D2 Paper D – Marking Guide

1.

		B			row minimum
		I	II	III	
A	I	$^-3$	4	0	$^-3$
	II	2	2	1	1
	III	3	$^-2$	$^-1$	$^-2$
column maximum		3	4	1	

M1

max (row min) = min (col max) = 1 $\therefore$ saddle point M1
$\therefore$ *A* should play II all the time, *B* should play III all the time A1
value of game = 1 A1 **(4)**

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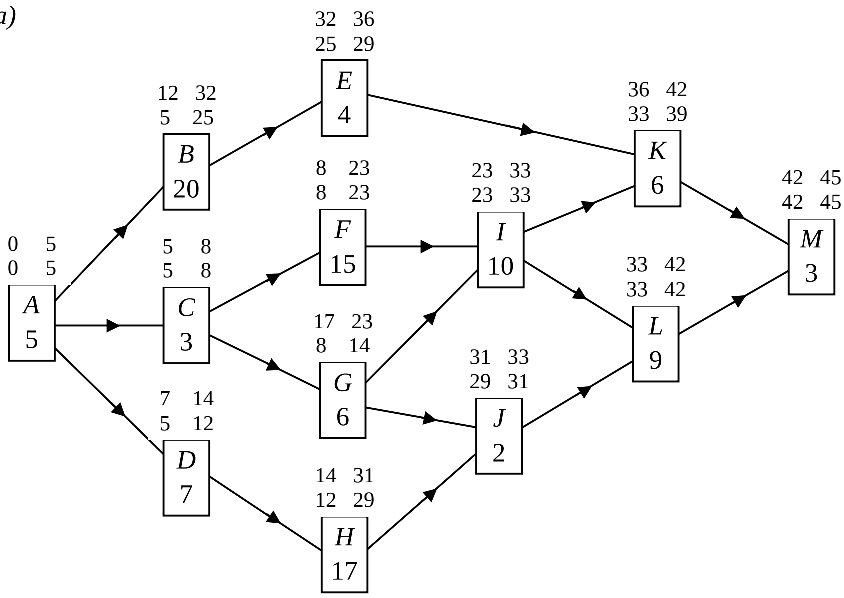
2. *(a)*



lower figures give forward scan M1 A1
minimum time is 45 days A1

(b) upper figures give backward scan M1 A1
critical path is *ACFILM* A1 **(6)**

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3. e.g. using stage, state approach:

Stage	State	Action	Destination	Value	
1	*I*	*IL*	*L*	19*	
	J	*JL*	*L*	18*	
	K	*KL*	*L*	26*	M1
2	*E*	*EI* *EJ*	*I* *J*	35 + 19 = 54 29 + 18 = 47*	
	F	*FI* *FJ* *FK*	*I* *J* *K*	17 + 19 = 36* 24 + 18 = 42 15 + 26 = 41	
	G	*GI* *GJ* *GK*	*I* *J* *K*	18 + 19 = 37* 26 + 18 = 44 14 + 26 = 40	
	H	*HJ* *HK*	*J* *K*	17 + 18 = 35* 39 + 26 = 65	M1 A2
3	*B*	*BE* *BF* *BG*	*E* *F* *G*	21 + 47 = 68 25 + 36 = 61* 28 + 37 = 65	
	C	*CE* *CF* *CG* *CH*	*E* *F* *G* *H*	28 + 47 = 75 30 + 36 = 66 40 + 37 = 77 28 + 35 = 63*	
	D	*DF* *DG* *DH*	*F* *G* *H*	38 + 36 = 74 24 + 37 = 61* 35 + 35 = 70	M1 A1
4	*A*	*AB* *AC* *AD*	*B* *C* *D*	19 + 61 = 80 12 + 63 = 75 7 + 61 = 68*	A1

giving route *ADGIL* A1 **(8)**

4. need to add dummy row giving — M1

				row min.
27	80	8	81	8
28	60	5	71	5
30	90	7	73	7
0	0	0	0	0

reducing rows gives:

19	72	0	73
23	55	0	66
23	83	0	66
0	0	0	0

M1 A1

reducing columns will make no difference — B1

2 lines required to cover all zeros, apply algorithm — B1

0	53	0	54
4	36	0	47
4	64	0	47
0	0	19	0

(N.B. a different choice of lines will lead to the same final assignment) — M1 A1

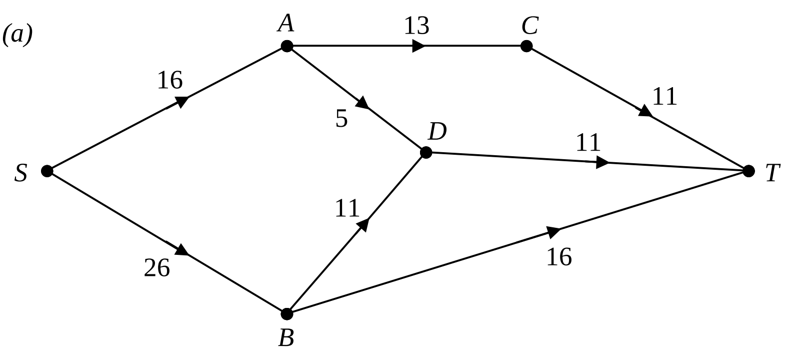
3 lines required to cover all zeros, apply algorithm

0*	17	0	18
4	0*	0	11
4	28	0*	11
36	0	55	0*

A1

4 lines required to cover all zeros so allocation is possible
team A does the windows — team B does the conservatory
team C does the doors — the greenhouse is not done — A1
total cost = $10 \times (27 + 60 + 7) = £940$ — A1 **(10)**

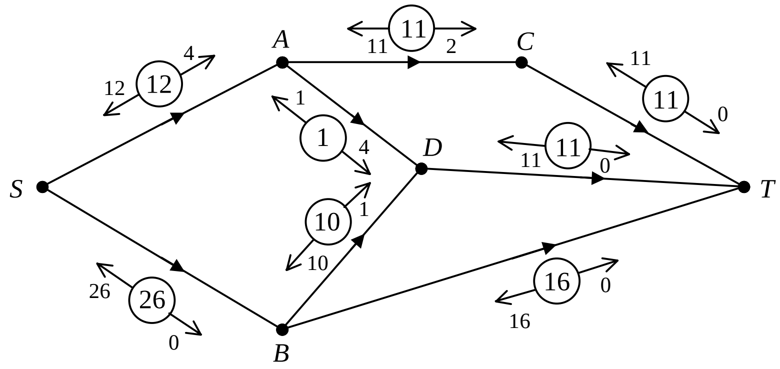
5. *(a)*



M1 A2

(b) minimum cut = $\{S, A, B, C, D\} \mid \{T\} = 38$ — M1 A1

(c) e.g. augment *SACT* by 11, *SBT* by 16, *SBDT* by 10 and *SADT* by 1, giving



maximum flow = 38 — M2 A3

(d) maximum as = to minimum cut — B1 **(11)**

6. *(a)* Change all signs and add 4 to make +ve

		B		
		I	II	III
A	I	6	1	5
	II	0	9	2

let *B* play strategies I, II and III with proportions p_1, p_2 and p_3

from *A* I, $6p_1 + 1p_2 + 5p_3 \leq$ v so v - 6p1 - 1p2 - 5p3 + s = 0
from *A* II, $0p_1 + 9p_2 + 2p_3 \leq$ v so v -9p2 -2p3 + t = 0
also, $p_1 + p_2 + p_3$ + r = 1

maximise P - v = 0

(b) Ignore the working below but the final answer is correct

tableau 1:

P	x_1	x_2	x_3	r	s	
1	$^-1$	$^-1$	$^-1$	0	0	0
0	4	9	5	1	0	1
0	[10]	1	8	0	1	1

A1

taking [10] as pivot M1

tableau 2:

P	x_1	x_2	x_3	r	s	
1	0	$-\frac{9}{10}$	$-\frac{1}{5}$	0	$\frac{1}{10}$	$\frac{1}{10}$
0	0	[$8\frac{3}{5}$]	$1\frac{4}{5}$	1	$-\frac{2}{5}$	$\frac{3}{5}$
0	1	$\frac{1}{10}$	$\frac{4}{5}$	0	$\frac{1}{10}$	$\frac{1}{10}$

M1 A2

taking [$8\frac{3}{5}$] as pivot M1

tableau 3:

P	x_1	x_2	x_3	r	s	
1	0	0	$-\frac{1}{86}$	$\frac{9}{86}$	$\frac{5}{86}$	$\frac{7}{43}$
0	0	1	$\frac{9}{43}$	$\frac{5}{43}$	$-\frac{2}{43}$	$\frac{3}{43}$
0	1	0	[$\frac{67}{86}$]	$-\frac{1}{86}$	$\frac{9}{86}$	$\frac{4}{43}$

M1 A1

taking $\boxed{\frac{67}{86}}$ as pivot

tableau 4:

P	x_1	x_2	x_3	r	s	
1	$\frac{1}{67}$	0	0	$\frac{7}{67}$	$\frac{4}{67}$	$\frac{11}{67}$
0	$-\frac{18}{67}$	1	0	$\frac{8}{67}$	$-\frac{5}{67}$	$\frac{3}{67}$
0	$1\frac{19}{67}$	0	1	$-\frac{1}{67}$	$\frac{9}{67}$	$\frac{8}{67}$

A1

tableau is optimal

$x_1 = 0,\ x_2 = \frac{3}{67},\ x_3 = \frac{8}{67},\ P = \frac{1}{v} = \frac{11}{67}\ \therefore\ v = \frac{67}{11}$ M1

giving $p_1 = 0,\ p_2 = \frac{67}{11} \times \frac{3}{67} = \frac{3}{11},\ p_3 = \frac{67}{11} \times \frac{8}{67} = \frac{8}{11}$ M1

$\therefore$ *B* should not play I, should play II $\frac{3}{11}$ of time and III $\frac{8}{11}$ of time A1

value of original game = $\frac{67}{11} - 6 = \frac{1}{11}$ A1 **(21)**

Total **(60)**