D2 Paper B – Marking Guide

1. *(a)* change all the signs to get B`s score and then add 6 to make them positive

		B		
		I	II	III
A	I	0	10	7
	II	8	1	3
	III	1	5	9

new value of game $v = V +$ 6

(b) let *B* play strategies I, II and III with proportions p_1, p_2 and p_3

p_1 + p2 + p3 + r = 1

(c) maximise *P* - v = 0 but must subtract 6 at the end as 6 has been added

(d) from *A* I, $0p_1 + 10p_2 + 7p_3 \leq v$ so v - 0p1 - 10p2 -7p3 + s = 0
from *A* II $3p_1 + 10p_2 + 8p_3 \leq v$ *so v - 3p1 - 10p2 -8p3 + t = 0*
from *A* III, $10p_1 + 6p_2 + 2p_3 \leq v$ so *v - 10p1 - 6p2 -2 p3 + u = 0*

(8)

2. e.g. using stage, state approach:

Stage	State	Action	Destination	Value	
1	*H*	*HK*	*K*	4*	
	I	*IK*	*K*	4*	M1
	J	*JK*	*K*	6*	
2	*E*	*EH*	*H*	6 + 4 = 10	
		EI	*I*	5 + 4 = 9*	
	F	*FH*	*H*	6 + 4 = 10	
		FI	*I*	5 + 4 = 9*	
		FJ	*J*	7 + 6 = 13	
	G	*GI*	*I*	4 + 4 = 8*	M1 A2
		GJ	*J*	4 + 6 = 10	
3	*B*	*BE*	*E*	7 + 9 = 16	
		BF	*F*	4 + 9 = 13*	
	C	*CE*	*E*	6 + 9 = 15	
		CF	*F*	6 + 9 = 15	
		CG	*G*	3 + 8 = 11*	
	D	*DF*	*F*	4 + 9 = 13*	M1 A1
		DG	*G*	5 + 8 = 13*	
4	*A*	*AB*	*B*	3 + 13 = 16*	
		AC	*C*	6 + 11 = 17	A1
		AD	*D*	6 + 13 = 19	

giving route *ABFIK* total distance 1600 miles A1 **(8)**

3. need to maximise so subtract all values from 55 giving M1

				row min.
18	26	11	4	4
10	25	12	14	10
23	28	16	5	5
12	30	4	0	0

reducing rows gives:

14	22	7	0
0	15	2	4
18	23	11	0
12	30	4	0
col min. 0	15	2	0

M1 A1

reducing columns gives:

14	7	5	0
~~0~~	~~0~~	~~0~~	~~4~~
18	8	9	0
12	15	2	0

A1

2 lines required to cover all zeros, apply algorithm B1

12	5	3	0
~~0~~	~~0~~	~~0~~	~~6~~
16	6	7	0
10	13	0	0

(N.B. a different choice of lines will lead to the same final assignment)

M1 A1

3 lines required to cover all zeros, apply algorithm

7	0^*	3	0
~~0^*~~	~~0~~	~~5~~	~~11~~
~~11~~	~~1~~	~~7~~	~~0^*~~
~~5~~	~~8~~	~~0^*~~	~~0~~

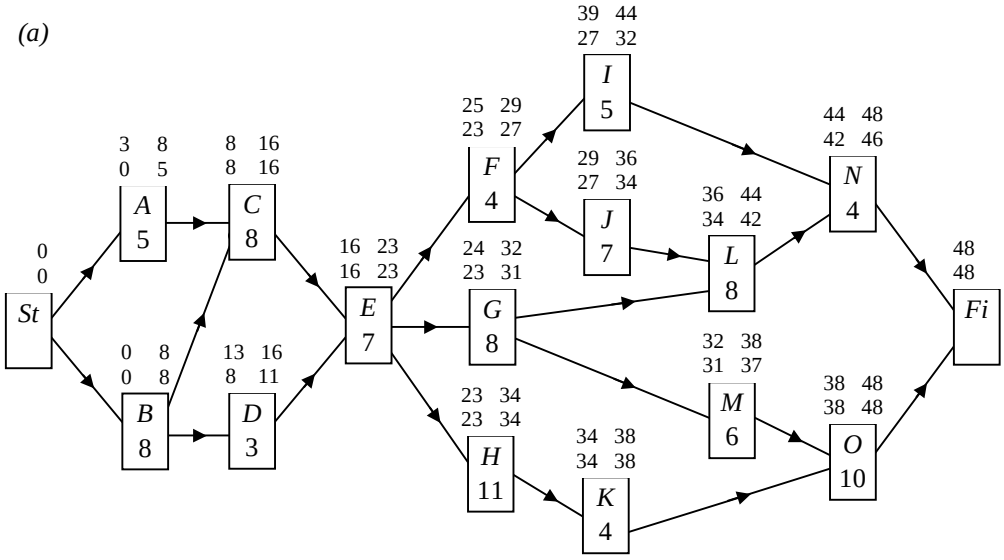
A1

4 lines required to cover all zeros so allocation is possible B1

R_1 goes to A_2
R_2 goes to A_1
R_3 goes to A_4
R_4 goes to A_3 A1 **(10)**

---

4. *(a)*

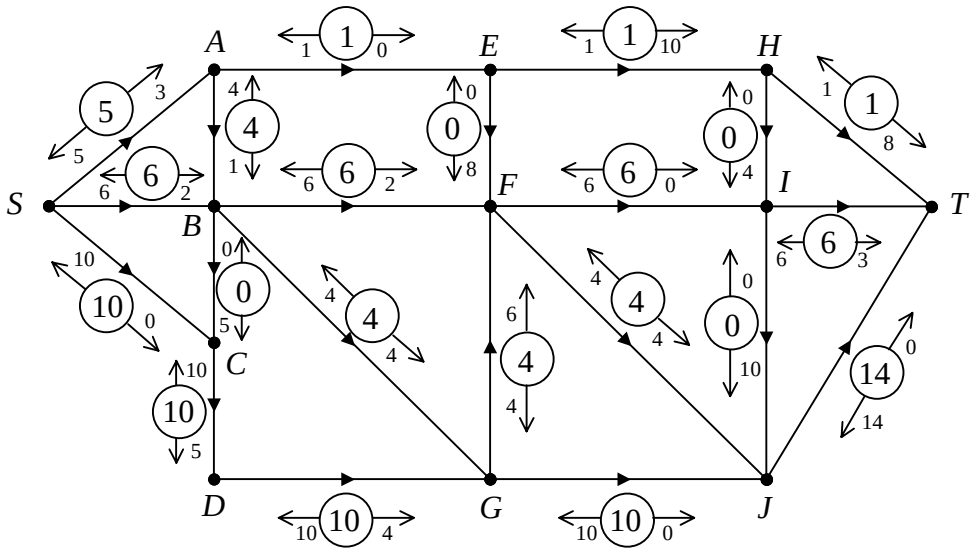


	lower figures give forward scan	M1 A1	
	minimum time is 48 days	A1	
(b)	upper figures give backward scan	M1 A1	
	critical path is *BCEHKO*	A1	
(c)	*E* on critical path ∴ £150 000 penalty		
	if reduce *K* by more than 1 day it is no longer on critical path		
	∴ only reduces penalty by £50 000 at cost of £90 000	B2	
(d)	*B*, *C* and *O*:		
	reducing any of these by 2 days reduces minimum time by 2 days		
	this reduces penalty by £100 000 at cost of £80 000 ∴ profitable	B3	**(11)**

---

5. *(a)* 1 + 8 + 8 + 15 = 32 B1

(b) (i), (ii) e.g. augment *SABGFJT* by 4 giving:



	max flow = 21	M2 A3	
(c)	max flow as = min cut of 21 {*S*, *A*, *B*, *C*, *D*, *F*, *G*, *J*} | {*E*, *H*, *I*, *T*}	M1 A1	
(d)	new min cut = 24 {*S*, *A*} | {*B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*, *T*}	M1 A1	
	∴ max flow could increase by 3	A1	**(11)**

---

6. *(a)*

P	x	y	z	r	s	
1	$^{-}3$	$^{-}1$	$^{-}1$	0	0	0
0	4	1	2	1	0	18
0	2	3	5	0	1	11

M1 A1

(b) θ values are $4\frac{1}{2}$ and $5\frac{1}{2}$ so pivot row is 2nd row

P	x	y	z	r	s	
1	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	0	$\frac{27}{2}$
0	1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{9}{2}$
0	0	$\frac{5}{2}$	4	$-\frac{1}{2}$	1	2

M2 A2

increase y next, θ values are 18 and $\frac{4}{5}$ so pivot row is 3rd row

P	x	y	z	r	s	
1	0	0	$\frac{9}{10}$	$\frac{7}{10}$	$\frac{1}{10}$	$\frac{137}{10}$
0	1	0	$\frac{1}{10}$	$\frac{3}{10}$	$-\frac{1}{10}$	$\frac{43}{10}$
0	0	1	$\frac{8}{5}$	$-\frac{1}{5}$	$\frac{2}{5}$	$\frac{4}{5}$

M2 A2

(c) $x = 4.3$, $y = 0.8$, $z = 0$, $P = 13.7$ A1

optimal solution as all values on the objective row are ≥ 0 B1 **(12)**

Total **(60)**