GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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D2 Paper B – Marking Guide

1.

start at *A*: tour is *AEDBCA*	length = 6 + 9 + 7 + 11 + 16 = 49 km	M1 A1	
start at *B*: tour is *BDEACB*	length = 7 + 9 + 6 + 16 + 11 = 49 km		
start at *C*: tour is *CBDEAC*	length = 11 + 7 + 9 + 6 + 16 = 49 km		
start at *D*: tour is *DBAECD*	length = 7 + 8 + 6 + 14 + 13 = 48 km		
start at *E*: tour is *EABDCE*	length = 6 + 8 + 7 + 13 + 14 = 48 km	M1 A2	
best upper bound = 48 km		A1	**(6)**

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2. *(a)*

$x_{11} = \begin{cases} 1 & \text{if team 1 is assigned to Maths} \\ 0 & \text{otherwise} \end{cases}$

$x_{12} = \begin{cases} 1 & \text{if team 1 is assigned to English} \\ 0 & \text{otherwise} \end{cases}$

$x_{13} = \begin{cases} 1 & \text{if team 1 is assigned to Verbal} \\ 0 & \text{otherwise} \end{cases}$

$x_{21} = \begin{cases} 1 & \text{if team 2 is assigned to Maths} \\ 0 & \text{otherwise} \end{cases}$

$x_{22} = \begin{cases} 1 & \text{if team 2 is assigned to English} \\ 0 & \text{otherwise} \end{cases}$

$x_{23} = \begin{cases} 1 & \text{if team 2 is assigned to Verbal} \\ 0 & \text{otherwise} \end{cases}$

$x_{31} = \begin{cases} 1 & \text{if team 3 is assigned to Maths} \\ 0 & \text{otherwise} \end{cases}$ B2

$x_{32} = \begin{cases} 1 & \text{if team 3 is assigned to English} \\ 0 & \text{otherwise} \end{cases}$

$x_{33} = \begin{cases} 1 & \text{if team 3 is assigned to Verbal} \\ 0 & \text{otherwise} \end{cases}$

(b) minimise

$z = 3x_{11} + 9x_{12} + 2x_{13} + 4x_{21} + 7x_{22} + x_{23} + 5x_{31} + 8x_{32} + 3x_{33}$ B2

(c)

$x_{11} + x_{12} + x_{13} = 1$	team 1 marks exactly one style of paper	
$x_{21} + x_{22} + x_{23} = 1$	team 2 marks exactly one style of paper	
$x_{31} + x_{32} + x_{33} = 1$	team 3 marks exactly one style of paper	
$x_{11} + x_{21} + x_{31} = 1$	Maths papers are marked by one team only	
$x_{12} + x_{22} + x_{32} = 1$	English papers are marked by one team only	M1 A1
$x_{13} + x_{23} + x_{33} = 1$	Verbal papers are marked by one team only	
$x_{ij} \geq 0$ for all i, j		
reference to balance		B1 **(7)**

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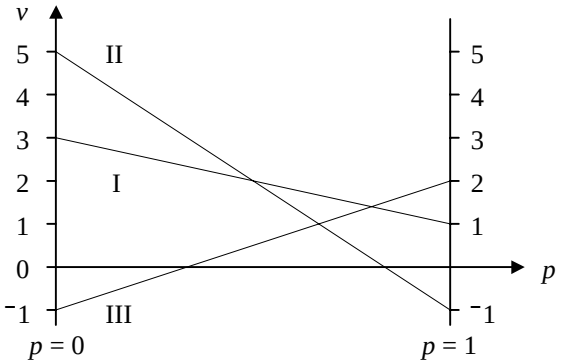
3. *(a)* let *A* play strategies I and II with proportions p and $(1 - p)$

expected payoff to *A* against each of *B*'s strategies:

B I $\quad p + 3(1 - p) = 3 - 2p$
B II $\quad -p + 5(1 - p) = 5 - 6p$ M1 A1
B III $\quad 2p - (1 - p) = 3p - 1$

giving



B2

it is not worth player *B* considering strategy I A1

(b) for optimal strategy $5 - 6p = 3p - 1$

$\therefore 9p = 6, \; p = \frac{2}{3}$

$\therefore$ *A* should play I $\frac{2}{3}$ of time and II $\frac{1}{3}$ of time M1 A1

value of original game $= 5 - (6 \times \frac{2}{3}) = 1$ M1 A1 **(9)**

4.

Stage	State	Action	Destination	Total Profit
1	*G*	*GI*	*I*	12*
	H	*HI*	*I*	10*
2	*D*	*DG*	*G*	14 + 12 = 26
		DH	*H*	17 + 10 = 27*
	E	*EG*	*G*	12 + 12 = 24
		EH	*H*	18 + 10 = 28*
	F	*FG*	*G*	13 + 12 = 25
		FH	*H*	19 + 10 = 29*
3	*A*	*AD*	*D*	8 + 27 = 35
		AE	*E*	10 + 28 = 38
		AF	*F*	14 + 29 = 43*
	B	*BD*	*D*	12 + 27 = 39
		BE	*E*	10 + 28 = 38
		BF	*F*	16 + 29 = 45*
	C	*CD*	*D*	9 + 27 = 36
		CE	*E*	13 + 28 = 41
		CF	*F*	15 + 29 = 44*
4	*Home*	*Home-A*	*A*	15 + 43 = 58*
		Home-B	*B*	11 + 45 = 56
		Home-C	*C*	13 + 44 = 57

A1

M1 A2

M1 A1

A1

giving route *HomeAFHI* M1 A1

expected profit = £580 A1 **(10)**

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5. need to add dummy row giving M1

				row min.
27	80	8	81	8
28	60	5	71	5
30	90	7	73	7
0	0	0	0	0

reducing rows gives:

19	72	0	73
23	55	0	66
23	83	0	66
~~0~~	~~0~~	~~0~~	~~0~~

M1 A1

reducing columns will make no difference B1

2 lines required to cover all zeros, apply algorithm B1

0	53	0	54
4	36	0	47
4	64	0	47
~~0~~	~~0~~	~~19~~	~~0~~

(N.B. a different choice of lines will lead to the same final assignment)

M1 A1

3 lines required to cover all zeros, apply algorithm

~~0*~~	~~17~~	~~0~~	~~18~~
~~4~~	~~0*~~	~~0~~	~~11~~
4	28	0*	11
~~36~~	~~0~~	~~55~~	~~0*~~

M1 A1

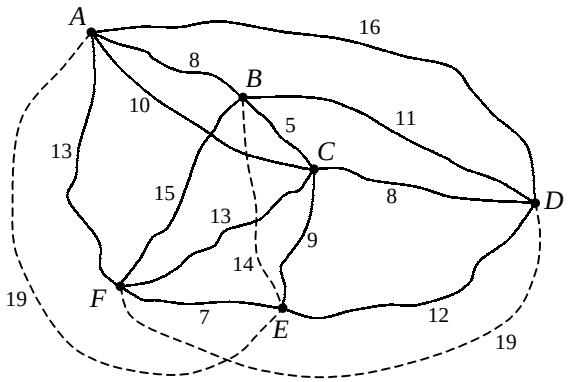
4 lines required to cover all zeros so allocation is possible B1

team A does the windows
team B does the conservatory
team C does the doors
the greenhouse is not done

M1 A1

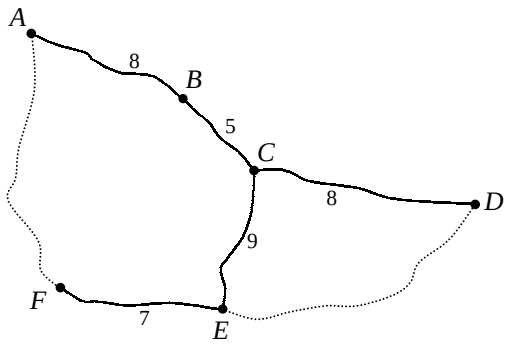
total cost = $10 \times (27 + 60 + 7) = £940$ A1 **(13)**

6. *(a)*



add $AE - 19$, $BE - 14$, $DF - 19$ M1 A2

(b)

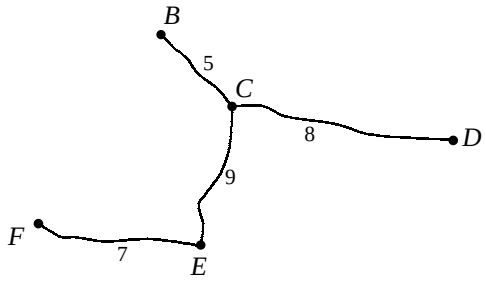


M1 A2

upper bound = $2 \times$ weight of MST
$= 2 \times (8 + 5 + 8 + 9 + 7) = 2 \times 37 = 74$ miles M1 A1

(c) use AF saving $8 + 5 + 9 + 7 - 13 = 16$
use DE saving $8 + 9 - 12 = 5$ M1 A1
new upper bound $= 74 - 16 - 5 = 53$ miles A1

(d)



M1

lower bound = weight of MST + two edges of least weight from A
$= (5 + 8 + 9 + 7) + 8 + 10 = 47$ miles M1 A1 **(14)**

7. *(a)* add dummy M1

	A	*B*	*Dummy*	Available
C	7			7
D	3	2		5
E		4	4	8
Required	10	6	4	

M1 A1

(b) taking $R_1 = 0$, $R_1 + K_1 = 2 \ \therefore K_1 = 2$ $R_2 + K_1 = 2 \ \therefore R_2 = 0$
$R_2 + K_2 = 5 \ \therefore K_2 = 5$ $R_3 + K_2 = 6 \ \therefore R_3 = 1$
$R_3 + K_3 = 0 \ \therefore K_3 = \bar{}1$ M1 A2

	$K_1 = 2$	$K_2 = 5$	$K_3 = \bar{}1$
$R_1 = 0$	(0)	3	0
$R_2 = 0$	(0)	(0)	0
$R_3 = 1$	7	(0)	(0)

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

$\therefore \ I_{12} = 3 - 0 - 5 = \bar{}2$
$I_{13} = 0 - 0 - (\bar{}1) = 1$
$I_{23} = 0 - 0 - (\bar{}1) = 1$ M1 A1
$I_{31} = 7 - 1 - 2 = 4$

(c) pattern not optimal $\therefore$ apply algorithm

	A	*B*	*Dummy*
C	$7 - \theta$	θ	
D	$3 + \theta$	$2 - \theta$	
E		4	4

M1

let $\theta = 2$

	A	*B*	*Dummy*
C	5	2	
D	5		
E		4	4

A1

taking $R_1 = 0$, $R_1 + K_1 = 2 \ \therefore K_1 = 2$ $R_1 + K_2 = 3 \ \therefore K_2 = 3$
$R_2 + K_1 = 2 \ \therefore R_2 = 0$ $R_3 + K_2 = 6 \ \therefore R_3 = 3$
$R_3 + K_3 = 0 \ \therefore K_3 = \bar{}3$ M1 A1

	$K_1 = 2$	$K_2 = 3$	$K_3 = \bar{}3$
$R_1 = 0$	(0)	(0)	0
$R_2 = 0$	(0)	5	0
$R_3 = 3$	7	(0)	(0)

$\therefore \ I_{13} = 0 - 0 - (\bar{}3) = 3$
$I_{22} = 5 - 0 - 3 = 2$
$I_{23} = 0 - 0 - (\bar{}3) = 3$ M1 A1
$I_{31} = 7 - 3 - 2 = 2$

all improvement indices are non-negative $\therefore$ pattern is optimal B1

$\therefore$ 5 from *C* go to *A*, 2 from *C* go to *B*, 5 from *D* go to *A*
4 from *E* go to *B*, 4 from *E* do not play A1 **(16)**

Total **(75)**

Performance Record – D2 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	nearest neighbour	allocation, formulate lin. prog.	game, graphical method	dynamic prog., max.	allocation, dummy	TSP, shortcuts	transport., n-w corner, stepping-stone	
Marks	6	7	9	10	13	14	16	75
Student								