Mark Scheme 4737
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1	(i)	4+4+8+7+6 = **29** litres per second	B1 [1]	For 29
	(ii)	**4-1-2+3+3+5** = 12 litres per second **0 – 5 – 4 + 3 + 0 + 5** = -1 So minimum flow across cut is **0**	M1 A1 M1 A1 [4]	For using upper and lower capacities correctly For showing how 12 (given) was worked out For a substantially correct calculation For 0, from an appropriate calculation
	(iii)	Flow in arc **CE ≥ 2** and flow in arc **CF ≥ 3**, so at least 5 litres per second must flow into *C* At most 4 litres per second flow into *A*, of which at least 1 flows out to *B* and 2 flow out to *E*, so at most 1 litre per second can flow along *AD*	M1 A1 M1 A1 [4]	For any reasonable attempt (eg *CE* = 2, *CF* = 3) For correct reasoning For identifying **≤ 4 in** and **≥ 3 out** or equivalent For a correct conclusion
	(iv)	Either a diagram or a description of a flow of 11 litres per second. Arcs *AD, AE, BE, CE, CF* must all be at their minimum capacities.	M1 A1 A1 [3]	For a flow of 11 litres per second from *S* to *T* Flow satisfies all lower capacities Flow satisfies all upper capacities
	(v)	**11 ≤** maximum **flow ≤ 12**	B1 B1 [2]	11 as lower bound 12 as upper bound (max flow = 12 ⇒ B0, B1) **14**

2	(i)	The route for which the minimum weight on the route is greatest		B1 B1 [2]	For identifying **route minima** For identifying **what has been maximised** ('maximises the minimum' ⇒ B0 B1)

(ii)

Stage	State	Action	Working	Maximin
1	0	0		18
	1	0		15
	2	0		15
2	0	0	min(16,18) =**16**	**16**
		1	min(13,15) =**13**	
		2	min(14,15) =**14**	
	1	0	min(19,18) =**18**	**18**
		1	min(13,15) =**13**	
		2	min(18,15) =**15**	
3	0	0	min(20,16) =**16**	**16**
		1	min(16,18) =**16**	

B1
B1
[2] — Stage and state columns completed correctly. Action column completed correctly

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M1 For calculating minima for stage 2 state 0
A1 For maximin values identified (may be implied from working seen for stage 3)
M1 For calculating minima for stage 2 state 1
A1 For maximin values identified (may be implied from working seen for stage 3)
M1 For calculating minima for stage 3
A1 For maximin value identified
[6] (Forwards working scores M0, M0, M0)

B1 For first correct route
B1 For second correct route
B1 For 16 tonnes (**with units**)
[3]

Maximin routes: **(3; 0) – (2; 0) – (1; 0) – (0; 0)**
        (3; 0) – (2; 1) – (1; 0) – (0; 0)
Maximum load = **16 tonnes**

	(iii)	**18** tonnes **(3; 0) – (2; 0) – (2; 1) – (1; 0) – (0; 0)**	B1 B1 [2]	For 18 For this route

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3	(i)	3 *Y*	M1 A1 [2]	For 3 (allow -3) For *Y* (cao)
	(ii)	**5 > 3, -2 > -4, 5 > -1 and 6 > 0** or using signs of differences +2, +2, +6, +6 **3 > -2, -5 > -6, 1 > 0, 4 > 2** or equivalent, or using differences Reduced matrix: Colin's strategy Rose's strategy 	M1 A1 M1 A1 B1 [5]	For an appropriate comparison, or implied For all four comparisons seen For an appropriate comparison, or implied For all four comparisons seen For correct reduced matrix, with rows and columns labelled *A*, *B*, *D* and *W*, *X*, *Y* Cao

Reduced matrix (part ii):

Colin's strategy

	W	**X**	**Y**
A	-1	4	-3
B	5	-2	5
D	-5	6	-4

Rose's strategy

	(iii)	Row minima are -3, -2, -5 Play-safe for Rose is **B** Column maxima are 5, 6, 5 Play-safes for Colin are **W** and **Y** **Not stable**	 M1 M1 A1 [3]	Follow through their 3×3 reduced matrix For identifying row *B* For identifying columns *W* and *Y* For 'no' or 'not stable'
	(iv)	**5 is added** throughout the matrix to make the entries non-negative. In this augmented reduced matrix, $9p_1 + 3p_2 + 11p_3$ is the expected number of points won by Rose when Colin plays **strategy *X***	M1 A1 [2]	For 'add 5' or equivalent For identifying that this is when Colin plays strategy *X*
	(v)	$p_1 = \frac{7}{48}$, $p_2 = \frac{27}{48}$, $p_3 = \frac{14}{48}$ $\Rightarrow m \leq \frac{298}{48}$ (or $6\frac{5}{24}, 6.2083, 6.21$) in all three cases $\Rightarrow M = \frac{58}{48}$ (or $\frac{29}{24}, 1\frac{5}{24}, 1.2083, 1.21$)	 M1 A1 [2]	 For attempting to evaluate *m* cao (in any appropriate form)

14

| 4 | (i) | | | | | | **ANSWERED ON INSERT** |

Activity	Duration	Immediate predecessors
A	6	-
B	4	-
C	5	A
D	1	A, B
E	5	A, D
F	4	D
G	2	C, E, F

B1 — For predecessors for activities *A*, *B* and *C* correct

B1 — For predecessors for activities *D*, *F* and *G* correct

B1 [3] — For predecessors for activity *E* correct

(ii)

6 | 6 C

A

0 | 0 E

12 |12

G **7 | 7**

14|14

B F

Minimum completion time = **14** hours
Critical activities: ***A, D, E, G***
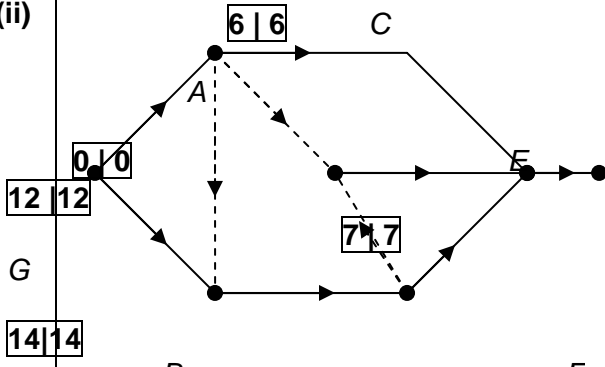
M1 — For carrying out forward pass (no more than one independent error)
A1 — For all early event times correct

M1 — For carrying out backwards pass (no more than one independent error)
A1 — For all late event times correct

B1 — For 14   cao
B1 [6] — For *A*, *D*, *E*, *G* only   cao

(iii) **Increased by 2** (hours)
Becomes 16 (hours)

B1 [1] — For stating that time increases by 2, or equivalent

(iv)

Workers

4

2

0

0   2   4   6   8   10   12   14

16

hours
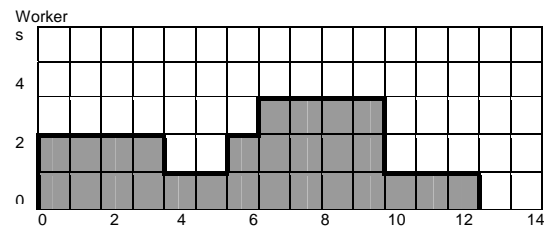
Number of workers required = **3**

B1 — For a resource histogram with no overhanging cells

M1 — For a reasonable attempt, ft their start times if possible
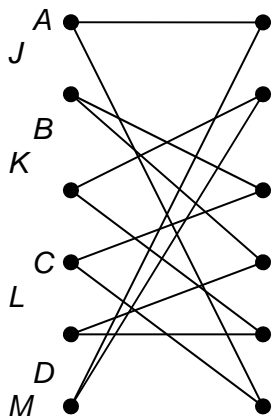A1 — For a completely correct histogram (cao)

B1 [4] — For 3 or follow through their histogram if possible

14

5	(i)								**ANSWERED ON INSERT**

			M1	For a substantially correct attempt

	A1 [2]	For a completely correct bipartite graph

(ii)	$C - N$ $E - M$ $F - K$ $A - J$ $B - L$ $D - O$	M1 A1 [2]	For pairing $F - K$, $C - N$, $E - M$ For all correct (Diagram only $\Rightarrow$ M1, A0)

(iii)

	J	K	L	M	N	O
A	2	5	2	2	5	2
B	2	5	2	0	5	5
C	5	0	5	5	2	2
D	2	5	0	5	5	2
E	5	2	5	2	0	5
F	2	2	5	5	2	2

B1   For '5' in all the entries that should be 5

B1   For '2' in all the entries that should be 2

B1 [3]   For '0' in all the entries that should be 0

(iv) Reduce rows

0	3	0	0	3	0
2	5	2	0	5	5
5	0	5	5	2	2
2	5	0	5	5	2
5	2	5	2	0	5
0	0	3	3	0	0

Columns are already reduced

Or, reduce columns

0	5	2	2	5	0
0	5	2	0	5	3
3	0	5	5	2	0
0	5	0	5	5	0
3	2	5	2	0	3
0	2	5	5	2	0

Rows are already reduced

Cannot cross out 0's using fewer than 6 lines so matching is complete

 $A - J$ $B - M$ $C - K$ $D - L$ $E - N$
 $F - O$
  $A - O$ $B - M$ $C - K$ $D - L$ $E - N$
 $F - J$

First matching: Fred and Jenny
 Second matching: Jenny and Olivia

M1   For a substantially correct attempt from their matrix

A1   For a correct reduction of rows and columns (or columns and rows) for their matrix

M1

A1
 B1
 B1

 B1
 B1
 [8]

For achieving a reduced cost matrix with a complete matching of zero cost (without unnecessary augmenting)
 0's in correct cells (not ft)
 For this matching or ft their reduced cost matrix
 For this matching or ft their reduced cost matrix

For the names for their first matching
 For the names for their second matching

		15	