OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4737

Decision Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 6 printed pages, 2 blank pages and an insert.

1 [Answer this question on the insert provided.]

Six neighbours have decided to paint their houses in bright colours. They will each use a different colour.

- Arthur wants to use lavender, orange or tangerine.
- Bridget wants to use lavender, mauve or pink.
- Carlos wants to use pink or scarlet.
- Davinder wants to use mauve or pink.
- Eric wants to use lavender or orange.
- Ffion wants to use mauve.

Arthur chooses lavender, Bridget chooses mauve, Carlos chooses pink and Eric chooses orange. This leaves Davinder and Ffion with colours that they do not want.

(i) Draw a bipartite graph on the insert, showing which neighbours (A, B, C, D, E, F) want which colours (L, M, O, P, S, T). On a separate diagram on the insert, show the incomplete matching described above. [3]

(ii) By constructing alternating paths obtain the complete matching between the neighbours and the colours. Give your paths and show your matching on the insert. [4]

(iii) Fill in the table on the insert to show how the Hungarian algorithm could have been used to find the complete matching. (You do **not** need to carry out the Hungarian algorithm.) [2]

2 A company has organised four regional training sessions to take place at the same time in four different cities. The company has to choose four of its five trainers, one to lead each session. The cost (£1000's) of using each trainer in each city is given in the table.

		City			
		London	Glasgow	Manchester	Swansea
Trainer	Adam	4	3	2	4
	Betty	3	5	4	2
	Clive	3	6	3	3
	Dave	2	6	4	3
	Eleanor	2	5	3	4

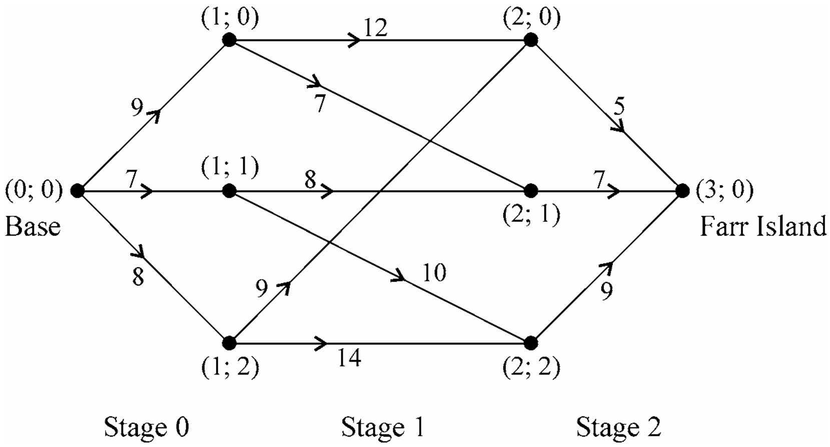
(i) Convert this into a square matrix and then apply the Hungarian algorithm, **reducing rows first**, to allocate the trainers to the cities at minimum cost. [7]

(ii) Betty discovers that she is not available on the date set for the training. Find the new minimum cost allocation of trainers to cities. [2]

3 [Answer this question on the insert provided.]

A flying doctor travels between islands using small planes. Each flight has a weight limit that restricts how much he can carry. A plague has broken out on Farr Island and the doctor needs to take several crates of medical supplies to the island. The crates must be carried on the same planes as the doctor.

The diagram shows a network with (stage; state) variables at the vertices representing the islands, arcs representing flight routes that can be used, and weights on the arcs representing the number of crates that the doctor can carry on each flight.



(i) It is required to find the route from (0; 0) to (3; 0) for which the minimum number of crates that can be carried on any stage is a maximum (the maximin route). The insert gives a dynamic programming tabulation showing stages, states and actions, together with columns for working out the route minimum at each stage and for indicating the current maximin.

Complete the table on the insert sheet and hence find the maximin route and the maximum number of crates that can be carried. [7]

(ii) It is later found that the number of crates that can be carried on the route from (2; 0) to (3; 0) has been recorded incorrectly and should be 15 instead of 5. What is the maximin route now, and how many crates can be carried? [3]

[Turn over

4 Henry is planning a surprise party for Lucinda. He has left the arrangements until the last moment, so he will hold the party at their home. The table below lists the activities involved, the expected durations, the immediate predecessors and the number of people needed for each activity. Henry has some friends who will help him, so more than one activity can be done at a time.

Activity	Duration (hours)	Preceded by	Number of people
A: Telephone other friends	2	–	3
B: Buy food	1	A	2
C: Prepare food	4	B	5
D: Make decorations	3	A	3
E: Put up decorations	1	D	3
F: Guests arrive	1	C, E	1

(i) Draw an activity network to represent these activities and the precedences. Carry out forward and reverse passes to determine the minimum completion time and the critical activities. If Lucinda is expected home at 7.00 p.m., what is the latest time that Henry or his friends can begin telephoning the other friends? [7]

(ii) Draw a resource histogram showing time on the horizontal axis and number of people needed on the vertical axis, assuming that each activity starts at its earliest possible start time. What is the maximum number of people needed at any one time? [3]

(iii) Now suppose that Henry's friends can start buying the food and making the decorations as soon as the telephoning begins. Construct a timetable, with a column for 'time' and a column for each person, showing who should do which activity when, in order than the party can be organised in the minimum time using a total of only six people (Henry and five friends). When should the telephoning begin with this schedule? [3]

5 [Answer this question on the insert provided.]

Fig. 1 shows a directed flow network. The weight on each arc shows the capacity in litres per second.

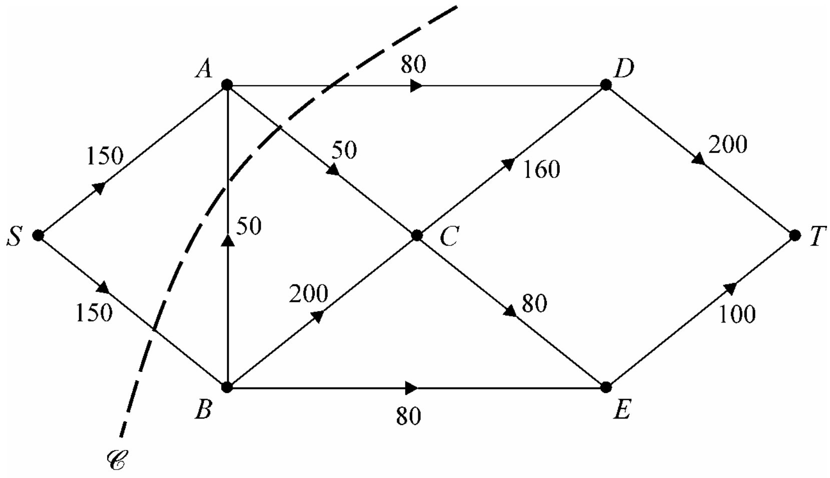


Fig. 1

(i) Find the capacity of the cut 𝒞 shown. [2]

(ii) Deduce that there is no possible flow from S to T in which both arcs leading into T are saturated. Explain your reasoning clearly. [2]

Fig. 2 shows a possible flow of 160 litres per second through the network.

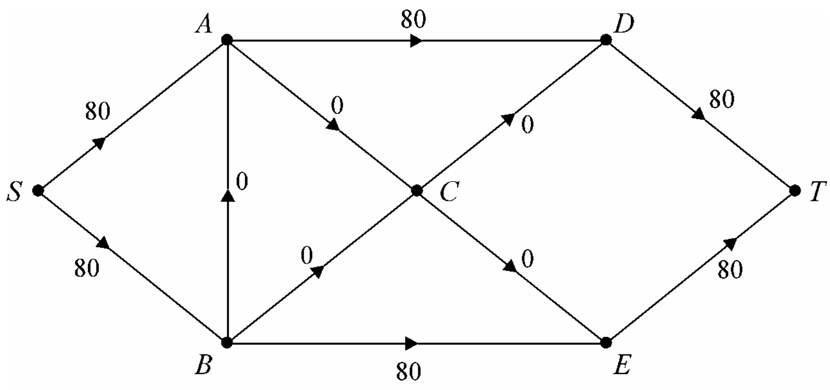


Fig. 2

(iii) On the diagram in the insert, show the excess capacities and potential backflows for this flow. [3]

(iv) Use the labelling procedure to augment the flow as much as possible. Show your working clearly, but do not obscure your answer to part **(iii)**. [4]

(v) Show the final flow that results from part **(iv)**. Explain clearly how you know that this flow is maximal. [3]

6 Rose is playing a game against a computer. Rose aims a laser beam along a row, *A*, *B* or *C*, and, at the same time, the computer aims a laser beam down a column, *X*, *Y* or *Z*. The number of points won by Rose is determined by where the two laser beams cross. These values are given in the table. The computer loses whatever Rose wins.

		Computer		
		X	*Y*	*Z*
Rose	*A*	1	3	4
	B	4	3	2
	C	3	2	1

(i) Find Rose's play-safe strategy and show that the computer's play-safe strategy is *Y*. How do you know that the game does not have a stable solution? [3]

(ii) Explain why Rose should never choose row *C* and hence reduce the game to a 2×3 pay-off matrix. [2]

(iii) Rose intends to play the game a large number of times. She decides to use a standard six-sided die to choose between row *A* and row *B*, so that row *A* is chosen with probability *a* and row *B* is chosen with probability $1-a$. Show that the expected pay-off for Rose when the computer chooses column *X* is $4-3a$, and find the corresponding expressions for when the computer chooses column *Y* and when it chooses column *Z*. Sketch a graph showing the expected pay-offs against *a*, and hence decide on Rose's optimal choice for *a*. Describe how Rose could use the die to decide whether to play *A* or *B*. [6]

The computer is to choose *X*, *Y* and *Z* with probabilities *x*, *y* and *z* respectively, where $x+y+z=1$. Graham is an AS student studying the D1 module. He wants to find the optimal choices for *x*, *y* and *z* and starts off by producing a pay-off matrix for the computer.

(iv) Graham produces the following pay-off matrix.

3	1	0
0	1	2

Write down the pay-off matrix for the computer and explain what Graham did to its entries to get the values in his pay-off matrix. [2]

(v) Graham then sets up the linear programming problem:

maximise $P = p - 4$,

subject to
$$p - 3x - y \leqslant 0,$$
$$p - y - 2z \leqslant 0,$$
$$x + y + z \leqslant 1,$$

and $p \geqslant 0,\ x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0.$

The Simplex algorithm is applied to the problem and gives $x = 0.4$ and $y = 0$. Find the values of *z*, *p* and *P* and interpret the solution in the context of the game. [4]

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Candidate Name	Centre Number	Candidate Number

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MATHEMATICS **4737**

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INSERT for Questions 1, 3 and 5

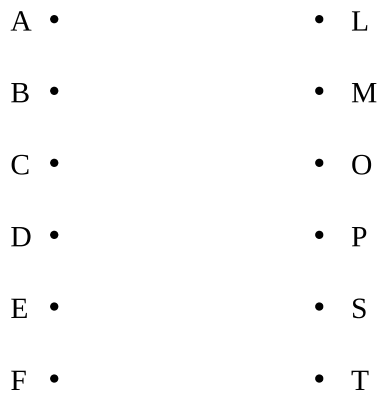
Specimen Paper

INSTRUCTIONS TO CANDIDATES

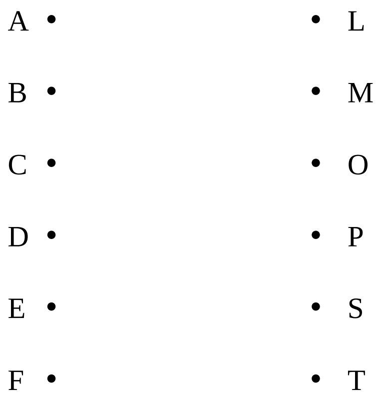
- This insert should be used to answer Questions **1**, **3** and **5**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Write your answers to Questions **1**, **3** and **5** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 4 printed pages.

1 **(i)**



Bipartite graph



Matching described in question

(ii) ....

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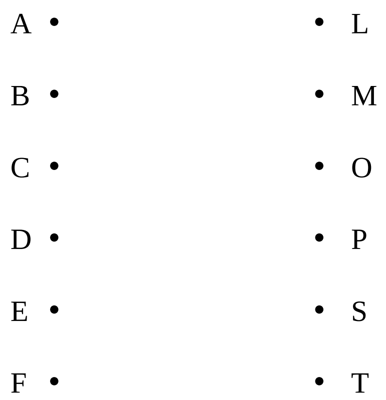


(iii)

	A	B	C	D	E	F
L						
M						
O						
P						
S						
T						

3 **(i)**

Stage	State	Action	Route minimum	Current maximin
2	0	0		
	1	0		
	2	0		
1	0	0		
		1		
	1	1		
		2		
	2	0		
		2		
0		0		
		1		
		2		

Route: ....

Maximum number of crates that can be carried: ....

(ii) ....

....

....

....

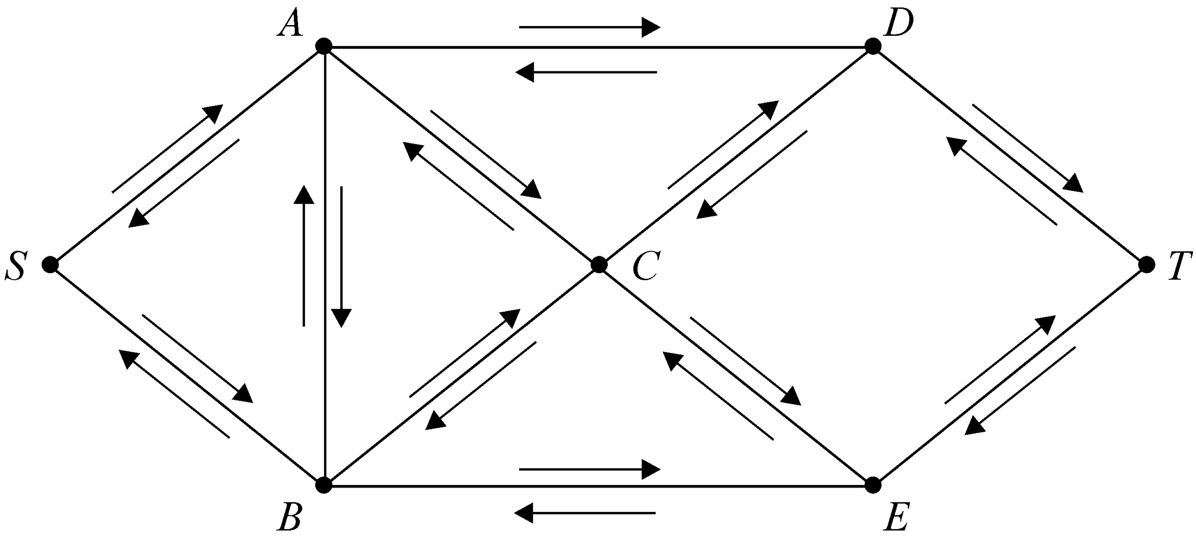
[Turn over

5 **(i)** Capacity of $\mathscr{C}$: ....

(ii) ....

....

(iii)



(iv) ....

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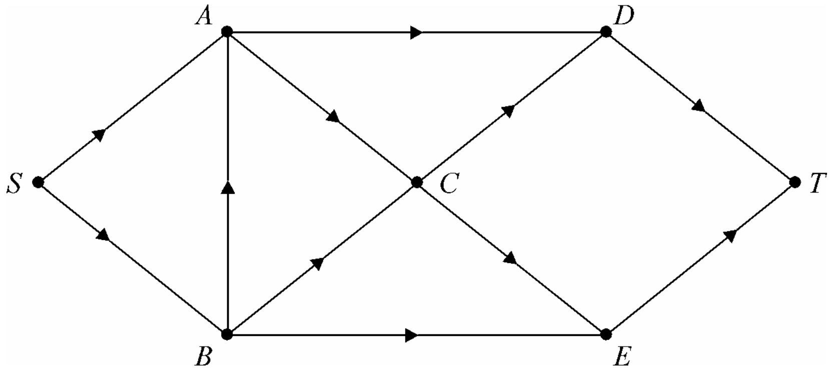
....

(v) Final flow:



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