OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Question **2**.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages and an insert.

1 **(i)** Use a truth table to prove $\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S)$. [8]

(ii) Prove that $(A \Rightarrow B) \Leftrightarrow (\sim A \vee B)$ and hence use Boolean algebra to prove that

$$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S).$$ [5]

(iii) A teacher wrote on a report "It is not the case that if Joanna doesn't try then she won't succeed." He meant to say that if Joanna were to try then she would have a chance of success. By letting T be "Joanna will try" and S be "Joanna will succeed", find the real meaning of what the teacher wrote. [3]

2 **Answer this question on the insert provided.**

Fig. 2 shows a network in which the weights on the arcs represent distances.

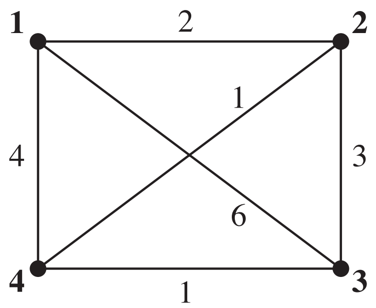

Fig. 2

(i) Apply Floyd's algorithm on the insert provided to find the complete network of shortest distances. [8]

(ii) Show how to use your final matrices to find the shortest route from vertex **1** to vertex **3**, together with the length of that route. [4]

(iii) Use the nearest neighbour algorithm, starting at vertex **1**, to find a Hamilton cycle in the complete network of shortest distances.

Give the corresponding cycle in the original network, together with its length. [4]

3 Emma has won a holiday worth £1000. She is wondering whether or not to take out an insurance policy which will pay out £1000 if she should fall ill and be unable to go on the holiday. The insurance company tells her that this happens to 1 in 200 people. The insurance policy costs £10. Thus Emma's monetary value if she buys the insurance and does not fall ill is £990.

(i) Draw a decision tree for Emma's problem. Use the EMV criterion in your calculations. [6]

(ii) Interpret your tree and say what the maximum cost of the insurance would have to be for Emma to consider buying it if she uses the EMV criterion. [2]

Suppose that Emma's utility function is given by $utility = \sqrt[3]{monetary\ value}$.

(iii) Using expected utility as the criterion, should Emma purchase the insurance?

Under this criterion what is the cost at which she will be indifferent to buying or not buying it? [3]

Emma could pay for a blood pressure check to help her to make her decision. Statistics show that 75% of checks are positive, and that when a check is positive the chance of missing a holiday through ill heath is 0.001. However, when a check is negative the chance of cancellation through ill health is 0.017.

(iv) Draw a decision tree to help Emma decide whether or not to pay for the check. Use EMV, not expected utility, in your calculations and assume that the insurance policy costs £10.

What is the maximum amount that she should pay for the blood pressure check? [9]

[Question 4 is printed overleaf.]

4 The "Cuddly Friends Company" produces soft toys. For one day's production run it has available $11\,m^2$ of furry material, $24\,m^2$ of woolly material and 30 glass eyes. It has three soft toys which it can produce:

The "Cuddly Aardvark", each of which requires $0.5\,m^2$ of furry material, $2\,m^2$ of woolly material and two eyes. Each sells at a profit of £3.

The "Cuddly Bear", each of which requires $1\,m^2$ of furry material, $1.5\,m^2$ of woolly material and two eyes. Each sells at a profit of £5.

The "Cuddly Cat", each of which requires $1\,m^2$ of furry material, $1\,m^2$ of woolly material and two eyes. Each sells at a profit of £2.

An analyst formulates the following LP to find the production plan which maximises profit.

$$\begin{aligned} \text{Maximise} \quad & 3a + 5b + 2c \\ \text{subject to} \quad & 0.5a + b + c \leqslant 11, \\ & 2a + 1.5b + c \leqslant 24, \\ & 2a + 2b + 2c \leqslant 30. \end{aligned}$$

(i) Explain how this formulation models the problem, and say why the analyst has not simplified the last inequality to $a + b + c \leqslant 15$. [4]

(ii) The final constraint is different from the others in that the resource is integer valued. Explain why that does not impose an additional difficulty for this problem. [1]

(iii) Solve this problem using the simplex algorithm.

Interpret your solution and say what resources are left over. [9]

On a particular day an order is received for two Cuddly Cats, and the extra constraint $c \geqslant 2$ is added to the formulation.

(iv) Set up an initial simplex tableau to deal with the modified problem using either the big-M approach or two-phase simplex. Do not perform any iterations on your tableau. [3]

(v) Show that the solution given by $a = 8$, $b = 2$ and $c = 5$ uses all of the resources, but that $a = 6$, $b = 6$ and $c = 2$ gives more profit.

What resources are left over from the latter solution? [3]