OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4737

Decision Mathematics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

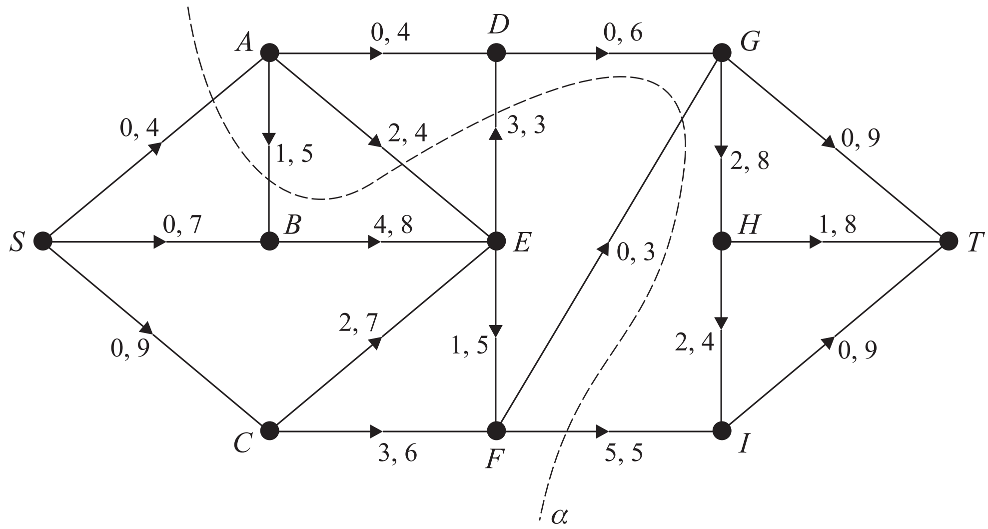
- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **4** and **5**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 6 printed pages, 2 blank pages and an insert.

1 The network represents a system of pipes along which fluid can flow from S to T. The values on the arcs are lower and upper capacities in litres per second.



(i) Calculate the capacity of the cut with X = $\{S, A, B, C\}$, Y = $\{D, E, F, G, H, I, T\}$. [1]

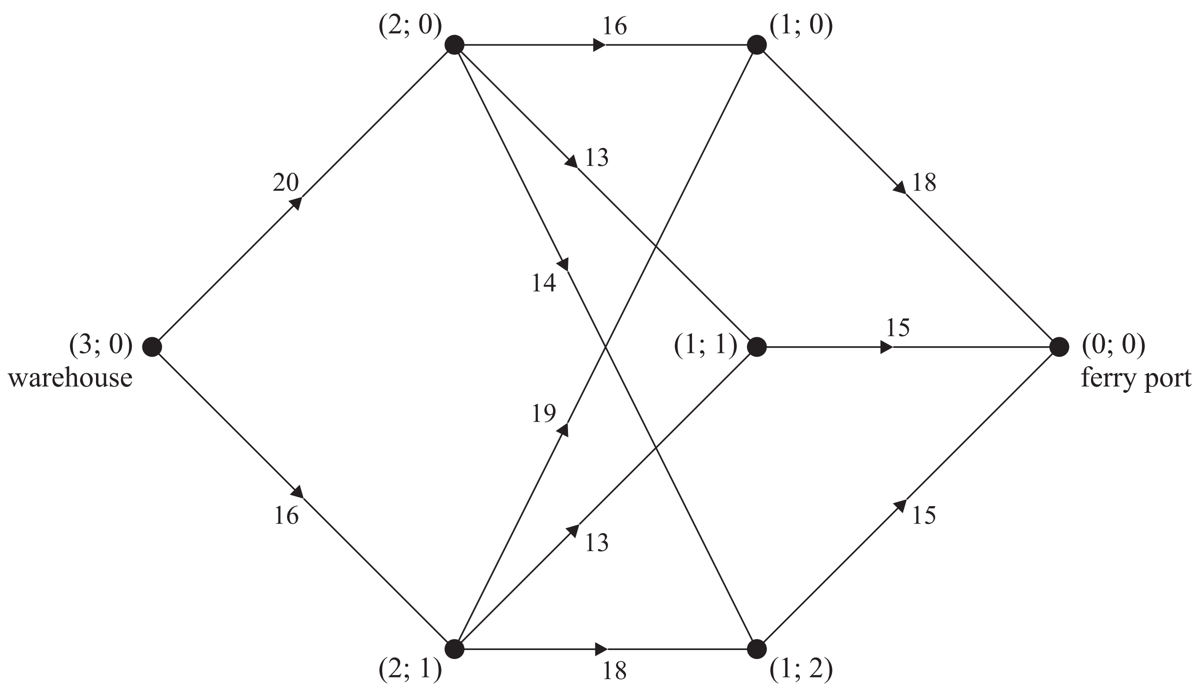
(ii) Show that the capacity of the cut α, shown on the diagram, is 12 litres per second and calculate the minimum flow across the cut α, from S to T, (without regard to the remainder of the diagram). [4]

(iii) Explain why the arc SC must have at least 5 litres per second flowing through it. By considering the flow through A, explain why AD cannot be full to capacity. [4]

(iv) Show that it is possible for 11 litres per second to flow through the system. [3]

(v) From your previous answers, what can be deduced about the maximum flow through the system? [2]

2 A delivery company needs to transport heavy loads from its warehouse to a ferry port. Each of the roads that it can use has a bridge with a maximum weight limit. The directed network below represents the roads that can be used to get from the warehouse to the ferry port. Road junctions are labelled with (stage; state) labels. The weights on the arcs represent weight limits in tonnes.



(i) Explain what a maximin route is. [2]

(ii) Set up a dynamic programming tabulation, working backwards from stage 1, to find the two maximin routes through the network. What is the maximum load that can be transported in one journey from the warehouse to the ferry port? [11]

(iii) A new road is opened connecting (2; 0) and (2; 1). There is no bridge on this road so it does not restrict the weight of the load that can be carried. Using the new road, what is the maximum load that can be transported in one journey from the warehouse to the ferry port? State the route that the delivery company should use. (Do **not** attempt to apply dynamic programming in this part.) [2]

3 Rose and Colin repeatedly play a zero-sum game. The pay-off matrix shows the number of points won by Rose for each combination of strategies.

Colin's strategy

Rose's strategy

	W	X	Y	Z
A	–1	4	–3	2
B	5	–2	5	6
C	3	–4	–1	0
D	–5	6	–4	–2

(i) What is the greatest number of points that Colin can win when Rose plays strategy A and which strategy does Colin need to play to achieve this? [2]

(ii) Show that strategy B dominates strategy C and also that strategy Y dominates strategy Z. Hence reduce the game to a 3×3 pay-off matrix. [5]

(iii) Find the play-safe strategy for each player on the reduced game. Is the game stable? [3]

Rose makes a random choice between the strategies, choosing strategy A with probability p_1, strategy B with probability p_2 and strategy D with probability p_3. She formulates the following LP problem to be solved using the Simplex algorithm:

maximise $M = m - 5$,

subject to
$m \leqslant 4p_1 + 10p_2$,
$m \leqslant 9p_1 + 3p_2 + 11p_3$,
$m \leqslant 2p_1 + 10p_2 + p_3$,
$p_1 + p_2 + p_3 \leqslant 1$,

and $p_1 \geqslant 0,\ p_2 \geqslant 0,\ p_3 \geqslant 0,\ m \geqslant 0$.

(You are **not** required to solve this problem.)

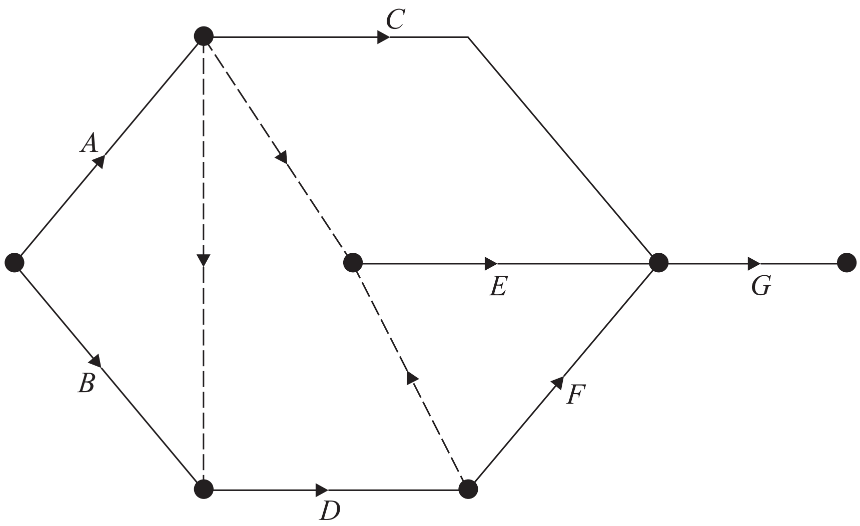
(iv) Explain how $9p_1 + 3p_2 + 11p_3$ was obtained. [2]

A computer gives the solution to the LP problem as $p_1 = \frac{7}{48}$, $p_2 = \frac{27}{48}$, $p_3 = \frac{14}{48}$.

(v) Calculate the value of M at this solution. [2]

4 Answer this question on the insert provided.

The diagram shows an activity network for a project. The table lists the durations of the activities (in hours).



Activity	Duration
A	6
B	4
C	5
D	1
E	5
F	4
G	2

(i) Complete the blank column of the table in the insert to show the immediate predecessors for each activity. [3]

(ii) Carry out a forward pass to find the early times for the events. Record these at the seven vertices on the copy of the network in the insert. Also calculate and record the late times for the events. Find the minimum completion time for the project and list the critical activities. [6]

(iii) What effect, if any, would a 3-hour delay on the start time of activity *C* have on the minimum completion time? [1]

Assume that each activity requires one worker and that each worker is able to do any of the activities. The activities may not be split. The start of activity *C* is not delayed.

(iv) Draw a resource histogram, assuming that each activity starts at its earliest possible time. How many workers are needed with this schedule? [4]

5 Answer parts (i), (ii) and (iii) of this question on the insert provided.

Six men and six women need to form pairs for a mixed doubles competition. The men were each asked to name two women that they would like to partner.

Alan named Jenny and Olivia
Ben named Lorna and Mandy
Charlie named Katie and Naomi
David named Lorna and Olivia
Edward named Mandy and Naomi
Fred named Jenny and Katie

(i) Draw a bipartite graph to show which men named which women. The men should be represented as A, B, C, D, E and F and the women as J, K, L, M, N and O. [2]

Initially it was suggested that Alan should be partnered with Jenny, Ben with Lorna, Charlie with Katie, David with Olivia and Edward with Naomi. Fred was not happy with this suggestion.

(ii) Use the alternating path $F-K-C-N-E-M$ to write down a complete matching between the men and the women. [2]

The women were each asked to name two men that they would like to partner.

Jenny named Ben and David
Katie named Charlie and Edward
Lorna named Alan and David
Mandy named Alan and Ben
Naomi named Edward and Fred
Olivia named Charlie and Fred

(iii) Complete the matrix on the insert with rows representing men and columns representing women.

If a cell corresponds to a pairing that **neither** person has named (such as row C, column J) give it the value 5.

If a cell corresponds to a pairing that **one** of the people has named but not the other (such as row A, column J or row B, column J) give it the value 2.

If a cell corresponds to a pairing that **both** people have named (such as row C, column K) give it the value 0. [3]

(iv) Apply the Hungarian algorithm to your matrix from part **(iii)** to find two possible minimum cost complete matchings. For each of these matchings, state which men are paired with a woman they did not name and which women are paired with a man they did not name. [8]

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Candidate Name	Centre Number	Candidate Number

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INSERT for Questions 4 and 5

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

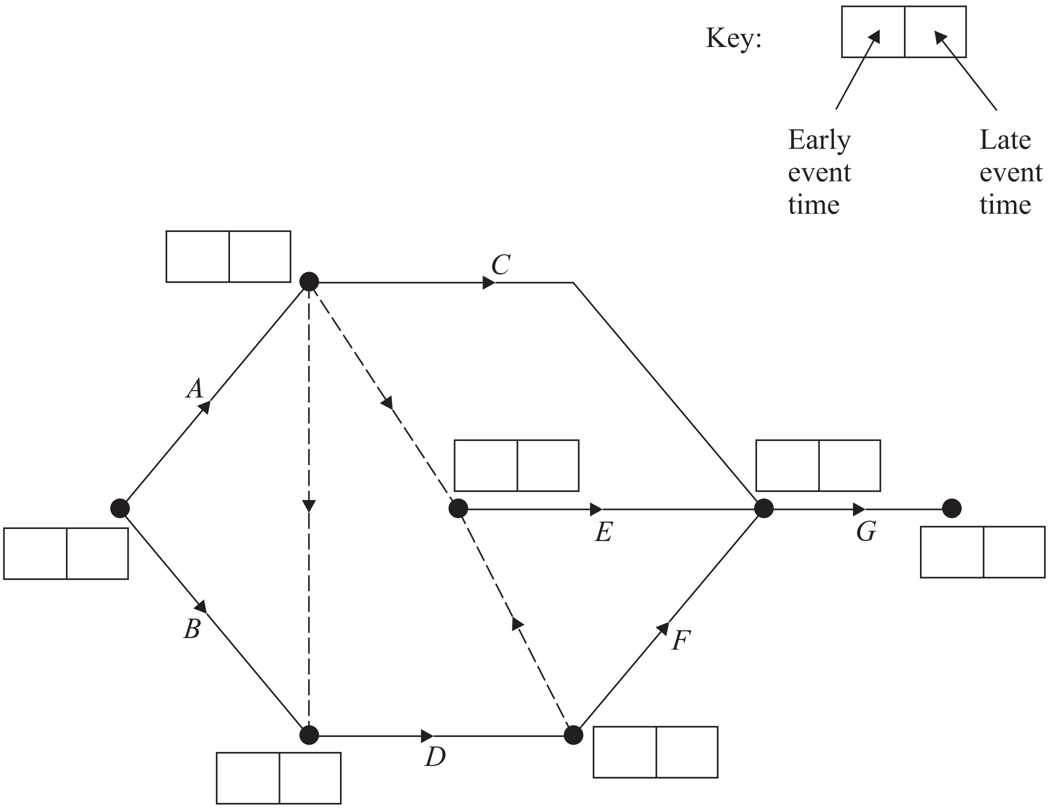
- This insert should be used to answer Question **4** and parts **(i)**, **(ii)** and **(iii)** of Question **5**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page.
- Write your answers to Question **4** and parts **(i)**, **(ii)** and **(iii)** of Question **5** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 4 printed pages.

4 (i)

Activity	Duration	Immediate predecessors
A	6	
B	4	
C	5	
D	1	
E	5	
F	4	
G	2	

(ii)



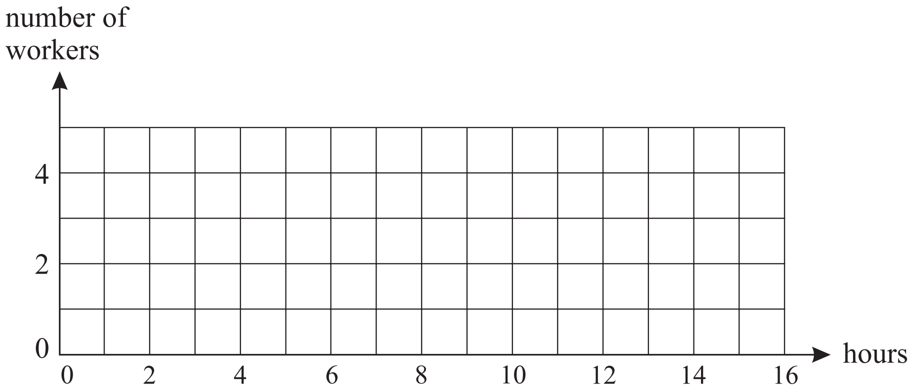
Minimum completion time = ... hours

Critical activities: ....

(iii) ....

....

(iv)

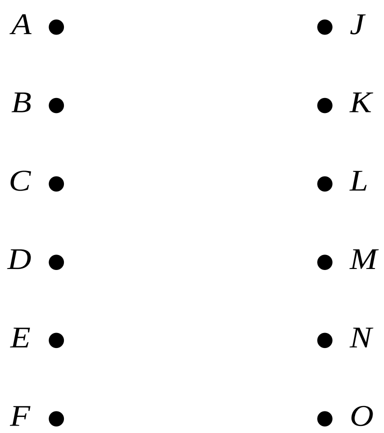


Number of workers required = ...

[Answer Question 5 (i), (ii), (iii) overleaf.]

[Turn over

5 **(i)**



(ii) ....

....

....

(iii)

	J	*K*	*L*	*M*	*N*	*O*
A	2	5	2	2	5	2
B	2	5	2	0	5	5
C	5	0	5	5	2	2
D						
E						
F						

Answer part **(iv)** in your answer booklet.