OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4737**

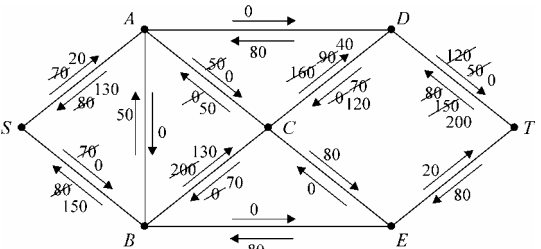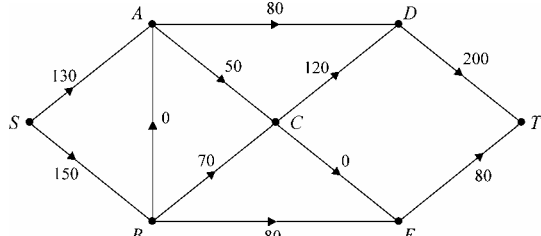Decision Mathematics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	**72**

This mark scheme consists of 5 printed pages and 3 blank pages.

Registered Charity Number: 1066969 **[Turn over**

1	**(i)**		M1 — For attempt at the bipartite graph
			A1 — For correct graph
			B1 **3** — For the correct incomplete matching

	(ii)	Alternating paths are:	
		S–C=P–D and T–A=L–B=M–F	
		or S–C=P–B=M–D and T–A=L–B=P–D=M–F	M1 — For attempt at an alternating path
		or S–C=P–B=M–F and T–A=L–B=P–D	A1 — For one correct path
		or T–A=L–B=M–D and S–C=P–D=M–F	A1 — For the second path correcct
		or T–A=L–B=M–F and S–C=P–D	
			B1 **4** — For correct matching

(iii)		A	B	C	D	E	F
	L	0	0	1	1	0	1
	M	1	0	1	0	1	0
	O	0	1	1	1	0	1
	P	1	0	0	0	1	1
	S	1	1	0	1	1	1
	T	0	1	1	1	1	1

M1 — For appropriate zeros and ones (e.g.) to correspond with minimum cost matching
A1 **2** — For a correct table

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2 **(i)** Adding a dummy column gives

$$\begin{array}{ccccc} 4 & 3 & 2 & 4 & 6 \\ 3 & 5 & 4 & 2 & 6 \\ 3 & 6 & 3 & 3 & 6 \\ 2 & 6 & 4 & 3 & 6 \\ 2 & 5 & 3 & 4 & 6 \end{array}$$

B1 — For a dummy column (equal entries, $\geqslant 6$)

Reducing:
$$\begin{array}{ccccc} 2 & 1 & 0 & 2 & 4 \\ 1 & 3 & 2 & 0 & 4 \\ 0 & 3 & 0 & 0 & 3 \\ 0 & 4 & 2 & 1 & 4 \\ 0 & 3 & 1 & 2 & 4 \end{array} \text{ then } \begin{array}{ccccc} 2 & 0 & 0 & 2 & 1 \\ 1 & 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 1 & 2 & 1 \end{array}$$

M1 — For reducing rows

M1 — For reducing columns

Four lines are needed to cover zeros

M1 — For covering zeros in the reduced matrix

Augmenting:
$$\begin{array}{ccccc} 3 & 0 & 0 & 3 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \end{array} \text{ or } \begin{array}{ccccc} 3 & 0 & 0 & 2 & 1 \\ 2 & 2 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{array}$$

M1 — For correct augmentation process

Hence Adam–Glasgow and Betty–Swansea
and either Clive–Manchester, Dave–London
   or Clive–Manchester, Eleanor–London
   or Dave–London, Eleanor–Manchester

A1 — For these two allocations correct

A1 **7** — For any one of the correct possibilities

(ii) Without Betty, reduced matrix is
$$\begin{array}{cccc} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 2 & 1 & 2 \end{array}$$

M1 — For new reduced matrix

Hence either Adam–Glasgow, Clive–Swansea,
   Dave–London, Eleanor–Manchester
   or Adam–Glasgow, Clive–Manchester,
   Dave–Swansea, Eleanor–London

A1 **2** — For either of the correct new possibilities

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3	(i)								

	0	0		5	5		
2	1	0		7	7		
	2	0		9	9		
	0	0	$\min(12, 5) = 5$	7			
		1	$\min(7, 7) = 7$				
1	1	1	$\min(8, 7) = 7$	9			
		2	$\min(10, 9) = 9$				
	2	0	$\min(9, 5) = 5$	9			
		2	$\min(14, 9) = 9$				
		0	$\min(9, 7) = 7$				
0	0	1	$\min(7, 9) = 7$	8			
		2	$\min(8, 9) = 8$				

M1		For dealing with route min column
A1		For at least 6 minima correct
M1		For dealing with maximin column
A1		For Stage 1 section of table all correct
A1		For completely correct table

Route is (0; 0)–(1; 2)–(2; 2)–(3; 0)   B1   For correct route
Maximum number of crates is 8   B1   **7**   For correct number

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	(ii)			
	New maximin values are 15, 7, 9, 12, 9, 9, 9	M1		For appropriate re-calculation
	Hence new route is (0; 0)–(1; 0)–(2; 0)–(3; 0)	A1		For correct new route
	New maximum number of crates is 9	A1	**3**	For correct number

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4	(i)			

Minimum completion time is 8 hours
Critical activities are *A*, *B*, *C*, *F*
Start telephoning at 11.00 am

B1		For correct arcs and activities (activity on arc network or equivalent with activity at node)
M1		For correct process for forward pass
M1		For correct process for reverse pass
A1		For all early and late times correct
A1		For correct minimum time stated
B1		For correct critical activities
B1✓	**7**	For stating the appropriate time of day

	(ii)			

Maximum number of people needed is 8

M1		For resource histogram with axes labelled
A1		For correct heights 3, 3, 5, 8, 8, 8, 5, 1
B1	**3**	For correct number stated

(iii)						
Time	H	1	2	3	4	5
10–11	*A*	*A*	*A*	*D*	*D*	*D*
11–12	*A*	*A*	*A*	*D*	*D*	*D*
12– 1		*B*	*B*	*D*	*D*	*D*
1– 2		*C*	*C*	*C*	*C*	*C*
2– 3		*C*	*C*	*C*	*C*	*C*
3– 4		*C*	*C*	*C*	*C*	*C*
4– 5		*C*	*C*	*C*	*C*	*C*
5– 6	*E*	*E*	*E*			
6– 7	*F*					

Start telephoning at 10.00 am

M1		For substantially correct attempt
A1		For a correct schedule
B1	**3**	For correct time stated

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5	(i)	Capacity is $150 + 0 + 50 + 80 = 280$ litres/sec	M1 A1	2	For correct use of zero from *AB* For correct value 280
	(ii)	Maximum flow is $\leqslant 280$ So flow of $200 + 100 = 300$ is not possible	M1 A1	2	For relevant use of max flow/min cut For completely correct proof
	(iii)		M1 A1 A1	3	For correct method for excess and backflow For all initial excess capacities correct For all initial backflows correct
	(iv)	Augment by 70 along *SBCDT* (e.g.) New excesses and backflows are as shown above Now augment by 50 along *SACDT* (e.g.) Final excesses and backflows are as shown above	B1 M1 M1 A1	4	For identifying a correct augmentation For modifying excesses and backflows For continuing the process as far as possible For a completely correct solution
	(v)	 The value of the augmented flow is 280 litres/sec, and so is the maximum possible	B1√ M1 A1	 3	For showing the augmented flow correctly For comparing with results from **(i)** or **(ii)** For a completely correct explanation (Or equivalent explanation based on the disconnectedness of *S* and *T* in **(iii)**)
				14	

6	(i)		X	Y	Z	row min			
		A	1	3	4	1			
		B	4	3	2	2	←		
		C	3	2	1	1			
		– col max	−4	−3	−4				

$\uparrow$

	Play-safe for Rose is B	B1		For correct statement
	Play-safe for computer is Y	B1		For correct statement
	Not stable as $-3+2 \neq 0$	B1	**3**	For use of max row min and min col max

(ii) Row C is dominated by row B — B1 — For correct statement or explanation

	X	Y	Z
A	1	3	4
B	4	3	2

B1   **2**   For new matrix

(iii) Expected pay-off with X is $1 \times a + 4(1-a) = 4 - 3a$ B1   For showing given answer correctly

and with Y is $3a + 3(1-a) = 3$ B1   For correct value 3

and with Z is $4a + 2(1-a) = 2 + 2a$ B1   For correct expression $2 + 2a$

B1✓   For correct diagram

Required value of a is 0.4   B1✓   For correct value

Score of 1 or 2: play A; score of 3, 4 or 5: play B;
Throw die again if it shows a six   B1✓   **6**   For a correct decision rule

(iv)

	X	Y	Z
A	−1	−3	−4
B	−4	−3	−2

B1   For correct pay-off matrix for the computer

Add 4 to each value   B1   **2**   For a correct explanation

(v) $z = 0.6$ B1   For the correct value of z

$p = 1.2 \Rightarrow P = -2.8$ B1   For the correct values of both p and P

The computer should choose X with probability 0.4 and Z with probability 0.6   B1   For a correct description of the strategy

On average the computer will lose no more than 2.8 points per game   B1   **4**   For a correct interpretation of P

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