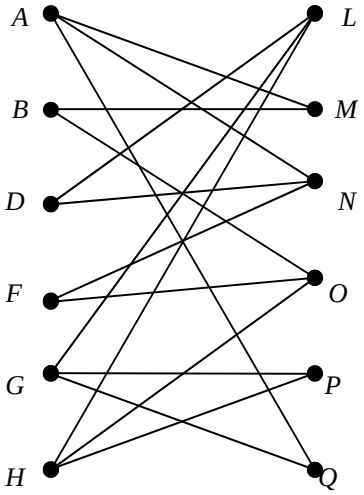
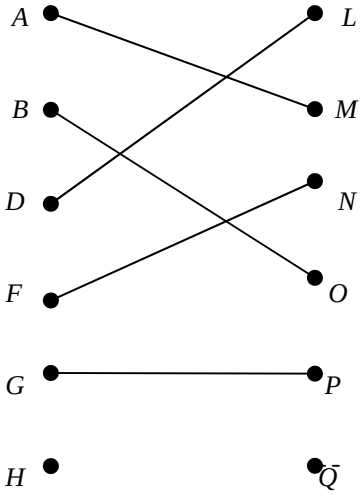
1	**(i)**		B1	A correct bipartite graph	**[1]**
	(ii)		B1	A second bipartite graph showing the incomplete matching correctly No augmentations made, even if in pencil. Ignore the addition of an *X* vertex though.	**[1]**
	(iii)	*H – P – G – Q* Axe handle = Prof Mulberry *A = M* Broomstick = Miss Olive *B = O* Drainpipe = Mrs Lemon *D = L* Fence post = Mr Nutmeg *F = N* Golf club = Rev Quince *G = Q* Hammer = Capt Peach *H = P*	B1 B1	This path in any reasonable form or in reverse. Accept *X-H-P-G-Q* Not any longer path from *H* to *Q* This complete matching written down (use initials of surnames if ambiguous, eg Rev Pineapple is interpreted as *P* = Capt Peach)	**[2]**
	(iv)	Axe handle = Rev Quince *A = Q* Broomstick = Prof Mulberry *B = M* Drainpipe = Mr Nutmeg *D = N* Fence post = Miss Olive *F = O* Golf club = Capt Peach *G = P* Hammer = Mrs Lemon *H = L*	M1 A1	A different complete matching in any form A valid complete matching in which none of the suspects uses the same weapon as in their solution to (iii)	**[2]**
				Total =	**6**

2 **(i)**

	1 pm	2 pm	3 pm	4 pm	5 pm
R	7	6	8	3	9
S	5	0	4	4	4
T	6	3	7	5	7
W	4	2	6	2	7
Y	2	2	3	6	7

M1 Modify table by subtracting each entry from a constant value

A1 Correct table (ie this $\pm$ a constant throughout, with no negative values) **[2]**

Reduce rows

4	3	5	0	6
5	0	4	4	4
3	0	4	2	4
2	0	4	0	5
0	0	1	4	5

M1 Substantially correct attempt to reduce rows (at most 2 independent errors)

Reduce columns

4	3	4	0	2
5	0	3	4	0
3	0	3	2	0
2	0	3	0	1
0	0	0	4	1

M1 Substantially correct attempt to reduce columns (at most 2 independent errors)

A1 Their reduced cost matrix **[3]**

Cross out 0's using minimum no. of lines

4	3	4	0	2
5	0	3	4	0
3	0	3	2	0
2	0	3	0	1
0	0	0	4	1

Augment

2	3	2	0	2
3	0	1	4	0
1	0	1	2	0
0	0	1	0	1
0	2	0	6	3

M1 Substantially correct attempt at augmenting (at most 2 errors)

A1 Their matrix augmented correctly to reach a complete matching **[2]**

	1 pm	2 pm	3 pm	4 pm	5 pm
R	2	3	2	0	2
S	3	0	1	4	0
T	1	0	1	2	0
W	0	0	1	0	1
Y	0	2	0	6	3

Mrs Rowan = 4 pm or = 4 pm
Dr Silverbirch = 2 pm or = 5 pm
Mr Thorn = 5 pm or = 2 pm
Ms Willow = 1 pm or = 1 pm
Sgt Yew = 3 pm or = 3 pm

B1 First matching, cao

B1 Second matching, cao

[2]

(ii) Mr Thorn

B1 Follow through their matchings (but not to S) **[1]**

Total = 10

3 (i)

Stage	State	Action	Working	Suboptimal minima
3	0	0	5	5
	1	0	4	4
	2	0	6	6
2	0	0	5 + 5 = 10	10
		1	6 + 4 = 10	10
	1	0	3 + 5 = 8	8
		1	5 + 4 = 9	
	2	1	3 + 4 = 7	7
		2	2 + 6 = 8	
1	0	0	2+10 = 12	
		1	3 + 8 = 11	11
	1	1	2 + 8 = 10	10
		2	3 + 7 = 10	10
	2	2	8 + 7 = 15	15
0	0	0	6 + 11 = 17	17
		1	8 + 10 = 18	
		2	3 + 15 = 18	

Minimum route = (0;0) - (1;0) - (2;1) - (3;0) – (4;0)
Weight = 17

B1 Structure of table correct (stage, state, action and 'working' columns)
M1
A1 Stage and state values correct
Action values correct **[3]**

M1 Working column substantially correct for stage 2 (calcs or totals) (at most 1 error)
A1 Suboptimal minima (10, 8, 7) correct for stage 2 (cao) **[2]**

M1 Working column substantially correct for stage 1(at most 1 error)
A1 Suboptimal minima (11, 10, 15) correct for stage 1 (cao) **[2]**

B1 Correct route from (0; 0) to (4; 0) **[2]**
B1 17 cao (written down, not just implied from table)

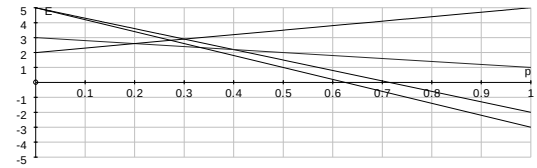
(ii)

Start at the bottom of the table at (0; 0)
Optimum for stage 0 comes from action 0,
so (0; 0) connects to (1; 0)
Optimum for (1; 0) comes from action 1,
so (1; 0) connects to (2; 1)
Optimum for (2; 1) comes from action 0
so (2; 1) connects to (3; 0) and hence to (4; 0)

M1 Start at (0; 0), action 0 or value 11 (theirs), hence (1; 0)

A1 (1; 0), action 1 (theirs), hence (2; 1)
Clearly relating <u>action</u> to state for stage above **[2]**

Total = **11**

4	**(i)**	In each game, whatever combination of strategies is chosen, the total number of points won is zero	B1	Points won by Euan equals points lost by Wai Mai, and vice versa, in every case	**[1]**
	(ii)	-2	B1	Loses 2	**[1]**
	(iii)	*Z* is dominated by *Y* In <u>each</u> row she loses more by choosing *Z* than *Y* -3 < 5, -4 < 3, -2 < 5 and 1 < 2 (or equivalent)	M1 A1	Idea of dominance by *Y* Four valid comparisons <u>and</u> a convincing explanation (or equivalent in words)	**[2]**
	(iv)	Wai Mai *X* *Y* row min *A* 2 -5 -5 Euan *B* -1 -3 -3 *C* 3 -5 -5 *D* 3 -2 -2 * col max 3 -2 * Play-safe for Euan is *D* Play-safe for Wai Mai is *Y* Game is stable, since row maximin = col minimax, -2 = -2	M1 A1 A1 B1	Determining row minima and column maxima, or equivalent (may be implied from both *D* and *Y* stated) *D*, stated (not just identified in table) *Y*, stated (not just identified in table) Stable, with a valid reason attempted (numerical or in words) (www)	**[4]**
	(v)	*A*: -2*p* + 5(1-*p*) = 5 – 7*p* *B*: *p* + 3(1-*p*) = 3 – 2*p* *C*: -3*p* + 5(1-*p*) = 5 – 8*p* *D*: 5*p* + 2(1-*p*) = 2 + 3*p* (note: leaving *DX* as 3 gives *D*: 2 - 5*p* = M1A0A0)	M1 A1 A1	Any one correct (or negative of correct), simplified or not All four correct (or negative of correct) and simplified All four correct and simplified	**[3]**
	(vi)	 2 + 3*p* = 3 – 2*p* ⇒ *p* = 0.2	M1 A1 M1 A1	Graph paper used with sensible scales Their equations plotted correctly Solving correct pair, or from graph 0.2, cao, from correct equations used (algebraically or from graph) (www)	**[2]** **[2]**
					Total = **15**

ANSWERED ON INSERT

5	**(i)**	21+36 +7 +18 = 82	M1 A1	Evidence of using the correct cut (eg 21 (+ 23) + 36 + 7 + 18 seen) 82	**[2]**
	(ii)	At most 17 can leave *C* so there cannot be as much as 20 or 18 entering it At most 17 can enter *E* so there cannot be 7 + 18 = 25 leaving it Maximum that can flow in arc *HT* is 33 Flow along arc *HG* = 0	B1 B1 B1 B1	17 < both 20 and 18 (NOT 17 < 38) 17 < 7 + 18 33 0	**[2]** **[2]**
	(iii)	A diagram showing a flow of 58 in which amount in equals amount out at each vertex, apart from *S* and *T* Arcs *CE*, *FH* and *GT* are saturated and other arc capacities are not exceeded Cut $X = \{S, A, B, C, D, F, G\}$, $Y = \{E, H, T\}$ Or cut through *GT, GH, FH, EF* and *CE*	M1 A1 B1	Assume that "blanks" mean 0 or full to capacity, provided consistent This cut presented in any form (accept it drawn on diagram)	**[3]**
	(iv)	Substantially correct attempt in which excess capacities and potential backflows marked correctly on arcs *CE*, *FH* and *GT* Their excess capacities and potential backflows marked correctly on arcs out of *S* and arcs into *T* and on *HG*	M1 A1	Assume that blanks mean 0 Accept <u>all</u> directions swapped Check directions on <u>*HG*</u> carefully If no flow in (iii), or ambiguous, then any valid flow > 0 labelled correctly gets M1, but must also be a flow of 58 to get A1	**[2]**
	(v)	Feasible route(s) written that send an additional 2 through system (or more on follow through) All route(s) valid with an additional 2 along *GH*	M1 A1	Routes must be written out properly eg route *SBFGHT* by 2	**[2]**
	(vi)	Their flow from part (iii) augmented by their routes in part (v) No more can flow across the cut $X = \{S, C\}$, $Y = \{A, B, D, E, F, G, H, T\}$	M1 A1	Follow through if possible Any reasonable explanation	**[2]**
				Total =	**15**

PARTS (i), (ii) AND (iii) ANSWERED ON INSERT

6 (i)

Activity	Duration	Predecessors
A	6	-
B	5	-
C	3	*A*, *B*
D	9	*A*
E	4	*A*, *B*
F	2	*A*, *B*
G	2	*E*, *H*
H	3	*C*, *F*
I	5	*D*, *G*
J	6	*E*, *H*
K	10	*C*, *F*
L	4	*I*
M	12	*I*
N	6	*J*, *K*, *L*

B1 Predecessors correct for *A* to *F* (entries for *A* and *B* may be blank)

M1 Substantially correct attempt at predecessors for other activities (at most 2 errors)

A1 Predecessors all correct for *G* to *N*

[3]

(ii)

Dummy is needed between **2** and **3** so that *C*, *E* and *F* follow both *A* and *B* but *D* follows *A* only

B1 *D* does not follow *B* (*D* follows *A* only)

Dummy is needed between **4** and **5** so that *C* and *F* do not share both a common start and a common finish

B1 Identifying *C* and *F* appropriately

[2]

(iii)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	6	6	9	9	12	15	20	24	32
0	6	7	10	10	13	15	20	26	32

B1 Early event times correct, in table

M1 Substantially correct backwards pass (at most 2 errors in total)

A1 Late event times correct, in table

Minimum project completion time = 32 minutes

B1 32, cao

Critical activities: *A*, *D*, *I* and *M*

B1 *A*, *D*, *I*, *M* and no others, cao

[5]

(iv)

Early event time at **9** becomes the larger of 24 and 9+*x*

M1 9+*x*

A1 Larger of 24 and 9+*x*

Early event time at **10** becomes the larger of 32 and 15+*x*, which then also becomes the late event time at **10**

M1 Considering the event times at **10**

Late event time at **9** then becomes 26 or 9+*x*

A1 Correct consideration of 26 and 9+*x*

[4]

(v) *x* = 17

B1 17

[1]

Total = 15