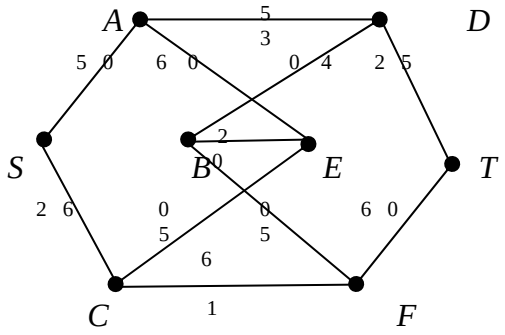
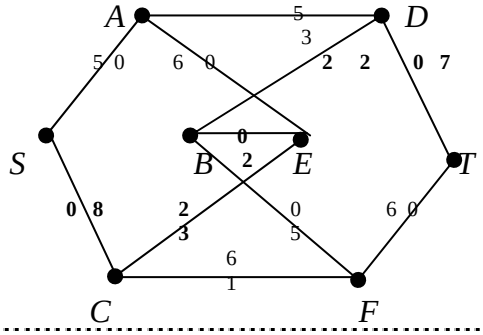
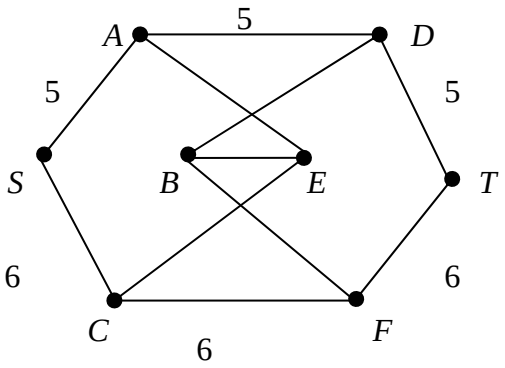
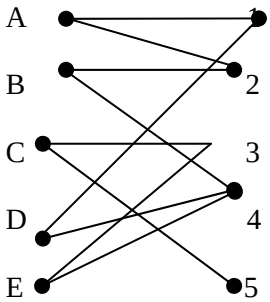
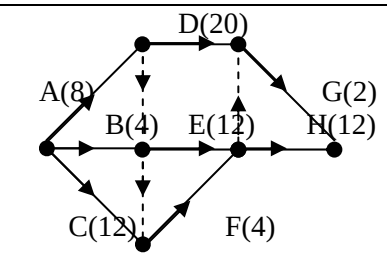
Mark Scheme 4737
June 2005

1	**(i)**	*AD*, *EB*, *CF* 8 + 2 + 7 = 17 litres per second	B1 M1 A1	For these three directed arcs and no others 8 + 7 + 0 or 8 + 7 – 2 seen ⇒ M1, A0 For 17
	(ii)		M1 A1 	Accept all arrows reversed For no more than three errors For a correct labelling
	(iii)	*SCEBDT* 	B1 B1	For this path only For a correct labelling
	(iv)	Cut {*S*, *A*, *B*, *C*, *D*, *E*, *F*} {*T*} = 13 Diagram in (iii) shows a flow of 13 litres/second	B1 B1	For a this cut or the cut {*S*} {*A*, *B*, *C*, *D*, *E*, *F*, *T*}, in any form, or for 'no more can flow into *T* ',or 'no more can flow out of *S* ', or equivalent. For flow shown = 13 <u>or</u> max flow ≥ 13 ≥ min cut (but NOT just stating max flow = min cut) Value 13 given in question
	(v)	 Max flow is 11 <u>litres per second</u> Cut {*S*, *C*, *E*, *F*} {*A*, *B*, *D*, *T* } = 11	B1 B1 B1 **12**	For showing this flow, or excess capacities and potential backflows equivalent to this For 11 <u>litres per second</u> (with units) For cut <u>or</u> a <u>convincing</u> explanation in words

2	**(i)**		B1	For a correct bipartite graph
	(ii)	Denny cannot have a song that she has chosen	B1	For this reasoning
	(iii)	5 C 3 D A-1, B-2, C-5, D-3, E-4	M1 A1	Follow through their bipartite graph, if possible For this path (or in reverse), not longer path - if shown on diagram, path must be obvious For this matching, not alternative
	(iv)	A-2, B-4, C-5, D-1, E-3	B1	For a different matching from their bipartite graph
	(v)	Hungarian algorithm finds minimum cost allocation, need to subtract each score from 10 to convert maximising into minimising. Dummy row is needed to make a square matrix.	B1 B1	For a valid reference to maximising/minimising For 'make it square' or equivalent
	(vi)	F G H J K A 6 1 3 10 3 B 4 2 7 2 10 C 3 6 5 8 3 D 4 4 8 3 9 X 0 0 0 0 0	B1	For setting up initial matrix as described
		Reduce rows 5 0 2 9 2 2 0 5 0 8 0 3 2 5 0 1 1 5 0 6 0 0 0 0 0	M1 A1	For reducing rows (to give a 0 in each row) For correct reduced matrix (cao)
		Cover 0's using four lines 5 0 2 9 2 2 0 5 0 8 0 3 2 5 0 1 1 5 0 6 0 0 0 0 0	M1	For covering 0's using minimum number of lines
		Augment 4 0 1 9 1 1 0 4 0 7 0 4 2 6 0 0 1 4 0 5 0 1 0 1 0	A1	For correct augmented matrix (cao)
		Complete matching A-G, B-J, C-K, D-F	B1 **13**	For correct matching (listed)

3 (i)

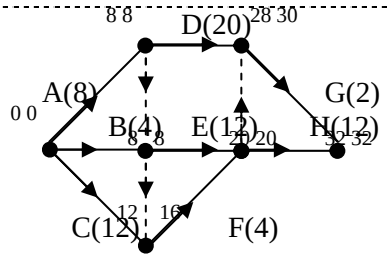


M1 For a correct activity network

A1 For directions indicated correctly

Durations not necessary

(ii)



Follow through their network if possible, provided not significantly simpler, for the passes

M1 For forward pass

A1 For forward pass correct

M1 For backwards pass

A1 For backwards pass correct

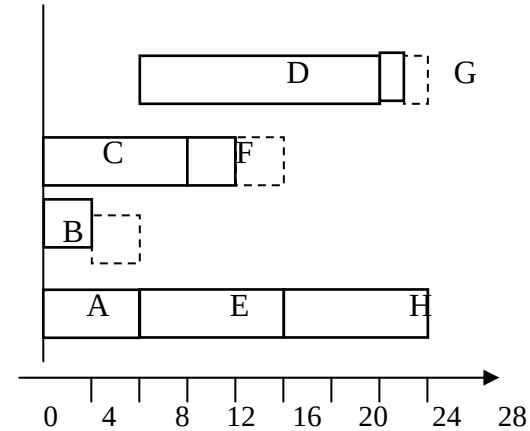
Minimum completion time = 32 minutes B1

Critical activities A, E, H B1

For 32 stated, not just on diagram (cao)

For A, E, H stated (not just on diagram (cao)

(iii)



32 time(mins)

Follow through their start times if possible

M1 For structure of chart correct, activities may be collected together or on individual rows

A1 For non-critical activities correct (floats optional)

A1 For critical activities correct

(iv) e.g.

Time	John	Kerry
0 – 4	A	
4 – 8	A	B
8 – 12	C	C
12 – 16	C	C
16 – 20	C	C
20 – 24	D	D
24 – 28	D	D
28 – 32	D	D
32 – 36	D	D
36 – 40	D	D
40 – 44	E	E
44 – 48	E	E
48 – 52	E	E
52 – 56	F	.
56 – 60	H	H
60 – 64	H	H
64 – 68	H	H
68 – 72		G (68 – 70)

M1 For structure of schedule correct and all activities shown (with H appearing twice)

A1 For activities A, B, C, D, E and F correct:
- A=8, B=4, C=12, D=20, E=12, F=4;
- D after A; E after A, B; F after A, B, C;
- C, D and E done by J and K at same time

A1 For activities G and H correct
- G = 2 (may see 4), H = 12
- G, H after (D), E, F (not alongside F)
- H done by each of J and K
- Total time taken = 70 (minutes)

14

4 **(i)**

stage	state	action	working	maximum
1	0	0	4	*
	1	0	4	*
2	0	0	4 + 4= 8	
		1	5 + 4= 9	*
	1	0	6 + 4=10	*
	2	1	7 + 4=11	*
	3	0	5 + 4= 9	
		1	6 + 4=10	*
3	0	1	8+10=18	*
		3	6+10=16	
	1	0	7+ 9=16	
		2	6+11=17	*
	2	0	7+ 9=16	
		2	6+11=17	
		3	8+10=18	*
4	0	0	5+18=23	
		1	8+17=25	*
	1	0	7+18=25	*
		2	5+18=23	
5	0	0	6+25=31	
		1	8+25=33	*

M1 For structure of table correct
A1 For stage and state columns correct
A1 For action values correct

M1 For all calculations correct for stages 1 and 2
(may be seen as an addition or the result and may be shown in final column)
A1 For suboptimal maxima identified correctly
(may be implied from next stage)

M1

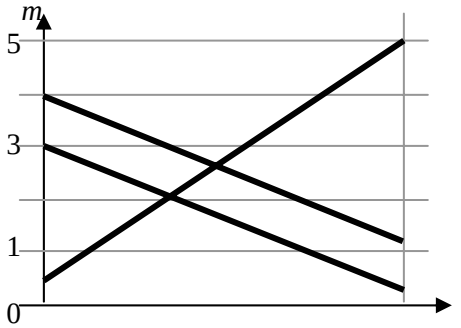
A1 For correct calculations for stage 3
(follow through from stage 2, if possible)
For suboptimal maxima correct (ft their totals)
M1 (may be implied from next stage)

A1 For correct calculations for stages 4 and 5
(follow through from stage 3)

Route: (0;0) – (1;0) – (2;1) – (3;0) – (4;1) – (5;0)
B1 Calculations correct for entire table
Giles will be able to see 33 plants
B1 (cao) or in reverse
For 33 (cao)

(ii) Minimax
B1 For 'minimax'
Route: (0;0) - (1;0) – (2;3) – (3;0) – (4;0) – (5;0)
M1 For a path with at most one path > 6 plants
A1
Or (0;0) - (1;1) – (2;3) – (3;0) – (4;0) – (5;0)
B1 For either correct path
Or stage 3 or any equivalent argument in words
At stage 5 all paths have at least 6 plants

15

5	**(i)**	What one player wins the other loses	B1	For a statement equivalent to 'total won each game is zero'
	(ii)	*S* and *T*: 3 > -2 but -2 < 1 (or -1 < 2) *S* and *U*: 3 > 1 (or -1 > -2) but -2 < 3 *T* and *U*: -2 < 1 (or 1 < 3) but 2 > -2	M1 A1	For considering differences, showing inequalities or considering rows where column maxima and/or minima occur For a valid explanation
		D and *E*: 3 > -2 but -2 < 1 (or 1 < 3) *D* and *F*: 3 > -1 (or 1 > -2) but -2 < 2 *E* and *F*: -2 < -1 (or 1 < 2) but 3 > -2	M1 A1	For considering differences, showing inequalities or considering columns where row maxima and/or minima occur For a valid explanation
	(iii)	Row minima are -2, -2, -2 ⇒ row maximin = -2 Col maxima are 3, 3, 2 ⇒ col minimax = 2 2 ≠ -2 ⇒ not stable	M1 M1 A1	For identifying -2 correctly or identifying all rows For identifying 2 correctly or identifying col *F* For a valid explanation, or equivalent in words
	(iv)	So that for p_1, p_2, $p_3 \geq 0$ we will have $m \geq 0$	B1	For explaining that this will make $m \geq 0$ (not sufficient to just say that we need to make all the entries non-negative)
	(v)	If Colin plays *D*, with the augmented payoffs Rhoda will expect to win $5p_1+0p_2+3p_3$, and similarly for when Colin chooses *E* or *F* *m* is the minimum of the augmented E(winnings)	 B1 B1	 For explaining any of the three expressions on the right hand side of the inequalities For explaining why $m \leq$ each expression
	(vi)	 $5p_1 = 3(1-p_1) \Rightarrow p_1 = \frac{3}{8}$ (and $p_2 = \frac{5}{8}$)	M1 A1 B1	For a graph of *m* against p_1 (or *m* against p_2) with three lines For lines (0,0)-(1,5), (0,3)-(1,0), (0,4)-(1,1) or equivalent For convincingly showing how values were obtained (ie identifying $5p_1 = 3p_2$ or equivalent Or reading off from correct point on graph) Note: $p_1 = \frac{3}{8}$ and $p_2 = \frac{5}{8}$ is given in the question
	(vii)	-0.125	B1	For $-\frac{1}{8}$, or equivalent (cao)
	(viii)	e.g. Toss the coin three times to give eight equally likely possible outcomes, allocate three outcomes to 'play *S*' and five to 'play *T*' In the long run she expects to lose $\frac{1}{8}$ per game	M1 A1 B1 **18**	For a specific example, or a description of any valid method eg HHT, HTH, THH → *S* all other outcomes → *T* For 'lose (at least) $\frac{1}{8}$ per game'