D2 Paper A – Marking Guide

1. e.g. using stage, state approach:

Stage	State	Action	Destination	Total Distance	
1	*G*	*GI*	*I*	12*	
	H	*HI*	*I*	10*	M1
2	*D*	*DG* *DH*	*G* *H*	14 + 12 = 26* 17 + 10 = 27	
	E	*EG* *EH*	*G* *H*	12 + 12 = 24* 18 + 10 = 28	
	F	*FG* *FH*	*G* *H*	13 + 12 = 25* 19 + 10 = 29	M1 A2
3	*A*	*AD* *AE* *AF*	*D* *E* *F*	8 + 26 = 34* 10 + 24 = 34* 14 + 25 = 39	
	B	*BD* *BE* *BF*	*D* *E* *F*	12 + 26 = 38 10 + 24 = 34* 16 + 25 = 41	
	C	*CD* *CE* *CF*	*D* *E* *F*	9 + 26 = 35* 13 + 24 = 37 15 + 25 = 40	M1 A1
4	*Home*	*Home-A* *Home-B* *Home-C*	*A* *B* *C*	15 + 34 = 49 11 + 34 = 45* 13 + 35 = 48	A1

giving route *HomeBEGI*, total distance 450 miles — A1 **(8)**

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2 need to add dummy column giving — M1

19	69	168	0
22	64	157	0
20	72	166	0
23	66	171	0
col min. 19	64	157	0

reducing rows will make no difference — B1

reducing columns gives:

0	5	11	0
3	0	0	0
1	8	9	0
4	2	14	0

(N.B. a different choice of lines will lead to the same final assignment) — M1 A1

3 lines required to cover all zeros, apply algorithm — B1

0	5	11	1
3	0	0	1
0	7	8	0
3	1	13	0

M1 A1

3 lines required to cover all zeros, apply algorithm

0*	4	10	1
4	0	0*	2
0	6	7	0*
3	0*	12	0

A1

4 lines required to cover all zeros so allocation is possible

stage 1 – Alex, stage 2 – Suraj, stage 3 – Darren, Leroy does not take part — A1 **(9)**

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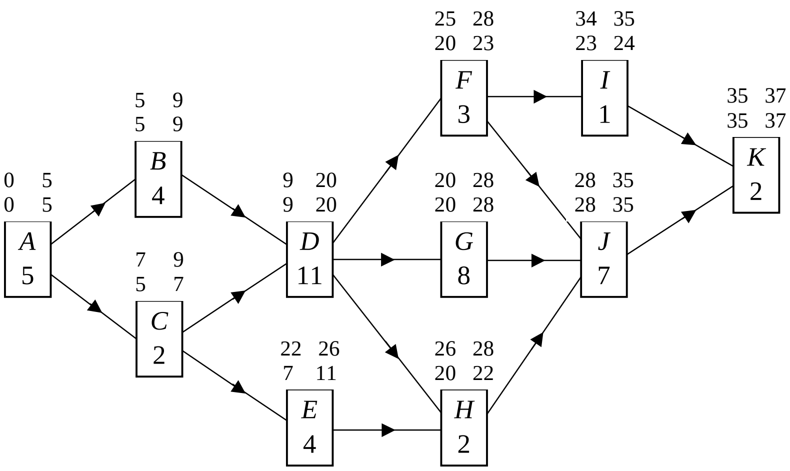
3. *(a)*



M2 A2

(b) lower figures give forward scan — M1
upper figures give backward scan — M1 A1
critical path is *ABDGJK* — A1
minimum time is 37 hours — A1

(c) 28 – 20 = 8 hours — B1 **(10)**

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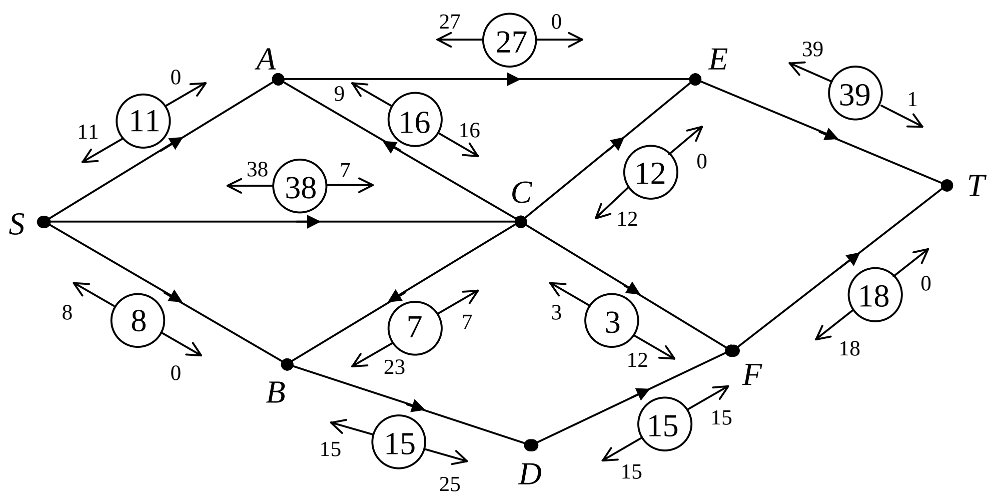
4. *(a)* $C_1 = 80$; $C_2 = 94$ — B2

(b) minimum cut: $\{S, A, B, C, D, F\} \mid \{E, T\} = 57$ — M1 A1

(c) $x = 15$, $y = 10$, $z = 36$ — B2

(d) augment *SCET* by 2 and *SCAET* by 1 giving maximum flow below



max. flow = 57 — M2 A2

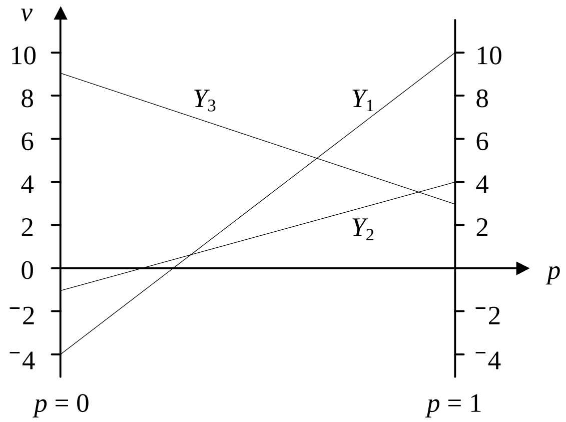
this is maximum flow as it is equal to the minimum cut — B1 **(11)**

5. *(a)* let X play strategies X_1 and X_2 with proportions p and $(1 - p)$

expected payoff to X against each of Y's strategies:

Y_1 $10p - 4(1 - p) = 14p - 4$
Y_2 $4p - (1 - p) = 5p - 1$ M1 A1
Y_3 $3p + 9(1 - p) = 9 - 6p$

giving



M1 A1

it is not worth player Y considering strategy Y_1 B1

for optimal strategy $5p - 1 = 9 - 6p$ M1

$\therefore 11p = 10, \ p = \frac{10}{11}$

$\therefore$ X should play X_1 $\frac{10}{11}$ of time and X_2 $\frac{1}{11}$ of time A1

(b) let Y play strategies Y_2 and Y_3 with proportions q and $(1 - q)$

expected loss to Y against each of X's strategies:

X_1 $4q + 3(1 - q) = q + 3$
X_2 $^{-}q + 9(1 - q) = 9 - 10q$ M1 A1

for optimal strategy $q + 3 = 9 - 10q$

$\therefore 11q = 6, \ q = \frac{6}{11}$

$\therefore$ Y should not play Y_1, should play Y_2 $\frac{6}{11}$ of time and Y_3 $\frac{5}{11}$ of time A1

(c) value $= (5 \times \frac{10}{11}) - 1 = 3\frac{6}{11}$ A1 **(11)**

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6. *(a)* maximise $R = 4x + 10y + 2z$
subject to $x - y \leq 5$
$^{-}y + 2z \leq 0$
$2x + 4y + z \leq 40$
$x \geq 0,\ \ y \geq 0,\ \ z \geq 0$ — M1 A2

(b) to change inequalities into equations — B1

(c) only one positive value so pivot row is 4th row

2nd tableau is:

R	x	y	z	r	s	t	
1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$	100
0	$\frac{3}{2}$	0	$\frac{1}{4}$	1	0	$\frac{1}{4}$	15
0	$\frac{1}{2}$	0	$\frac{9}{4}$	0	1	$\frac{1}{4}$	10
0	$\frac{1}{2}$	1	$\frac{1}{4}$	0	0	$\frac{1}{4}$	10

M2 A2

(d) final tableau as all values on the objective row are ≥ 0 — B1

(e) centre provides 10 courses for adults (not pensioners)
and gets £100 revenue per day — B1

(f) no. e.g. the slack variable s associated with this constraint is not zero
so optimal solution without this constraint would be the same — B1 **(11)**

Total **(60)**