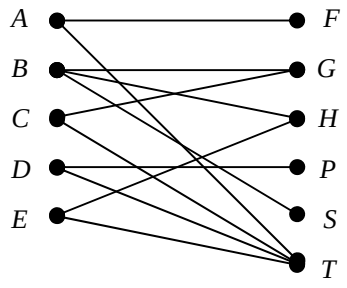
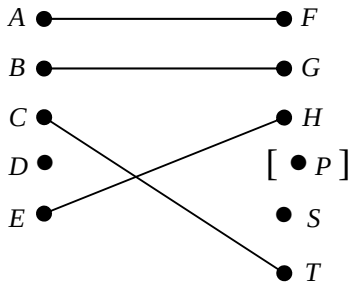
4737 Decision Mathematics 2

1	(i)		B1	Bipartite graph correct	[1]
	(ii)		B1	A new bipartite graph showing the pairings *AF*, *BG*, *CT* and *EH* but not *DS*	
		$D = T - C = G - B = S$	M1	This alternating path written down, not read off from labels on graph	
		Andy = food Beth = science Chelsey = geography Dean = television Elly = history	A1	$B = S$, $C = G$ and $D = T$ written down	
			B1	$A = F$, $E = H$ written down	[4]
	(iii)	Andy = food Beth = television Chelsey = geography Dean = politics Elly = history	B1	$A = F$, $C = G$, $D = P$ and $E = H$ (cao) ($B = T$ may be omitted)	
		Science did not arise	B1	S (cao)	[2]
				Total =	**7**

2

Add a dummy row

	P	R	S	T
April	30	28	32	25
May	32	34	32	35
June	40	40	39	38
Dummy	40	40	40	40

B1 Adding a dummy row of all equal values

Reduce rows

5	3	7	0
0	2	0	3
2	2	1	0
0	0	0	0

M1 Substantially correct attempt to reduce matrix (condone 1 numerical slip)

A1 Correct reduced cost matrix from reducing rows first and statement of how table was formed, including reference to columns (cao) **[3]**

Columns are already reduced

Incomplete matching, cross through zeros

5	3	7	0
0	2	0	3
2	2	1	0
0	0	0	0

B1 Cross through zeros using minimum number of lines

Augment by 1

4	2	6	0
0	2	0	4
1	1	0	0
0	0	0	1

B1 Correct augmented matrix and statement of how table was formed (cao) **[2]**

Complete matching

	P	R	S	T
April	4	2	6	0
May	0	2	0	4
June	1	1	0	0
Dummy	0	0	0	1

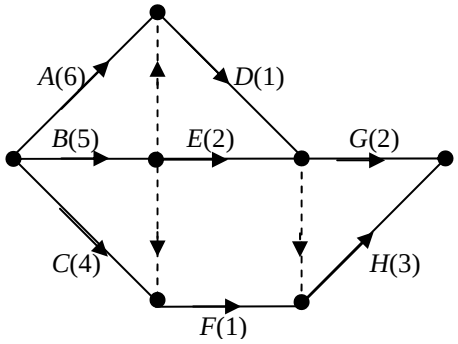
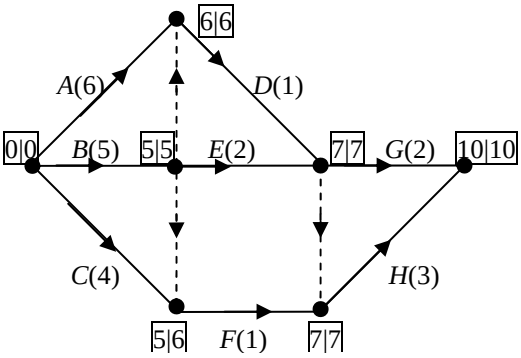
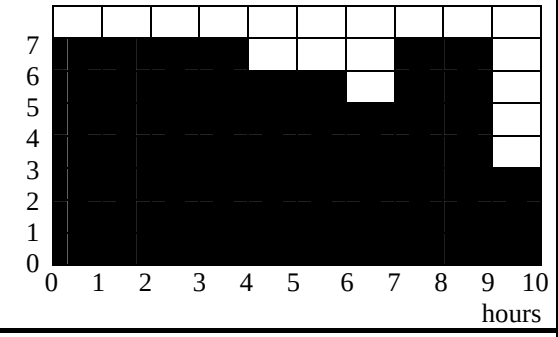
April = Tall Trees £2500
May = Palace £3200
June = Sunnyside £3900

B1 A = T, M = P, J = S (cao)

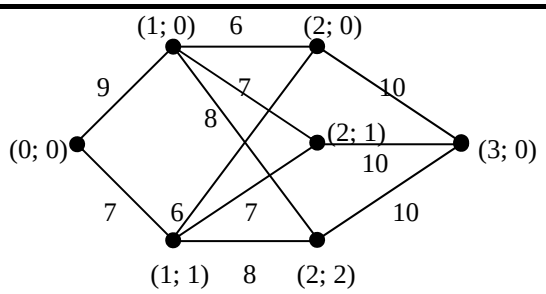
Total cost = £9600

B1 £9600 (cao) with units **[2]**

Total = 7

3	**(i)**		M1 A1	Durations not necessary Correct structure, even without directions shown Activities must be labelled Completely correct, with exactly three dummies and all arcs directed	**[2]**
	(ii)			Follow through their activity network if possible	
		Minimum project completion time = 10 hours	M1 M1 A1ft B1	Substantially correct attempt at forward pass (at most 1 independent error) Substantially correct attempt at backward pass (at most 1 independent error) Both passes wholly correct 10 hours (with units) cao	**[3]**
		Critical activities *A*, *B*, *D*, *E*, *H*	M1 A1	Either *B*, *E*, *H* or *A*, *D*, *H* (possibly with other critical activities, but *C*, *F*, *G* not listed). Not follow through. *A*, *B*, *D*, *E*, *H* (and no others) cao	**[3]**
	(iii)			On graph paper	
			M1 A1	A plausible resource histogram with no holes or overhangs Axes scaled and labelled and histogram completely correct, cao	**[2]**
	(iv)	1 hour	B1	Accept 1 (with units missing) cao	**[1]**
	(v)	No need to change start times for *A*, *B*, *C*, *D* and *E* Activities *G* and *H* cannot happen at the same time, so they must follow one another This causes a 2 hour delay	M1 A1	*G* and *H* cannot happen together (stated, not just implied from a diagram) 2 cao	**[2]**
		F could be delayed until 1 hour before *H* starts *H* should be started as late as possible ⇒ a maximum delay of 3 hours	B1 B1	Diagram or explaining that for max delay on *F* need *H* to happen as late as possible 3 cao	**[2]**
				Total =	**15**

4 (i)



B1	Correct structure (vertex labels and graph correct)	
M1	Assigning weights to their graph (no more than 1 error or no more than 2 arcs missing/extra)	
A1	Completely correct network	**[3]**

(ii) Maximin

B1	cao	**[1]**

(iii)

Stage	State	Action	Working	Suboptimal maximin
2	0	0	10	10
	1	0	10	10
	2	0	10	10
1	0	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8
	1	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8
0	0	0	min(9,8) = 8	8
		1	min(7,8) = 7	

B1	Four or five columns, including 'stage', 'state' and 'action'	
B1	Stage and state columns completed correctly	
B1	Action column completed correctly	**[3]**
M1	Min values correct for stage 1	
A1	Suboptimal maximin values correct for stages 2 and 1 (follow through their network if possible, no more than 2 arcs missing/extra)	**[2]**
M1	Min values correct for stage 0	
A1	Maximin value for stage 0 (follow through their network if possible, no more than 2 arcs missing/extra)	**[2]**
B1	8, cao	
B1	Correct route, or in reverse	**[2]**

Weight of heaviest truck = 8 tonnes
Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)

Total = 13

SR Special ruling for working forwards

(iii)

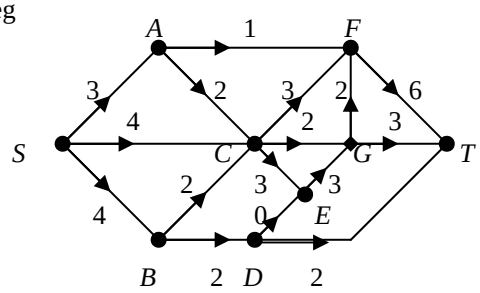
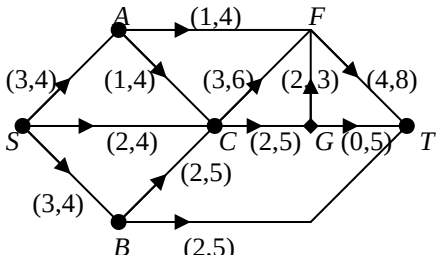
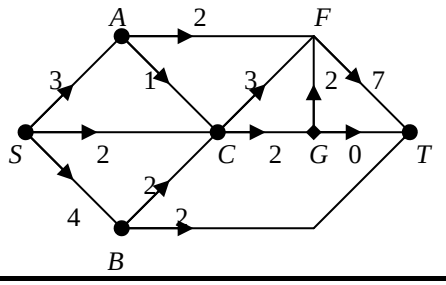
Stage	State	Action	Working	Suboptimal maximin
1	0	0	9	9
	1	0	7	7
2	0	0	min(9, 6) = 6	6
		1	min(7, 6) = 6	
	1	0	min(9, 7) = 7	7
		1	min(7, 7) = 7	
	2	0	min(9, 8) = 8	8
		1	min(7, 8) = 7	
3	0	0	min(6,10) = 6	
		1	min(7,10) = 7	
		2	min(8,10) = 8	8

B1	Four or five columns, including 'stage', 'state' and 'action'	
B0		
B0	No follow through from incorrect networks	**[3]**
M1	Min values correct for stage 2 and suboptmal maximin values correct for stages 1 and 2 (cao)	
A0	No follow through from incorrect networks	**[2]**
M1	Correct min values for stage 3 and maximin value for stage 3 (cao)	
A0		**[2]**
B1	8, cao	
B1	Correct route, or in reverse	**[2]**

Weight of heaviest truck = 8 tonnes
Maximin route = (0; 0) – (1; 0) – (2; 2) – (3; 0)

Maximum = B1 M1 M1 B1 B1 = 5 marks out of 9

5	**(i)**	Conan Robbie *D* -1 -4 2 -4 *E* 3 1 -4 -4 *F* 1 -1 1 -1 * col max 3 1 2 * Play-safe for Robbie is fairy Play-safe for Conan is hag Robbie should choose the elf	M1 M1 A1 A1 B1	Calculating row minima (cao) Calculating column maxima (or their negatives) (cao) Fairy or *F* (not just -1 or identifying row) Hag or *H* (not just ± 1 or identifying column) Follow through their play-safe for Conan Elf or *E*	**[5]**
	(ii)	Dwarf: $\frac{1}{3}[(-1) + (-4) + (2)] = -1$ Elf: $\frac{1}{3}[(3) + (1) + (-4)] = 0$ Fairy: $\frac{1}{3}[(1) + (-1) + (1)] = \frac{1}{3}$	M1 A1	$D = -1$ or $F = \frac{1}{3}$ or -3, 0, 1 All three correct	**[2]**
	(iii)	Goblin: $3p + (1-p) = 1 + 2p$ Hag: $p - (1-p) = 2p - 1$ Imp: $-4p + (1-p) = 1 - 5p$ $2p - 1 = 1 - 5p$ $\Rightarrow p = \frac{2}{7}$	M1 A1 M1 A1	Any one correct (in any form) All three correct (in any form) Appropriate equation seen for their expressions $\frac{2}{7}$ or 0.286 (or better) from method seen	**[2]** **[2]**
	(iv)	4 is added throughout the table to make all the entries non-negative If Conan chooses the goblin, this gives an expected value (in the new table) of $3x + 7y + 5z$	B1 B1	Add 4 to remove negative values Expected value when Conan chooses the goblin	**[2]**
	(v)	$z = \frac{5}{7} \Rightarrow m \leq 5.571$, $m \leq 3.571$, $m \leq 3.571$ $\Rightarrow m \leq 3.571$ $(3\frac{4}{7})$ $(\frac{25}{7})$ Hence, maximum value for *M* is 3.571 – 4 = -0.429 or $-\frac{3}{7}$	M1 M1 A1	Using $z = \frac{5}{7}$ to find a value for *m* (or implied) Subtacting 4 from their *m* value cao	**[3]**
					Total = 16

6	**(i)**	α = 12 litres per second β = 15 litres per second	B1 B1	12 15	**[2]**
	(ii)	At least 3 litres per second must flow into *A*, so *AC* and *AF* cannot both have flows of 1	B1	At least 3 flows along *SA*	**[1]**
	(iii)	At most 4 litres per second can flow into *B*, and at least 4 must flow out, so *BC* and *BD* must have flows of 2 Hence, only 2 litres per second flows into *D* and at least 2 litres per second must flow out, so *DE* and *DT* must both be at their lower capacities	B1 B1	At *B*: flow in $\leq$ 4 (and flow out $\geq$ 4) hence given flows in *BC* and *BD* Stating that flow into *D* is 2 and hence given flows in *DE* and *DT*	**[2]**
	(iv)	Flow across {*S*, *A*, *B*, *C*}, {*D*, *E*, *F*, *G*, *T*} $\geq$ 11 (so 10 litres per second is impossible) Minimum = 11 eg  Maximum = 12 No more than 12 can cross cut α and 12 is possible, eg augment flow shown above by 1 litre per second along *SAFT*	M1 A1 M1 A1 M1 A1	Or any equivalent reasoning (eg flow through *C*) Wholly convincing argument 11 Showing that 11 is possible (check *C*) 12 Showing that 12 is possible but 13 is not	**[2]** **[2]** **[2]**
	(v)	 e.g  	B1 M1 A1	A correct reduced network (vertex *E* and all arcs incident on *E* deleted), including arc capacities Or putting E_{in} and E_{out} with a capacity of 0 between them Or giving *CE*, *EG* and *DE* upper and lower capacities of 0 On same diagram or a new diagram *SA* = 3, *SC* = 2, *SB* = 4, *BC* = 2 and *BT* = 2 (and nothing through *E*, if shown) A valid flow of 9 litres per second through the network	**[3]**
					Total = 14