GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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D2 Paper F – Marking Guide

1. *(a)*

$x_{11} = \begin{cases} 1 & \text{if Alan is assigned to the lawns} \\ 0 & \text{otherwise} \end{cases}$

$x_{12} = \begin{cases} 1 & \text{if Alan is assigned to the hedgerows} \\ 0 & \text{otherwise} \end{cases}$

$x_{13} = \begin{cases} 1 & \text{if Alan is assigned to the flower beds} \\ 0 & \text{otherwise} \end{cases}$

$x_{21} = \begin{cases} 1 & \text{if Beth is assigned to the lawns} \\ 0 & \text{otherwise} \end{cases}$

$x_{22} = \begin{cases} 1 & \text{if Beth is assigned to the hedgerows} \\ 0 & \text{otherwise} \end{cases}$

$x_{23} = \begin{cases} 1 & \text{if Beth is assigned to the flower beds} \\ 0 & \text{otherwise} \end{cases}$

$x_{31} = \begin{cases} 1 & \text{if Colin is assigned to the lawns} \\ 0 & \text{otherwise} \end{cases}$ B2

$x_{32} = \begin{cases} 1 & \text{if Colin is assigned to the hedgerows} \\ 0 & \text{otherwise} \end{cases}$

$x_{33} = \begin{cases} 1 & \text{if Colin is assigned to the flower beds} \\ 0 & \text{otherwise} \end{cases}$

(b) minimise

$z = 4x_{11} + 4.5x_{12} + 6x_{13} + 3x_{21} + 4x_{22} + 5x_{23} + 3.5x_{31} + 5x_{32} + 6x_{33}$ B2

(c)

$x_{11} + x_{12} + x_{13} = 1$	Alan has exactly one job	
$x_{21} + x_{22} + x_{23} = 1$	Beth has exactly one job	
$x_{31} + x_{32} + x_{33} = 1$	Colin has exactly one job	
$x_{11} + x_{21} + x_{31} = 1$	lawns are done by one gardener	
$x_{12} + x_{22} + x_{32} = 1$	hedgerows are done by one gardener	M1 A1
$x_{13} + x_{23} + x_{33} = 1$	flower beds are done by one gardener	

$x_{ij} \geq 0$ for all i, j

reference to balance B1 **(7)**

2.

Stage	Previous tournament	Current tournament	Value
1	*G*	*J* *K* *L*	2 4* 1
	H	*J* *K* *L*	3* 2 2
	I	*J* *K* *L*	2 5* 3
2	*D*	*G* *H* *I*	min(5, 4) = 4* min(3, 3) = 3 min(3, 5) = 3
	E	*G* *H* *I*	min(3, 4) = 3 min(5, 3) = 3 min(6, 5) = 5*
	F	*G* *H* *I*	min(3, 4) = 3 min(6, 3) = 3 min(5, 5) = 5*
3	*A*	*D* *E* *F*	min(6, 4) = 4 min(3, 5) = 3 min(7, 5) = 5*
	B	*D* *E* *F*	min(5, 4) = 4 min(5, 5) = 5* min(4, 5) = 4
	C	*D* *E* *F*	min(7, 4) = 4 min(5, 5) = 5* min(5, 5) = 5*
4	*None*	*A* *B* *C*	min(5, 5) = 5* min(3, 5) = 3 min(3, 5) = 3

M1 A1 (stage 1)

M1 A2 (stage 2)

M1 A1 (stage 3)

A1 (stage 4)

he should play *A*, *F*, *I* and *K* M1 A1 **(10)**

3.

				row min.
5	20	12	18	5
6	18	15	16	6
4	21	9	15	4
5	16	11	13	5

reducing rows gives:

	0	15	7	13
	0	12	9	10
	0	17	5	11
	0	11	6	8
col min.	0	11	5	8

M1 A1

reducing columns gives:

0	4	2	5
0	1	4	2
0	6	0	3
1	0	1	0

(N.B. a different choice of lines will lead to the same final assignment) A1

3 lines required to cover all zeros, apply algorithm B1

0^*	3	1	4
0	0^*	3	1
1	6	0^*	3
2	0	1	0^*

M1 A1

4 lines required to cover all zeros so allocation is possible B1

Andrew reviews a film
Betty reviews a musical
Carlos reviews a ballet
Davina reviews a concert M1 A1

total cost = 5 + 18 + 9 + 13 = £45 A1 **(10)**

4. *(a)*

		B I	B II	row minimum
A	I	4	$^{-}8$	$^{-}8$
	II	2	$^{-}4$	$^{-}4$
	III	$^{-}8$	2	$^{-}8$
column maximum		4	2	

M1 A1

max (row min) = $^{-}4$ min (col max) = 2

max (row min) ≠ min (col max) ∴ no saddle point B1

(b) let *B* play strategies I and II with proportions q and $(1 - q)$

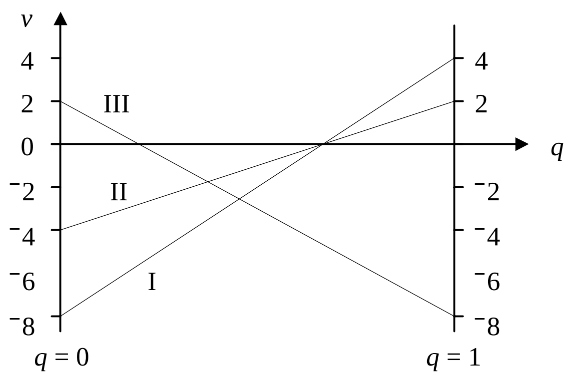
expected loss for *B* against each of *A*'s strategies:

A I $4q - 8(1 - q) = 12q - 8$

A II $2q - 4(1 - q) = 6q - 4$ M1 A1

A III $^{-}8q + 2(1 - q) = 2 - 10q$

giving



B2

it is not worth player *A* considering strategy I

for optimal strategy $6q - 4 = 2 - 10q$

∴ $16q = 6$, $q = \frac{3}{8}$

∴ *B* should play I $\frac{3}{8}$ of time and II $\frac{5}{8}$ of time M1 A1

(c) let *A* play strategies II and III with proportions p and $(1 - p)$

expected payoff to *A* against each of *B*'s strategies:

B I $2p - 8(1 - p) = 10p - 8$

B II $^{-}4p + 2(1 - p) = 2 - 6p$ M1 A1

for optimal strategy $10p - 8 = 2 - 6p$

∴ $16p = 10$, $p = \frac{5}{8}$

∴ *A* should play I never, II $\frac{5}{8}$ of time and III $\frac{3}{8}$ of time M1 A1

(d) value of game = $(6 \times \frac{3}{8}) - 4 = {}^{-}1\frac{3}{4}$ M1 A1 **(15)**

5. *(a)* add dummy M1

	S_1	S_2	S_3	Available
W_1	40	5		45
W_2		18	22	40
Dummy			15	15
Required	40	23	37	

M1 A1

(b) taking $R_1 = 0$, $R_1 + K_1 = 8 \therefore K_1 = 8$ $R_1 + K_2 = 7 \therefore K_2 = 7$
$R_2 + K_2 = 10 \therefore R_2 = 3$ $R_2 + K_3 = 11 \therefore K_3 = 8$ M1 A2
$R_3 + K_3 = 0 \therefore R_3 = ^-8$

	$K_1 = 8$	$K_2 = 7$	$K_3 = 8$
$R_1 = 0$	(0)	(0)	11
$R_2 = 3$	9	(0)	(0)
$R_3 = ^-8$	0	0	(0)

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

$\therefore$ $I_{13} = 11 - 0 - 8 = 3$
$I_{21} = 9 - 3 - 8 = ^-2$
$I_{31} = 0 - (^-8) - 8 = 0$ M1 A1
$I_{32} = 0 - (^-8) - 7 = 1$

(c) applying algorithm

	S_1	S_2	S_3
W_1	$40 - \theta$	$5 + \theta$	
W_2	θ	$18 - \theta$	22
Dummy			15

M1

let $\theta = 18$, giving

	S_1	S_2	S_3
W_1	22	23	
W_2	18		22
Dummy			15

A1

taking $R_1 = 0$, $R_1 + K_1 = 8 \therefore K_1 = 8$ $R_1 + K_2 = 7 \therefore K_2 = 7$
$R_2 + K_1 = 9 \therefore R_2 = 1$ $R_2 + K_3 = 11 \therefore K_3 = 10$ M1 A1
$R_3 + K_3 = 0 \therefore R_3 = ^-10$

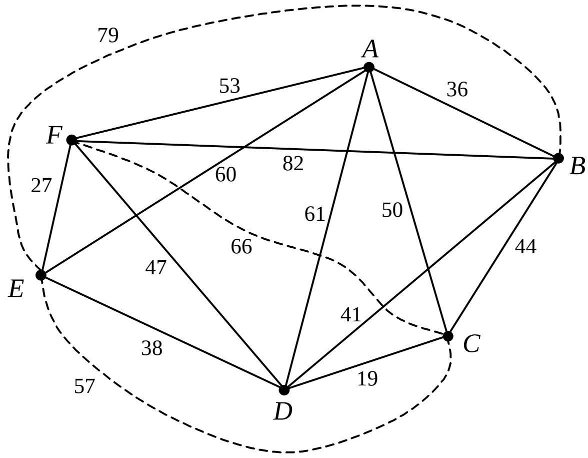
	$K_1 = 8$	$K_2 = 7$	$K_3 = 10$
$R_1 = 0$	(0)	(0)	11
$R_2 = 1$	(0)	10	(0)
$R_3 = ^-10$	0	0	(0)

$\therefore$ $I_{13} = 11 - 0 - 10 = 1$
$I_{22} = 10 - 1 - 7 = 2$
$I_{31} = 0 - (^-10) - 8 = 2$ M1 A1
$I_{32} = 0 - (^-10) - 7 = 3$

all improvement indices are non-negative $\therefore$ pattern is optimal B1

22 rolls from W_1 to S_1, 23 rolls from W_1 to S_2, 18 rolls from W_2 to S_1,
22 rolls from W_2 to S_3, S_3 still requires 15 rolls A1 **(16)**

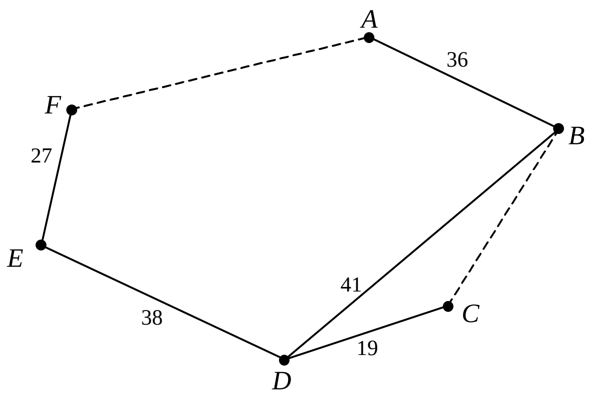
6. *(a)*



add BE – 79, CE – 57, CF – 66 M1 A2

(b) AB (36), BD (41), DC (19), CE (57), EF (27), FA (53) M1 A1
tour: $ABDCEFA$
upper bound = 36 + 41 + 19 + 57 + 27 + 53 = 233 miles A1

(c) (i)

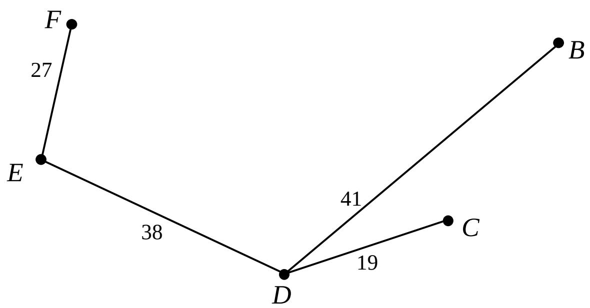


M1 A1

upper bound = 2 × weight of MST
= 2 × (36 + 41 + 19 + 38 + 27) = 2 × 161 = 322 miles M1 A1

(ii) use AF saving 36 + 41 + 38 + 27 – 53 = 89
use BC saving 41 + 19 – 44 = 16 M1 A1
new upper bound = 322 – 89 – 16 = 217 miles A1

(d)



B1

lower bound = weight of MST + two edges of least weight from A
= (41 + 19 + 38 + 27) + 36 + 50 = 211 miles M1 A1

(e) $211 \le d \le 217$ B1 **(17)**

Total **(75)**

Performance Record – D2 Paper F

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	Total
Topic(s)	allocation, formulate lin. prog.	dynamic prog., maximin.	allocation	game, graphical method	transport., dummy, n-w corner, stepping-stone	TSP, nearest neighbour, shortcuts	
Marks	7	10	10	15	16	17	75
Student							