D2 Paper C – Marking Guide

1. *(a)*

				row min.
5	3	5	4	3
7	5	6	4	4
8	4	7	6	4
5	3	2	3	2

reducing rows gives:

2	0	2	1
3	1	2	0
4	0	3	2
3	1	0	1
col min. 2	0	0	0

M1 A1

reducing columns gives:

~~0*~~	~~0~~	~~2~~	~~1~~
~~1~~	~~1~~	~~2~~	~~0*~~
2	0*	3	2
~~1~~	~~1~~	~~0*~~	~~1~~

M1 A1

4 lines are required to cover all zeros so allocation is possible B1

strip wallpaper – Alice; paint – Dieter;
hang wallpaper – Bhavin; replace fittings – Carl M1 A1

(b) 5 + 4 + 4 + 2 = 15 hours A1 **(8)**

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2. *(a)* (i) strategy III dominates II since $9 \geq 7, \ ^-4 \geq \ ^-4, 8 \geq \ ^-1$
player *A* can ignore strategy II B1

(ii) strategy III dominates I since $^-2 \leq 3, \ ^-1 \leq 7, 8 \leq 9$
player *B* can ignore strategy I B1

(b) reduced table:

		B	
		II	III
A	I	5	$^-2$
	III	$^-4$	8

(i) let *A* play strategies I and III with proportions p and $(1 - p)$

expected payoff to *A* against each of *B*'s strategies:

B II $5p - 4(1 - p) = 9p - 4$
B III $^-2p + 8(1 - p) = 8 - 10p$ M1 A1

for optimal strategy $9p - 4 = 8 - 10p$ M1

$\therefore 19p = 12, \ p = \frac{12}{19}$

$\therefore$ *A* should play I $\frac{12}{19}$ of time, II never and III $\frac{7}{19}$ of time A1

(ii) let *B* play strategies II and III with proportions q and $(1 - q)$

expected loss to *B* against each of *A*'s strategies:

A I $5q - 2(1 - q) = 7q - 2$
A III $^-4q + 8(1 - q) = 8 - 12q$ M1 A1

for optimal strategy $7q - 2 = 8 - 12q$

$\therefore 19q = 10, \ q = \frac{10}{19}$

$\therefore$ *B* should play I never, II $\frac{10}{19}$ of time and III $\frac{9}{19}$ of time A1 **(9)**

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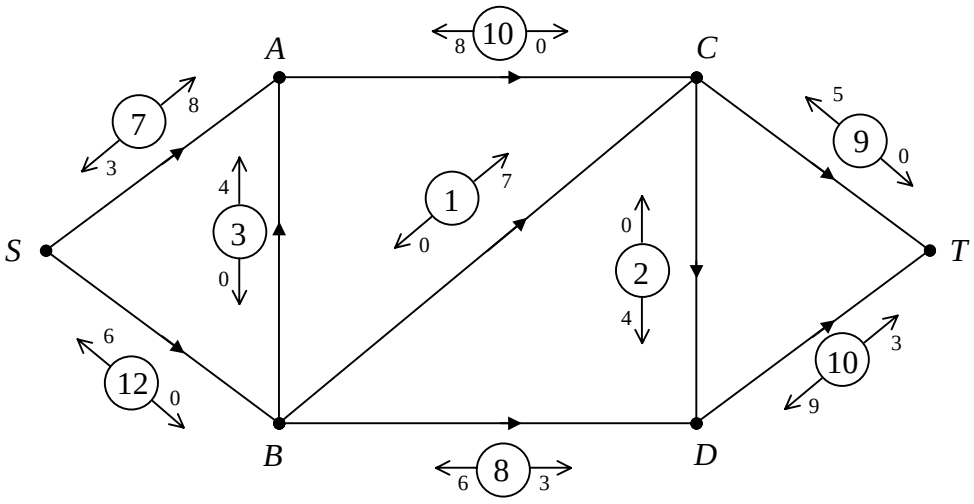
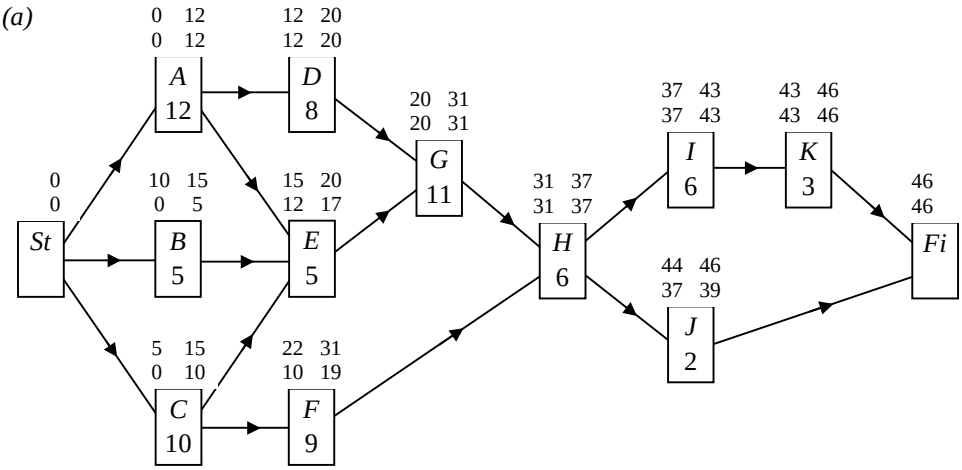
3. *(a)* e.g. augment *SBDT* by 2 and *SABDT* by 2 giving maximum flow below



max. flow = 19 M2 A4

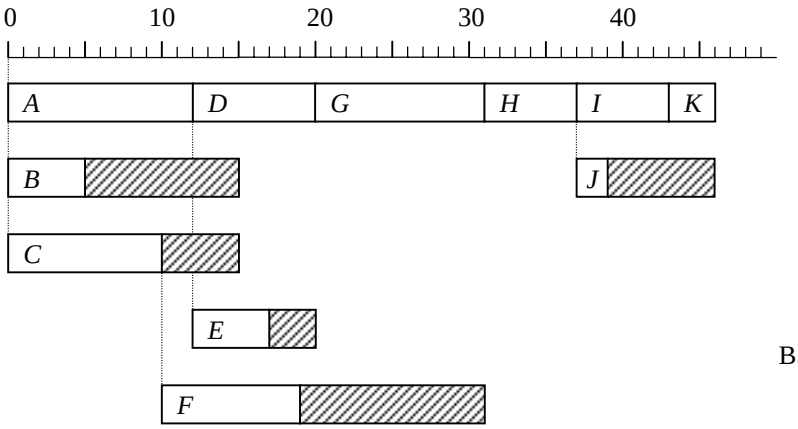
(b) cut through arcs *AC*, *AB* and *SB* = 10 – 3 + 12 = 19 M1 A1
proves max. flow as no more flow is possible across this cut B1 **(9)**

4. *(a)*



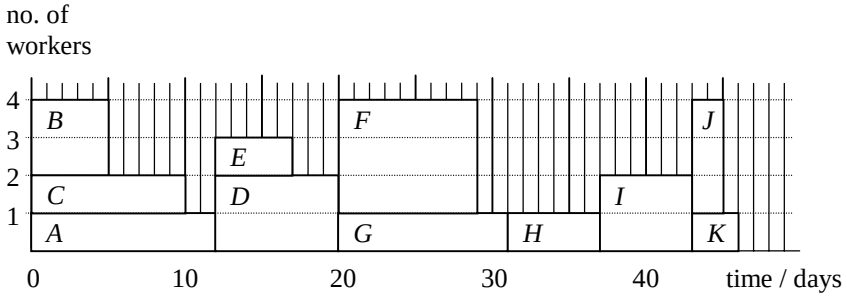
	lower figures give forward scan	M1
	upper figures give backward scan	M1 A1
	critical path is *ADGHIK*	A1
(b)	minimum time is 46 days	A1

(c)



B3

(d) e.g.



M1 A2 **(11)**

5. e.g. using stage, state approach:

Stage	State in store	Action	To store	Total Cost	
1	0	1	0	0 + 5500 = 5500*	M1 A1
	1	0	0	400 + 0 = 400*	
2	1	3	0	400 + 13100 + 5500 = 19000*	
	2	2	0	800 + 9700 + 5500 = 16000	
		3	1	800 + 13100 + 400 = 14300*	
	3	1	0	1200 + 5500 + 5500 = 12200	M1 A2
		2	1	1200 + 9700 + 400 = 11300*	
3	0	3	1	0 + 13100 + 19000 = 32100*	
	1	2	1	400 + 9700 + 19000 = 29100	
		3	2	400 + 13100 + 14300 = 27800*	
	2	1	1	800 + 5500 + 19000 = 25300	M1 A2
		2	2	800 + 9700 + 14300 = 24800*	
		3	3	800 + 13100 + 11300 = 25200	
4	0	1	0	0 + 5500 + 32100 = 37600	
		2	1	0 + 9700 + 27800 = 37500*	M1 A1
		3	2	0 + 13100 + 24800 = 37900	

$\therefore$ should make 2 in March, 3 in April, 3 in May and 0 in June A1 **(11)**

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6. *(a)* $6x + 15y + 12z \leq 185$
$3x + 3y + z \leq 30$
$x + 4y + 4z \leq 60$ B2

(b) θ values are $12\frac{1}{3}$, 10 and 15 so pivot row is 3rd row

P	x	y	z	r	s	t	
1	5	0	$^{-}3$	0	3	0	90
0	$^{-}9$	0	7	1	$^{-}5$	0	35
0	1	1	$\frac{1}{3}$	0	$\frac{1}{3}$	0	10
0	$^{-}3$	0	$\frac{8}{3}$	0	$-\frac{4}{3}$	1	20

M2 A2

increase z next, θ values are 5, 30 and $7\frac{1}{2}$ so pivot row is 2nd row

P	x	y	z	r	s	t	
1	$\frac{8}{7}$	0	0	$\frac{3}{7}$	$\frac{6}{7}$	0	105
0	$-\frac{9}{7}$	0	1	$\frac{1}{7}$	$-\frac{5}{7}$	0	5
0	$\frac{10}{7}$	1	0	$-\frac{1}{21}$	$\frac{4}{7}$	0	$8\frac{1}{3}$
0	$\frac{3}{7}$	0	0	$-\frac{8}{21}$	$\frac{4}{7}$	1	$6\frac{2}{3}$

M1 A2

optimal solution as all values on the objective row are ≥ 0 B1

(c) 0 of X, $8\frac{1}{3}$ of Y and 5 of Z, giving $P = 105$ so profit = £1050 A1

(d) try integer coordinates around the optimal solution
e.g. (0, 8, 5) (1, 8, 5) (0, 9, 5) etc. checking feasible and seeking optimum B1 **(12)**

Total **(60)**