + 6 = 8* 2 + 10 = 12	
	G	*GI* *GJ* *GK*	*I* *J* *K*	8 + 5 = 13* 9 + 6 = 15 3 + 10 = 13*	
	H	*HI* *HJ* *HK*	*I* *J* *K*	10 + 5 = 15 2 + 6 = 8* 9 + 10 = 19	M1 A2
3	*B*	*BF* *BG* *BH*	*F* *G* *H*	8 + 8 = 16 11 + 13 = 24 4 + 8 = 12*	
	C	*CF* *CH*	*F* *H*	5 + 8 = 13* 10.5 + 8 = 18.5	
	D	*DF* *DH*	*F* *H*	9 + 8 = 17 6 + 8 = 14*	
	E	*EF* *EG* *EH*	*F* *G* *H*	12 + 8 = 20* 7 + 13 = 20* 15 + 8 = 23	M1 A2
4	*A*	*AB* *AC* *AD* *AE*	*B* *C* *D* *E*	5 + 12 = 17* 4.5 + 13 = 17.5 13 + 14 = 27 10 + 20 = 30	M1 A1

giving route *ABHJL* A1 **(10)**

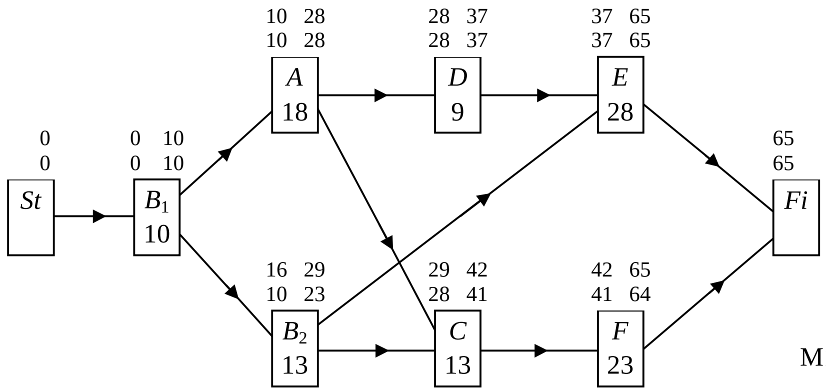
5. *(a)*



M1 A2

(b) lower figures give forward scan — M1
upper figures give backward scan — M1
critical path is *BCF* — A1
minimum time is 59 minutes — A1

(c)



M1 A1

(d) new minimum time is 65 minutes — M1 A1
new critical path is B_1ADE — A1 **(12)**

6. *(a)*

		B I	*B* II	row minimum
A	I	4	$^{-}8$	$^{-}8$
	II	2	$^{-}4$	$^{-}4$
	III	$^{-}8$	2	$^{-}8$
column maximum		4	2	

M1

max (row min) = $^{-}4$ min (col max) = 2

max (row min) $\neq$ min (col max) $\therefore$ no saddle point A1

(b) let *B* play strategies I and II with proportions q and $(1 - q)$

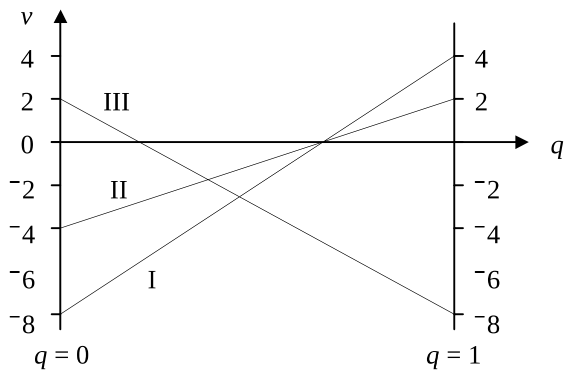
expected loss for *B* against each of *A*'s strategies:

A I $4q - 8(1 - q) = 12q - 8$

A II $2q - 4(1 - q) = 6q - 4$ M1 A1

A III $^{-}8q + 2(1 - q) = 2 - 10q$

giving



M1

it is not worth player *A* considering strategy I A1

for optimal strategy $6q - 4 = 2 - 10q$ M1

$\therefore 16q = 6, \ q = \frac{3}{8}$

$\therefore$ *B* should play I $\frac{3}{8}$ of time and II $\frac{5}{8}$ of time A1

(c) let *A* play strategies II and III with proportions p and $(1 - p)$

expected payoff to *A* against each of *B*'s strategies:

B I $2p - 8(1 - p) = 10p - 8$

B II $^{-}4p + 2(1 - p) = 2 - 6p$ M1 A1

for optimal strategy $10p - 8 = 2 - 6p$

$\therefore 16p = 10, \ p = \frac{5}{8}$

$\therefore$ *A* should play I never, II $\frac{5}{8}$ of time and III $\frac{3}{8}$ of time A1

(d) value of game = $(6 \times \frac{3}{8}) - 4 = {}^{-}1\frac{3}{4}$ A1 **(12)**

Total **(60)**