OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4772

Decision Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Mickey ate the last of the cheese. Minnie was put out at this. Mickey's defence was "There wasn't enough left not to eat it all".

Let "c" represent "there is enough cheese for two" and "e" represent "one person can eat all of the cheese".

(i) Which of the following best captures Mickey's argument?

$c \Rightarrow e$ $\quad$ $c \Rightarrow \sim e$ $\quad$ $\sim c \Rightarrow e$ $\quad$ $\sim c \Rightarrow \sim e$ **[1]**

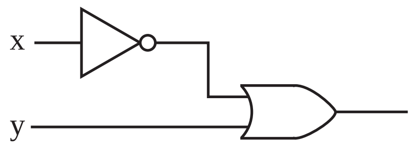
In the ensuing argument Minnie concedes that if there's a lot left then one should not eat it all, but argues that this is not an excuse for Mickey having eaten it all when there was not a lot left.

(ii) Prove that Minnie is right by writing down a line of a truth table which shows that

$(c \Rightarrow \sim e) \Leftrightarrow (\sim c \Rightarrow e)$

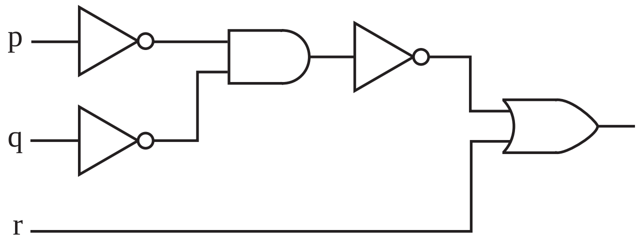
is false.
(You may produce the whole table if you wish, but you need to indicate a specific line of the table.) **[4]**

(b) **(i)** Show that the following combinatorial circuit is modelling an implication statement. Say what that statement is, and prove that the circuit and the statement are equivalent.



[5]

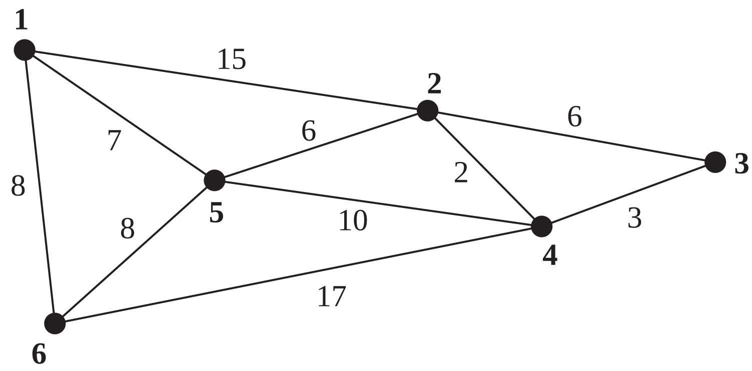
(ii) Express the following combinatorial circuit as an equivalent implication statement.



[2]

(iii) Explain why $(\sim p \wedge \sim q) \Rightarrow r$, together with $\sim r$ and $\sim q$, give p. **[4]**

2 The network is a representation of a show garden. The weights on the arcs give the **times** in minutes to walk between the six features represented by the vertices, where paths exist.



(i) Why might it be that the time taken to walk from vertex **2** to vertex **3** via vertex **4** is less than the time taken by the direct route, i.e. the route from **2** to **3** which does not pass through any other vertices? **[1]**

The matrices shown below are the results of the first iteration of Floyd's algorithm when applied to the network.

	1	**2**	**3**	**4**	**5**	**6**
1	∞	15	∞	∞	7	8
2	15	30	6	2	6	23
3	∞	6	∞	3	∞	∞
4	∞	2	3	∞	10	17
5	7	6	∞	10	14	8
6	8	23	∞	17	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	1	1	3	4	5	1
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	1	6
6	1	1	3	4	5	1

(ii) Complete the second iteration of Floyd's algorithm. **[4]**

The matrices below are the final matrices resulting from Floyd's algorithm.

	1	**2**	**3**	**4**	**5**	**6**
1	14	13	18	15	7	8
2	13	4	5	2	6	14
3	18	5	6	3	11	19
4	15	2	3	4	8	16
5	7	6	11	8	12	8
6	8	14	19	16	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	5	5	5	5	5	6
2	5	4	4	4	5	5
3	4	4	4	4	4	4
4	2	2	3	2	2	2
5	1	2	2	2	2	6
6	1	5	5	5	5	1

(iii) Explain what the algorithm has achieved.

Show how to find the shortest time and the quickest route from vertex **3** to vertex **6**.

Give the shortest time and the quickest route from vertex **3** to vertex **6**. **[5]**

A visitor to the garden wishes to visit all six features, starting from the feature represented by vertex **1**.

(iv) Use the final matrices to find an upper bound for the minimum time for which the visitor must walk, and give a route through the garden corresponding to this. **[2]**

(v) By deleting vertex **1** and its arcs construct a lower bound for the time for which the visitor must walk. You may construct a minimum connector for the reduced network without using an algorithm. **[3]**

(vi) Given that the sum of the times taken to walk the paths is 82 minutes, find the minimum time that could be taken by a member of staff to start at vertex **1**, walk along every path, and return to vertex **1**. **[1]**

3 It is Ken's 59th birthday, and he is considering whether or not to retire early. He can retire and take his pension now, when he reaches 60, or when he reaches 65.

Ken's pension is computed by taking the number of years for which he has worked, multiplying by his final salary, and dividing by 80. He has currently worked for 35 years. He is at the top of his grade and is earning £50 000 per annum. (Ignore any changes which might occur due to inflation or pay rises.)

Ken estimates that, at age 59, he has a 0.6 probability of getting a part-time (50%) contract on his current grade; at age 60 the probability will be 0.5; at age 65 the probability will be 0.25. (His part-time earnings will not affect his pension.)

Ken intends to retire completely when he reaches 70.

(i) Draw up a decision tree showing Ken's options. **[4]**

(ii) Find the EMV of Ken's gross income (before tax and other stoppages) from age 59 to age 70 in each scenario, and indicate the course of action which will maximise his EMV. **[9]**

Ken values his time and decides to apply a utility function to his gross incomes to reflect this. In each 11-year scenario he computes his utility as (gross income × 3^{-p}) where p is the proportion of working time for which he is working. Thus, in the scenario in which he retires at 65 and succeeds in securing a part-time contract thereafter, $p = \frac{6 + 2.5}{11} = \frac{17}{22}$.

(iii) Find Ken's expected utilities and indicate the course of action which will maximise his expected utility. **[7]**

4 A craft workshop produces three products, xylophones, yodellers and zithers. The times taken to make them and the total time available per week are shown in the table. Also shown are the costs and the total weekly capital available.

	xylophones	yodellers	zithers	resource availability
time (hours)	2	5	3	30
cost (£00s)	4	1	2	24

Profits are £180 per xylophone, £90 per yodeller and £110 per zither.

(i) Formulate a linear programming problem to find the weekly production plan which maximises profit within the resource constraints. **[3]**

(ii) Use the simplex algorithm to solve the problem, pivoting first on the column of your tableau containing the variable which represents the number of xylophones produced. Explain how your final tableau shows that the workshop should produce 5 xylophones and 4 yodellers. **[8]**

If, when applying the simplex algorithm, the first pivot is on the column containing the variable which represents the number of zithers produced, then the final solution produced is for the workshop to produce 1.5 xylophones and 9 zithers per week.

(iii) How can this production plan be implemented? **[1]**

(iv) Explain how the simplex algorithm can lead to different solutions. **[2]**

To satisfy demand an extra constraint has to be added to the problem – that the number of xylophones plus the number of yodellers produced per week must total exactly 7.

(v) Show how to adapt this problem for solution either by the two-stage simplex method or the big-M method. In either case you should show the initial tableau and describe what has to be done next. You are not required to apply your method. **[4]**

(vi) The simplex solution to the revised problem is for the workshop to produce $4\frac{1}{15}$ xylophones, $2\frac{14}{15}$ yodellers and $2\frac{2}{5}$ zithers. Ignoring the practicalities explain how this solution relates to the two solutions referred to in part **(iv)**. **[2]**