OCR
RECOGNISING ACHIEVEMENT

Friday 22 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4772   Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
* Printed Answer Book 4772
* MEI Examination Formulae and Tables (MF2)

Other materials required:
* Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

* The Question Paper will be found in the centre of the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
* **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

* The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **72**.
* The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

* Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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1 **(a)** When marking coursework, a teacher has to complete a form which includes the following:

> In your opinion is this the original work of the pupil? (tick as appropriate)
>
> I have no reason to believe that it is not ☐
>
> I cannot confirm that it is ☐

    (i) The teacher suspects that a pupil has copied work from the internet. For each box, state whether the teacher should tick the box or not. **[2]**

  (ii) The teacher has no suspicions about the work of another pupil, and has no information about how the work was produced. Which boxes should she tick? **[2]**

(iii) Explain why the teacher must always tick at least one box. **[2]**

(b) Angus, the ski instructor, says that the class will have to have lunch in Italy tomorrow if it is foggy or if the top ski lift is not working. On the next morning Chloe, one of Angus's students, says that it is not foggy, so they can have lunch in Switzerland.

Produce a line of a truth table which shows that Chloe's deduction is incorrect. You may produce a complete truth table if you wish, but you must indicate a row which shows that Chloe's deduction is incorrect. **[6]**

(c) You are given that the following two statements are true.

$$(X \vee \sim Y) \Rightarrow Z$$
$$\sim Z$$

Use Boolean algebra to show that Y is true. **[4]**

2   Adrian is considering selling his house and renting a flat.

Adrian still owes £150 000 on his house. He has a mortgage for this, for which he has to pay £4800 annual interest. If he sells he will pay off the £150 000 and invest the remainder of the proceeds at an interest rate of 2.5% per annum. He will use the interest to help to pay his rent.

His estate agent estimates that there is a 30% chance that the house will sell for £225 000, a 50% chance that it will sell for £250 000, and a 20% chance that it will sell for £275 000.

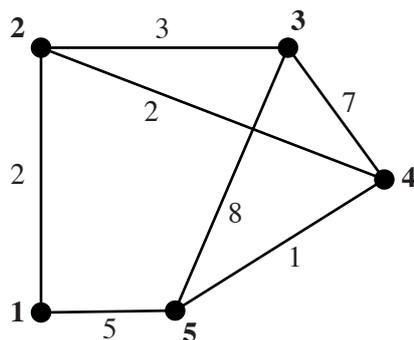A flat will cost him £7500 per annum to rent.

   (i) Draw a decision tree to help Adrian to decide whether to keep his house, or to sell it and rent a flat. Compare the EMVs of Adrian's annual outgoings, and ignore the costs of selling.   **[6]**

   (ii) Would the analysis point to a different course of action if Adrian were to use a square root utility function, instead of EMVs?   **[3]**

Adrian's circumstances change so that he has to decide now whether to sell or not in one year's time. Economic conditions might then be less favourable for the housing market, the same, or more favourable, these occurring with probabilities 0.3, 0.3 and 0.4 respectively. The possible selling prices and their probabilities are shown in the table.

Economic conditions and probabilities		Selling prices (£) and probabilities					
less favourable	0.3	200 000	0.2	225 000	0.3	250 000	0.5
unchanged	0.3	225 000	0.3	250 000	0.5	275 000	0.2
more favourable	0.4	250 000	0.3	300 000	0.5	350 000	0.2

   (iii) Draw a decision tree to help Adrian to decide what to do. Compare the EMVs of Adrian's annual outgoings. Assume that he will still owe £150 000 in one year's time, and that the cost of renting and interest rates do not change.   **[7]**

4

3 The weights on the network represent distances.

(i) The answer book shows the initial tables and the results of iterations 1, 2, 3 and 5 when Floyd's algorithm is applied to the network.

   (*A*) Complete the two tables for iteration 4. **[6]**

   (*B*) Use the final route table to give the shortest route from vertex **3** to vertex **5**. **[1]**

   (*C*) Use the final distance table to produce a complete network with weights representing the shortest distances between vertices. **[2]**

(ii) Using the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson Problem by deleting vertex **5** and its arcs, and by finding the length of a minimum connector for the remainder. (You may find the minimum connector by inspection.) **[3]**

(iii) Use the nearest neighbour algorithm, starting at vertex **1**, to produce a Hamilton cycle in the complete network. Give the length of your cycle. **[3]**

(iv) Interpret your Hamilton cycle in part **(iii)** in terms of the original network. **[2]**

(v) Give a walk of minimum length which traverses every arc on the original network at least once, and which returns to the start. Give the length of your walk. **[3]**

4   A publisher is considering producing three books over the next week: a mathematics book, a novel and a biography. The mathematics book will sell at £10 and costs £4 to produce. The novel will sell at £5 and costs £2 to produce. The biography will sell at £12 and costs £5 to produce. The publisher wants to maximise profit, and is confident that all books will be sold.

There are constraints on production. Each copy of the mathematics book needs 2 minutes of printing time, 1 minute of packing time, and $300 \, cm^3$ of temporary storage space.

Each copy of the novel needs 1.5 minutes of printing time, 0.5 minutes of packing time, and $200 \, cm^3$ of temporary storage space.

Each copy of the biography needs 2.5 minutes of printing time, 1.5 minutes of packing time, and $400 \, cm^3$ of temporary storage space.

There are 10 000 minutes of printing time available on several printing presses, 7500 minutes of packing time, and $2 \, m^3$ of temporary storage space.

(i)   Explain how the following initial feasible tableau models this problem. **[5]**

P	x	y	z	$s1$	$s2$	$s3$	RHS
1	−6	−3	−7	0	0	0	0
0	2	1.5	2.5	1	0	0	10 000
0	1	0.5	1.5	0	1	0	7500
0	300	200	400	0	0	1	2 000 000

(ii)   Use the simplex algorithm to solve your LP, and interpret your solution. **[8]**

(iii)   The optimal solution involves producing just one of the three books. By how much would the price of each of the other books have to be increased to make them worth producing? **[2]**

There is a marketing requirement to provide at least 1000 copies of the novel.

(iv)   Show how to incorporate this constraint into the initial tableau ready for an application of the two-stage simplex method.

Briefly describe how to use the modified tableau to solve the problem. You are NOT required to perform the iterations. **[5]**

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