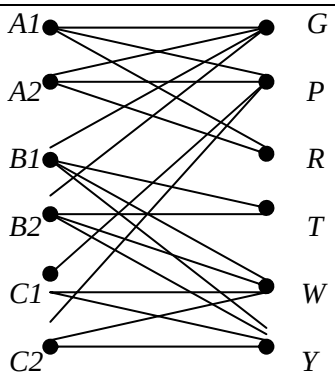
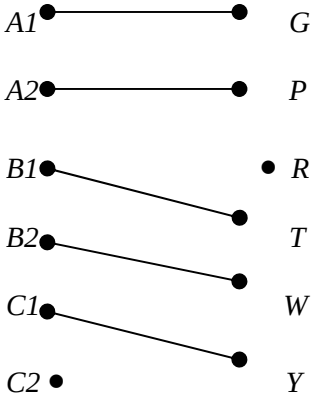
Mark Scheme 4737
January 2006

1	**(i)**		M1	For *A1* or *A2* or both joined to *G*, *P* and *R* *B1* or *B2* or both joined to *G*, *T*, *W* and *Y* and *C1* or *C2* or both joined to *P*, *W* and *Y*
			A1	For both *A1* and *A2* joined to *G*, *P* and *R* both *B1* and *B2* joined to *G*, *T*, *W* and *Y* and both *C1* and *C2* joined to *P*, *W* and *Y*
	(ii)		B1	For *A1* paired with *G* and *A2* with *P*, or vice versa; *B1* paired with *T* and *B2* with *W*, or vice versa; one of *C1*, *C2* paired with *Y* and the other left unpaired
	(iii)	*R – A2 – P – C2* or in reverse	M1	For a valid alternating path for their diagram (need not be minimum)
		Accept *R – A – P – C* or in reverse	A1	For this alternating path
		Amanda gets green and red Ben gets turquoise and white Carrie gets yellow and pink	B1	For this matching (may use *G*, *R*, etc.)
	(iv)	Amanda gets pink and red Ben gets green and turquoise Carrie gets white and yellow	B1 7	For this matching (may use *G*, *R*, etc.)

2

Stage	State	Action	Cost	Minimum
1	0	0	3	3
	1	0	4	4
2	0	0	6+3 = 9	
		1	4+4 = 8	8
	1	0	6+3 = 9	9
		1	7+4 = 11	
3	0	0	5+8 = 13	
		1	3+9 = 12	12

Shortest route: (3; 0) – (2; 1) – (1; 0) – (0; 0) B1 For this route (not in reverse)
Length of shortest route = 12 km B1 **6** For 12

M1 For completing cost column for stage 2
A1 For completing cost column for stage 3
M1 For completing minimum column for stage 2
A1 For completing minimum column for stage 3

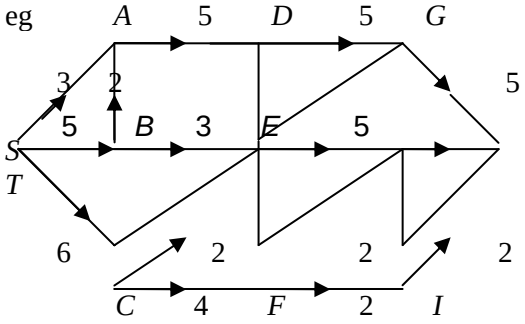
3 **(i)** 6+3+2+4 = 15 litres per second B1 For 15

(ii) 6+2+3+6+2+2 = 21 litres per second M1 For showing how 21 (given) was worked out
Do not use arc *DG* as it is crossed twice A1 For explaining why arc *DG* is not used
or X = {*S*,*A*,*B*,*C*,*E*,*F*} Y = {*D*,*G*,*H*,*I*,*T*}

(iii) *SC* cannot be full since the most that can leave it is 2+4 = 6 litres per second B1 For 6 and 'out' or equivalent
AD cannot be full since the most that can enter it is 2+3 = 5 litres per second B1 For 5 and 'in' or equivalent
The most that can flow in *SB* is 2+3 = 5 litres per second B1 For *SB* = 5

(iv) Maximum flow = 14 litres per second B1 For 14

eg



M1 For a feasible flow (may imply vertex labels)
A1 For a feasible flow of 14 litres per second (directions must be shown for the A mark)

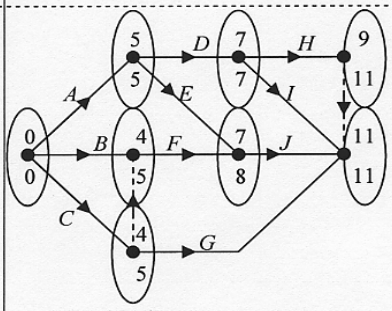
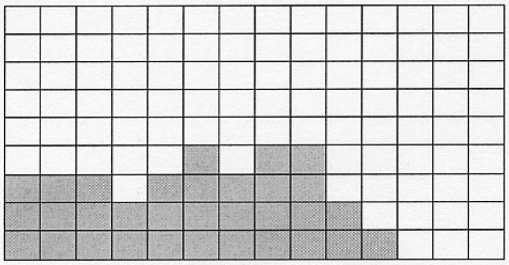
M1 For this cut described or drawn on diagram
Cut X = {*S*,*B*,*C*} Y = {*A*,*D*,*E*,*F*,*G*,*H*,*I*,*T*} A1 For explicitly stating that this cut = 14
This cut has capacity 14 litres per second

Maximum flow ≥ this flow = 14
Minimum cut ≤ this cut = 14
But maximum flow = minimum cut
so 14 is the maximum flow and the minimum cut.

B1 For explaining how maximum flow = minimum cut shows that 14 is the maximum here (at least referring to 'this flow' and 'this cut', not just stating 'max flow = min cut)

12

4	**(i)**	£260	B1	For correct answer with units
	(ii)	Reduce rows 0 60 30 10 0 90 90 60 0 80 70 40 10 10 20 0	M1	Or scaled throughout by 10 For correct method for reducing rows
		Reduce columns 0 50 10 10 0 80 70 60 0 70 50 40 10 0 0 0	M1 A1	For correct method for reducing columns For a correct reduced cost matrix
		Cross through 0's using as few lines as possible 0 50 10 10 0 80 70 60 0 70 50 40 10 0 0 0	M1	For correct crossing out for their reduced cost matrix. Likely to be shown on reduced cost matrix.
		Augment by 10 0 40 0 0 0 70 60 50 0 60 40 30 20 0 0 0	M1 A1	For correct augmenting from their reduced cost matrix and their crossing out For a correct solution after first augmenting
		Cross through 0's using as few lines as possible 0 40 0 0 0 70 60 50 0 60 40 30 20 0 0 0	M1	For correct crossing out for their augmented matrix. Likely to be shown on matrix.
		Augment by 30 30 40 **0** 0 **0** 40 30 20 0 30 10 **0** 50 **0** 0 0	M1 A1	For correctly augmenting their matrix in one step by an amount greater than 10 (or greater than 1 if scaling has been used) For correct final matrix from completely correct method
		Allocation $A = Y$ $B = W$ $C = Z$ $D = X$ Cost = £180	B1 B1	For this allocation For £180
	(iii)	Hungarian algorithm finds the minimum cost complete matching	B1 **13**	For 'minimum cost' or equivalent

5	(i)	The dummy activity after C is needed because G follows C only but F follows both B and C. The dummy activity after H is needed because both H and I directly join the same pair of vertices (events).	B1 B1	For explaining the relationships between B,C,F,G For identifying H and I
	(ii)	<table><tr><th>Activity</th><th>Duration</th><th>Immediate predecessors</th></tr><tr><td>A</td><td>5</td><td>-</td></tr><tr><td>B</td><td>3</td><td>-</td></tr><tr><td>C</td><td>4</td><td>-</td></tr><tr><td>D</td><td>2</td><td>A</td></tr><tr><td>E</td><td>1</td><td>A</td></tr><tr><td>F</td><td>3</td><td>B,C</td></tr><tr><td>G</td><td>5</td><td>C</td></tr><tr><td>H</td><td>2</td><td>D</td></tr><tr><td>I</td><td>4</td><td>D</td></tr><tr><td>J</td><td>3</td><td>E,F</td></tr></table>	B1 B1 B1	For A,B,C having no predecessors For D,E,F,G correct For H,I,J correct
	(iii)	 Minimum completion time = 11 days Critical activities: A, D, I	M1 A1 M1 A1 B1 B1	For carrying out forward pass For all early event times correct For carrying out backwards pass For all late event times correct For 11 with units For A, D, I only
	(iv)	1 day	B1	For 1 (not 'more than 1')
	(v)	 Number of workers required = 4	B1 M1 A1 A1	For the first four days correct For a resource histogram with no 'hanging' cells For days five onwards correct For 4 or follow through their histogram if possible
	(vi)	Delay activity E by 2 days, so the start changes from 5 to 7 (it happens on day 8). This causes activities H and J to shift. J shifts by 1 day and H by 2 days.	B1 M1 A1 **19**	For stating how much to move E For saying that H and J(only) will need to shift For describing how much H and J move

6	**(i)**	In column *Y*: -3 < 5 so *A* does not dominate *B* In column *X*: -3 < 2 (or column *Z*: 1 < 4) so *B* does not dominate *A*	B1 B1	For -3 < 5 or equivalent For -3 < 2 or 1 < 4 or equivalent
	(ii)	The worst outcomes for Maria are: *X* lose 4, *Y* lose 5, *Z* lose 4	M1 A1	For finding column maxima For rejecting 5 as being bigger than 4, or using a word like 'lose' or '-4, -5, -4'
	(iii)	If Lucy plays *B* she could win as much as 5	B1	For '5 is the most she can win' or equivalent
	(iv)	Need to add 3 throughout matrix to make values non-negative, this removes the 3 again	B1	For 'add 3 throughout matrix' or equivalent
	(v)	Having added 3 throughout, the expected number of points win by Lucy when Maria chooses strategy *X* is $5p_1 + 0p_2 + 7p_3$, and similarly the second expression is the expected number of points won by Lucy when Maria chooses strategy *Y* and the third expression is the expected number of points won by Lucy if Maria chooses strategy *Z*	M1 A1	For showing where one of the expressions came from, or for referring to 'when Maria plays each of her strategies' or equivalent in a non-specific way For specifically linking the expressions to Maria choosing strategy *X*, strategy *Y* and strategy *Z* in that order
	(vi)	The number of points that Lucy can expect to win cannot be less than the worst of the three expressions, so it is less than or equal to each of them.	M1 A1	For reference to 'number of points won by Lucy' or equivalent For reference to 'the worst outcome' or equivalent
	(vii)	$2(p_1) + 4(p_3) = 2p_1 + 4(1-p_1) = 4-2p_1$ (given) $4p_1 - 3(1-p_1) = 7p_1 - 3$	B1 B1	For $2p + 4(1-p)$ For $7p - 3$
	(viii)	$4 - 2p_1 = 7p_1 - 3 \Rightarrow p_1 = \frac{7}{9}$ $p_1 = \frac{7}{9} \Rightarrow 2\frac{4}{9}$, $p_1 = 0 \Rightarrow \min(4, -3) = -3$, $p_1 = 1 \Rightarrow \min(2, 4) = 2$ Maximin is when $p_1 = \frac{7}{9} \Rightarrow$ choose randomly between *A* and *C* so that *A* is chosen with probability $\frac{7}{9}$	B1 M1 A1 **15**	For solving $4 - 2p_1$ = their expression to get a probability For evaluating $4-2p_1$ at their p_1 and the values -3 and 2 For reference to maximin, or equivalent, leading to selection of $p = \frac{7}{9}$, or in context