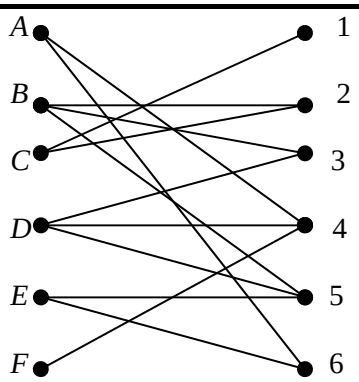
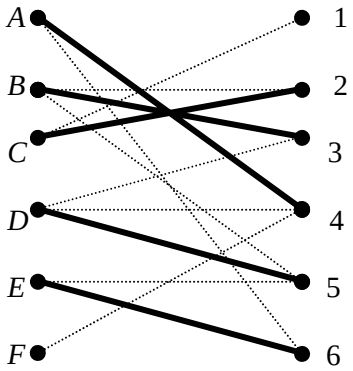
4737 Decision Mathematics 2

1	**(i)**		M1 A1	Any three stars paired to the correct rooms All correct $A \rightarrow 4, 6$ $D \rightarrow 3, 4, 5$ $B \rightarrow 2, 3, 5$ $E \rightarrow 5, 6$ $C \rightarrow 1, 2$ $F \rightarrow 4$	**[2]**
	(ii)	Faye 	B1 B1	Accept *F* Incomplete matching shown correctly on a second diagram (need not see other arcs) Arc $F \rightarrow 1$ must NOT be shown as part of the matching	**[2]**
	(iii)	$F=4 - A=6 - E=5 - D=3 - B=2 - C=1$ Arnie = Room 6 Diana = Room 3 Brigitte = Room 2 Edward = Room 5 Charles = Room 1 Faye = Room 4	B1 B1	This path indicated clearly This matching <u>listed</u> in any form (but NOT just shown as a bipartite graph)	**[2]**

(iv)

	1	2	3	4	5	6
A	3	6	4	1	5	2
B	5	3	2	4	1	6
C	2	1	3	4	5	6
D	5	4	1	3	2	6
E	5	6	4	3	2	1
F	5	6	4	1	3	2

For reference only

Reduce rows

2	5	3	0	4	1
4	2	1	3	0	5
1	0	2	3	4	5
4	3	0	2	1	5
4	5	3	2	1	0
4	5	3	0	2	1

Then reduce columns

1	5	3	0	4	1
3	2	1	3	0	5
0	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

M1

Or reduce columns

1	4	3	0	4	1
3	2	1	3	0	5
2	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

M1

Then reduce rows

1	4	3	0	4	1
3	2	1	3	0	5
2	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

A1 cao with rows reduced first

Follow through their reasonable reduced cost matrix if possible **[3]**

Cross out 0's using 5 lines
Augment by 1 to get a complete allocation

M1 Any valid choice of lines (max for theirs)
M1 Augmenting appropriately
A1 Augmentation completely correct (ft) **[3]**

A = 1 *B* = 5 *C* = 2 *D* = 3 *E* = 6 *F* = 4

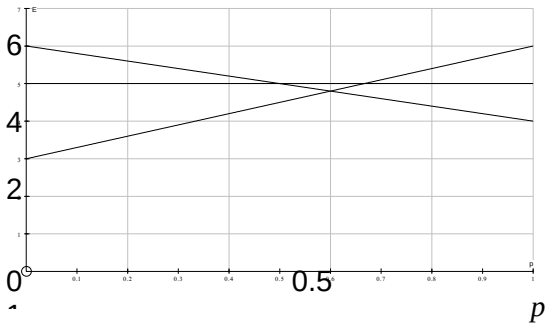
B1 This allocation <u>listed</u> in any form, cao

Arnie

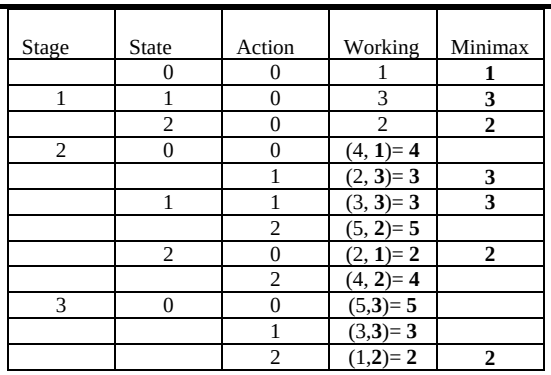
B1 Arnie named (not just *A*), cao **[2]**

Total = 14

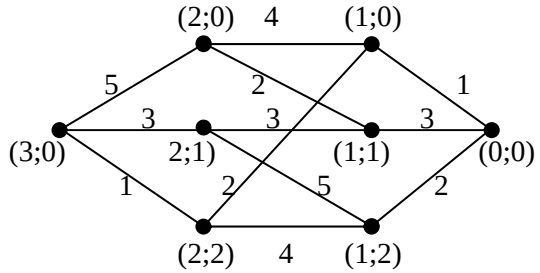
2	**(i)**	6	B1	6 **[1]**
	(ii)	The total number of points for each combination is 10, subtracting 5 from each entry gives a total of 0 for each entry.	B1	Total = 10 changes to total = 0 or subtracting 5 gives total = 0 for every cell **[1]**
	(iii)	<table><tr><th></th><th>Liam</th><th>Mike</th><th>Nicola</th><th>row min</th></tr><tr><td>Philip</td><td>-1</td><td>0</td><td>1</td><td>-1</td></tr><tr><td>Sanjiv</td><td>-2</td><td>-3</td><td>-1</td><td>-3</td></tr><tr><td>Tina</td><td>1</td><td>0</td><td>-2</td><td>-2</td></tr><tr><td>col max</td><td>1</td><td>0</td><td>1</td><td></td></tr></table> Play-safe for R is Philip Play-safe for C is Mike Not stable since $-1 \neq 0$ If Team R play safe then Team C should choose Liam	M1 B1 A1 B1 B1	Row for Sanjiv is optional Writing out pay-off matrix for zero-sum game (or explaining that the given matrix will give the same play safes since each entry is a constant 5 more than in the zero-sum game P, cao, row minima need not be seen M, cao, col maxima need not be seen Accept any reasonable identification Any equivalent reasoning Their row maximin ≠ their col minimax 'Liam' or 'L', or follow through their choice of play safe for Team R **[5]**
	(iv)	If the entry for row P column L is increased the col max for Liam is at least as big as at present so column M is still the column minimax and the row min for Philip is at least as big as at present so row P is still the row maximin.	M1 A1	Using either original values or augmented values. A reasonable explanation of either part A correct explanation of both (in play safe row and not in play safe column, without further explanation ⇒ M1, A0) **[2]**
	(v)	Sanjiv's scores are dominated by Philip's. Sanjiv scores fewer hits than Philip <u>for each choice</u> of captains from Team C	B1	Identifying dominance by *P* <u>and</u> explaining it or showing the three comparisons **[1]**
	(vi)	$4p + 6(1-p)$ or $-1p + 1(1-p) + 5$ $= 6-2p$ M: $5p + 5(1-p)$ or $0(p) + 0(1-p) + 5 = 5$ N: $6p + 3(1-p)$ or $1p + -2(1-p) + 5 = 3p+3$	M1 A1 B1	Using original or reduced values correctly Achieving given expression from valid working 5 and $3p+3$, cao **[3]**

	(vii)	E  $3p + 3 = 6 - 2p \Rightarrow p = 0.6$ Expect at least 4.8 hits	 M1 A1 B1 B1	MAY BE ON GRAPH PAPER Appropriate scales and line E = 6-2*p* drawn correctly (Their) other lines drawn correctly Solving for their *p* or from graph Their E for chosen value of *p* or from graph	 **[2]** **[2]**
				Total =	**17**

ANSWERED ON INSERT

3	**(i)**				

Stage	State	Action	Working	Minimax
	0	0	1	**1**
1	1	0	3	**3**
	2	0	2	**2**
2	0	0	(4, **1**)= **4**	
		1	(2, **3**)= **3**	**3**
	1	1	(3, **3**)= **3**	**3**
		2	(5, **2**)= **5**	
	2	0	(2, **1**)= **2**	**2**
		2	(4, **2**)= **4**	
3	0	0	(5,**3**)= **5**	
		1	(3,**3**)= **3**	
		2	(1,**2**)= **2**	**2**

	B1	Minimax column for stage 1 shows 1, 3, 2 identified in some way	
	M1	1, 3, 2 transferred to working column for stage 2 correctly	
	M1	Calculating maximum values in working column for stage 2	
	A1	Minimax column for stage 2 shows 3, 3, 2 identified in some way (cao)	**[4]**
	M1	Calculating maximum values in working column for stage 3, correct method	
	A1	Minimax column for stage 3 shows 2 identified in some way (cao)	**[2]**
(ii)	Minimax value = 2 Minimax route = (3;0) – (2;2) – (1;0) – (0;0) (or in reverse)	B1 M1 A1	2, cao Tracing their route (whatever problem solved) This route from correct working (using network ⇒ M0)	**[3]**
(iii)	 	B1 M1 A1	All vertices labelled correctly Arcs correct, need not be directed Condone stage boundaries shown Arc weights correct (be generous in interpretation of which weight is attached to which arc)	 **[3]**
			Total =		**12**

ANSWERED ON INSERT

4	**(i)**	A single source that joins to S_1 and S_2 Directed arcs with weights of at least 90 and 110, respectively T_1 and T_2 joined to a single sink Directed arcs with weights of at least 100 and 200, respectively	 B1 B1	 Condone no directions shown Condone no directions shown	**[2]**
	(ii)	If *AE* and *BE* were both full to capacity there would be 50 gallons per hour flowing into *E*, but the most that can flow out of *E* is 40 gallons per hour.	M1 A1	Considering what happens at *E* (50 into *E*) At most 40 out	**[2]**
	(iii)	40 + 60 + 60 + 140 = 300 gallons per hour	B1	300	**[1]**
	(iv)	30 + 20 + 30 + 20 + 40 + 40 + 20 + 40 = 240 gallons per hour	M1 A1	Evidence of using correct cut 240	**[2]**
	(v)	A feasible flow through network Flow = 200 gallons per hour Cut through arcs S_1A, S_1B, S_1C, S_2B, S_2C and S_2D or cut X = { S_1, S_2}, Y = {*A*, *B*, *C*, *D*, *E*, *F*, *G*, T_1, T_2}	M1 A1 B1	 Cut indicated in any way (May be on diagram for part (i))	**[3]**
	(vi)	 Flows into *C* go to C_{IN}, arc of capacity 20 from C_{IN} to C_{OUT}, and flows out of *C* go from C_{OUT}. Cut X = {S_1, S_2, C_{IN}}or X = {S_1, S_2, C_{IN}, *D*} shows max flow = 140 gallons per hour	 B1 B1 B1 B1	May have working or cut shown on diagram Into *C* (S_1 = 40, S_2 = 40, *D* = 20) Through *C* Out of *C* (*F* = 60, *G* = 60) 140 (cut not necessary)	**[4]**
					Total = 14

ANSWERED ON INSERT

5 (i)

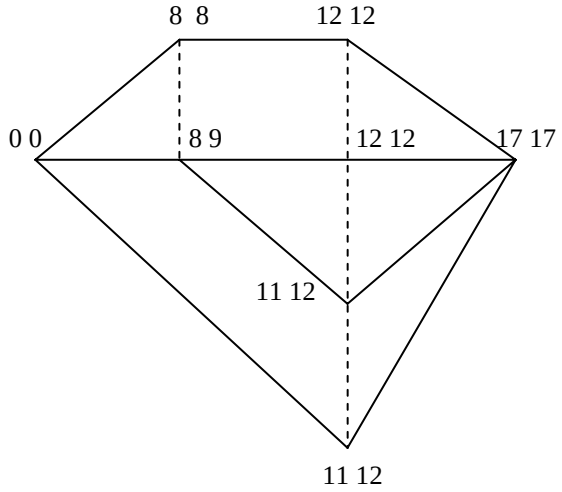
Activity	Duration (days)	Immediate predecessors
A	8	-
B	6	-
C	4	-
D	4	***A***
E	2	***A B***
F	3	***A B***
G	4	***D***
H	5	***D E F***
I	3	***F***
J	5	***C F***

B1 Precedences correct for *A*, *B*, *C*, *D*

B1 Precedences correct for *E*, *F*, *G*

B1 Precedences correct for *H*, *I*, *J* **[3]**

(ii)



Minimum project duration = 17 days

Critical activities = *A D H*

M1 Forward pass, no more than one independent error

A1 Forward pass correct (cao)

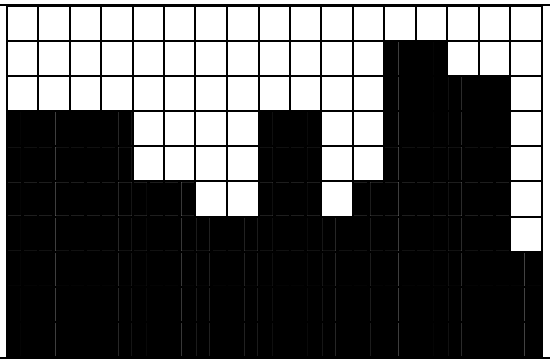
M1 Backward pass, no more than one independent error

A1 Backward pass correct (cao) **[4]**

B1 17, cao

B1 *A D H*, cao **[2]**

(iii)



ANSWERED ON GRAPH PAPER

M1 A plausible histogram, with no holes or overhanging blocks

A1 Correct shape

[2]

(iv)

Example:

Start *A* and *B* as before but delay *C* to day 6

Start *D* and *F* as before but delay *E* to day 11

Then, for example, start *G* on day 12, *H* on day 13, and *I* and *J* on day 16

Precedences not violated, durations correct

B1 Dealing with *A*, *B* and *C*

B1 Dealing with *D*, *E* and *F*

M1 Dealing with *G*, *H I* and *J*

A1 A valid solution using 6 workers for 21 days **[4]**

Total = 15