GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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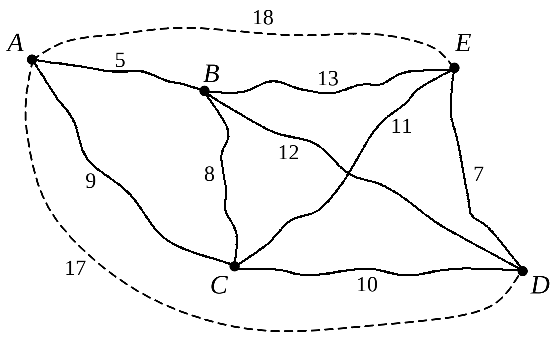
D2 Paper C – Marking Guide

1. *(a)*



add $AD - 17$, $AE - 18$ — M1 A1

(b) AB (5), BC (8), CD (10), DE (7), EA (18) — M1
tour: $ABCDEA$
upper bound = 5 + 8 + 10 + 7 + 18 = 48 miles — A1

(c) actual tour is $ABCDEBA$ as EA is not in original network — M1 A1 **(6)**

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2. *(a)* adding 4 to all entries to make them positive gives — M1

		B		
		I	II	III
A	I	3	8	1
	II	1	11	5
	III	9	2	3

new value of game $v = V + 4$ — A1

(b) let B play strategies I, II and III with proportions p_1, p_2 and p_3 — M1
let $x_1 = \frac{p_1}{v}$, $x_2 = \frac{p_2}{v}$, $x_3 = \frac{p_3}{v}$ — A1

(c) $p_1 + p_2 + p_3 = 1$ — M1
dividing by v gives $x_1 + x_2 + x_3 = \frac{1}{v}$
we wish to minimise v $\therefore$ maximise $\frac{1}{v}$
objective function is maximise $P = x_1 + x_2 + x_3$ — A1

(d) from A I, $3p_1 + 8p_2 + p_3 \leq v$
from A II, $p_1 + 11p_2 + 5p_3 \leq v$ — M1
from A III, $9p_1 + 2p_2 + 3p_3 \leq v$

dividing by v gives the constraints

$3x_1 + 8x_2 + x_3 \leq 1$
$x_1 + 11x_2 + 5x_3 \leq 1$
$9x_1 + 2x_2 + 3x_3 \leq 1$ — A1 **(8)**
also $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$

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3.

Stage	State	Action	Destination	Value	
1	*I*	*IL*	*L*	19*	
	J	*JL*	*L*	18*	
	K	*KL*	*L*	26*	A1
2	*E*	*EI* *EJ*	*I* *J*	max(35, 19) = 35 max(29, 18) = 29*	
	F	*FI* *FJ* *FK*	*I* *J* *K*	max(17, 19) = 19* max(24, 18) = 24 max(15, 26) = 26	
	G	*GI* *GJ* *GK*	*I* *J* *K*	max(18, 19) = 19* max(26, 18) = 26 max(14, 26) = 26	
	H	*HJ* *HK*	*J* *K*	max(17, 18) = 18* max(39, 26) = 39	M1 A2
3	*B*	*BE* *BF* *BG*	*E* *F* *G*	max(21, 29) = 29 max(25, 19) = 25* max(28, 19) = 28	
	C	*CE* *CF* *CG* *CH*	*E* *F* *G* *H*	max(28, 29) = 29 max(30, 19) = 30 max(40, 19) = 40 max(28, 18) = 28*	
	D	*DF* *DG* *DH*	*F* *G* *H*	max(38, 19) = 38 max(24, 19) = 24* max(35, 18) = 35	M1 A1
4	*A*	*AB* *AC* *AD*	*B* *C* *D*	max(19, 25) = 25 max(12, 28) = 28 max(7, 24) = 24*	A1

giving route *ADGIL* M1 A1 **(9)**

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4.

	W_1	W_2	W_3	Available
S_1	20	10		30
S_2		5	20	25
S_3			10	10
Required	20	15	30	

M1 A1

taking $R_1 = 0$, $R_1 + K_1 = 12 \therefore K_1 = 12$
$R_1 + K_2 = 11 \therefore K_2 = 11$
$R_2 + K_2 = 5 \therefore R_2 = {}^-6$
$R_2 + K_3 = 10 \therefore K_3 = 16$
$R_3 + K_3 = 8 \therefore R_3 = {}^-8$ M1 A2

	$K_1 = 12$	$K_2 = 11$	$K_3 = 16$
$R_1 = 0$	0	0	17
$R_2 = {}^-6$	7	0	0
$R_3 = {}^-8$	5	6	0

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

$\therefore$ $I_{13} = 17 - 0 - 16 = 1$
$I_{21} = 7 - ({}^-6) - 12 = 1$
$I_{31} = 5 - ({}^-8) - 12 = 1$ M1 A1
$I_{32} = 6 - ({}^-8) - 11 = 3$

pattern is optimal as there are no negative improvement indices B1

optimal pattern:
20 rolls from S_1 to W_1, 10 rolls from S_1 to W_2, 5 rolls from S_2 to W_2,
20 rolls from S_2 to W_3, 10 rolls from S_3 to W_3 A1

total cost = $(20 \times 12) + (10 \times 11) + (5 \times 5) + (20 \times 10) + (10 \times 8)$ = £655 M1 A1 **(11)**

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5. need to maximise so subtract all values from 9 giving — M1

$$\begin{array}{cccc|c} & & & & \text{row min.} \\ 2 & 1 & 4 & 3 & 1 \\ 3 & 0 & 3 & 4 & 0 \\ 0 & 1 & 4 & 2 & 0 \\ 2 & 2 & 3 & 3 & 2 \end{array}$$

reducing rows gives:

$$\begin{array}{rcccc} & 1 & 0 & 3 & 2 \\ & 3 & 0 & 3 & 4 \\ & 0 & 1 & 4 & 2 \\ & 0 & 0 & 1 & 1 \\ \hline \text{col min.} & 0 & 0 & 1 & 1 \end{array}$$

M1 A1

reducing columns gives:

$$\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 3 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

(N.B. a different choice of lines will lead to the same final assignment) — A1

3 lines required to cover all zeros, apply algorithm — B1

$$\begin{array}{cccc} 1 & 0 & 1 & 0^* \\ 3 & 0^* & 1 & 2 \\ 0^* & 1 & 2 & 0 \\ 1 & 1 & 0^* & 0 \end{array}$$

M1 A1

4 lines are required to cover all zeros so allocation is possible — B1

stage 1 – *C*
stage 2 – *B*
stage 3 – *D*
stage 4 – *A*

M1 A1

total number of days = 9 + 9 + 6 + 6 = 30 days — A1 **(11)**

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6. *(a)* (i) strategy III dominates II since $9 \geq 7,\ ^{-}4 \geq {}^{-}4,\ 8 \geq {}^{-}1$
player A can ignore strategy II — M1 A1

(ii) strategy III dominates I since $^{-}2 \leq 3,\ ^{-}1 \leq 7,\ 8 \leq 9$
player B can ignore strategy I — A1

(b) reduced table:

		B	
		II	III
A	I	5	$^{-}2$
	III	$^{-}4$	8

(i) let A play strategies I and III with proportions p and $(1 - p)$

expected payoff to A against each of B's strategies:

B II $\quad 5p - 4(1 - p) = 9p - 4$
B III $\quad {}^{-}2p + 8(1 - p) = 8 - 10p$ — M1 A1

for optimal strategy $\quad 9p - 4 = 8 - 10p$
$\therefore\ 19p = 12,\ \ p = \frac{12}{19}$

$\therefore$ A should play I $\frac{12}{19}$ of time, II never and III $\frac{7}{19}$ of time — M1 A1

(i) let B play strategies II and III with proportions q and $(1 - q)$

expected loss to B against each of A's strategies:

A I $\quad 5q - 2(1 - q) = 7q - 2$
A III $\quad {}^{-}4q + 8(1 - q) = 8 - 12q$ — M1 A1

for optimal strategy $\quad 7q - 2 = 8 - 12q$
$\therefore\ 19q = 10,\ \ q = \frac{10}{19}$

$\therefore$ B should play I never, II $\frac{10}{19}$ of time and III $\frac{9}{19}$ of time — M1 A1

(c) value of game $= (9 \times \frac{12}{19}) - 4 = 1\frac{13}{19}$ — M1 A1 **(13)**

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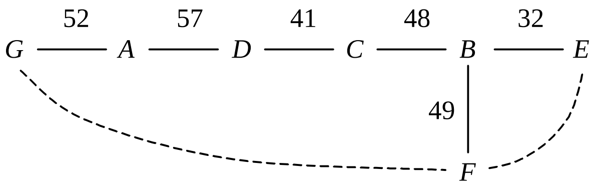
7. *(a)* e.g. starting at *A*

order:	1	5	4	3	6	7	2
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	63	75	57	81	102	52
B	63	–	$\boxed{48}$	83	32	49	61
C	75	48	–	$\boxed{41}$	72	65	109
D	$\boxed{57}$	83	41	–	49	79	70
E	81	$\boxed{32}$	72	49	–	51	88
F	102	$\boxed{49}$	65	79	51	–	90
G	$\boxed{52}$	61	109	70	88	90	–

M1 A2



A1

upper bound = 2 × weight of MST

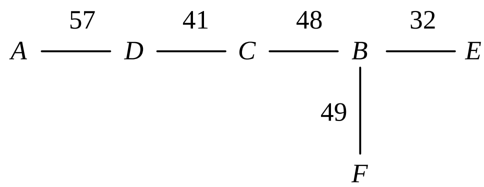
= 2 × (52 + 57 + 41 + 48 + 32 + 49) = 2 × 279 = 558 km M1 A1

(b) use *FG* saving 52 + 57 + 41 + 48 + 49 – 90 = 157

use *EF* saving 32 + 49 – 51 = 30 M1 A1

new upper bound = 558 – 157 – 30 = 371 km A1

(c)



M1

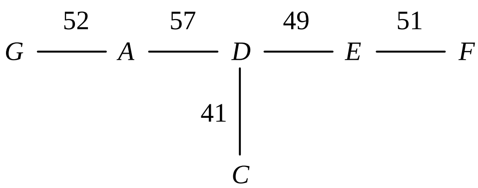
lower bound = weight of MST + two edges of least weight from *G*

= (57 + 41 + 48 + 32 + 49) + 52 + 61 = 340 km M1 A1

(d) e.g. starting at *A*

order:	1		4	3	5	6	2
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	63	75	57	81	102	52
B	63	–	48	83	32	49	61
C	75	48	–	$\boxed{41}$	72	65	109
D	$\boxed{57}$	83	41	–	49	79	70
E	81	32	72	$\boxed{49}$	–	51	88
F	102	49	65	79	$\boxed{51}$	–	90
G	$\boxed{52}$	61	109	70	88	90	–

M1 A1



lower bound = weight of MST + two edges of least weight from *B*

= (52 + 57 + 49 + 51 + 41) + 32 + 48 = 330 km M1 A1

(e) 340 km, from *(c)* is better as it is higher B1 **(17)**

Total **(75)**

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Performance Record – D2 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	nearest neighbour	game, formulate lin. prog.	dynamic prog., minimax	transport., n-w corner, improv. indices	allocation, maximum	game, dominance	TSP, shortcuts	
Marks	6	8	9	11	11	13	17	75
Student								

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