OCR RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

Wednesday **25 MAY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

**This question paper consists of 5 printed pages and 3 blank pages.**


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1 The switching circuit in Fig. 1.1 shows switches, s for a car's sidelights, h for its dipped headlights and f for its high-intensity rear foglights. It also shows the three sets of lights.

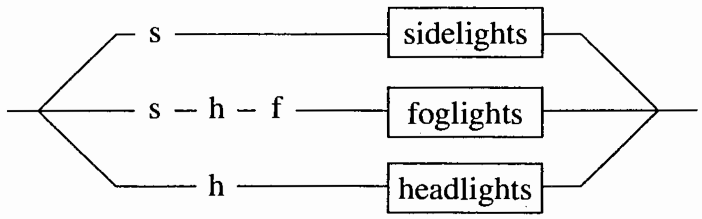


**Fig. 1.1**

(Note: s and h are each "ganged" switches. A ganged switch consists of two connected switches sharing a single switch control, so that both are either on or off together.)

**(a)** **(i)** Describe in words the conditions under which the foglights will come on. [2]

Fig. 1.2 shows a combinatorial circuit.

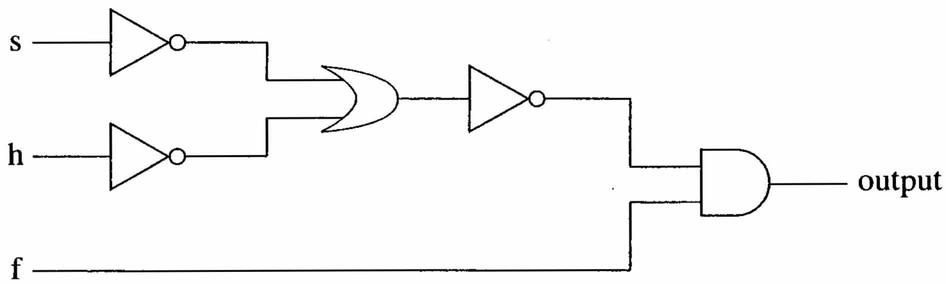


**Fig. 1.2**

**(ii)** Write the output in terms of a Boolean expression involving s, h and f. [2]

**(iii)** Use a truth table to prove that $s \wedge h \wedge f = \sim(\sim s \vee \sim h) \wedge f$. [3]

**(b)** A car's first gear can be engaged (g) if either both the road speed is low (r) and the clutch is depressed (d), or if both the road speed is low (r) and the engine speed is the correct multiple of the road speed (m).

**(i)** Draw a switching circuit to represent the conditions under which first gear can be engaged. Use two ganged switches to represent r, and single switches to represent each of d, m and g. [2]

**(ii)** Draw a combinatorial circuit to represent the Boolean expression $r \wedge (d \vee m) \wedge g$. [4]

**(iii)** Use Boolean algebra to prove that $r \wedge (d \vee m) \wedge g = ((r \wedge d) \vee (r \wedge m)) \wedge g$. [2]

**(iv)** Draw another switching circuit to represent the conditions under which first gear can be selected, but without using a ganged switch. [1]

**2** Karl is considering investing in a villa in Greece. It will cost him 56 000 euros (€ 56 000). His alternative is to invest his money, £35 000, in the United Kingdom.

He is concerned with what will happen over the next 5 years. He estimates that there is a 60% chance that a house currently worth € 56 000 will appreciate to be worth € 75 000 in that time, but that there is a 40% chance that it will be worth only € 55 000.

If he invests in the United Kingdom then there is a 50% chance that there will be 20% growth over the 5 years, and a 50% chance that there will be 10% growth.

**(i)** Given that £1 is worth € 1.60, draw a decision tree for Karl, and advise him what to do, using the EMV of his investment (in thousands of euros) as his criterion. [4]

In fact the £/€ exchange rate is not fixed. It is estimated that at the end of 5 years, if there has been 20% growth in the UK then there is a 70% chance that the exchange rate will stand at 1.70 euros per pound, and a 30% chance that it will be 1.50. If growth has been 10% then there is a 40% chance that the exchange rate will stand at 1.70 and a 60% chance that it will be 1.50.

**(ii)** Produce a revised decision tree incorporating this information, and give appropriate advice. [3]

A financial analyst asks Karl a number of questions to determine his utility function. He estimates that for $x$ in cash (in thousands of euros) Karl's utility is $x^{0.8}$, and that for $y$ in property (in thousands of euros), Karl's utility is $y^{0.75}$.

**(iii)** Repeat your computations from part **(ii)** using utility instead of the EMV of his investment. Does this change your advice? [3]

**(iv)** Using EMVs, find the exchange rate (number of euros per pound) which will make Karl indifferent between investing in the UK and investing in a villa in Greece. [2]

**(v)** Show that, using Karl's utility function, the exchange rate would have to drop to 1.277 euros per pound to make Karl indifferent between investing in the UK and investing in a villa in Greece. [4]

PMT



3 The distance and route matrices shown in Fig. 3.1 are the result of applying Floyd's algorithm to the incomplete network on 4 vertices shown in Fig. 3.2.

Distance Matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 4 | 2 | 3 | 9 |
| 2 | 2 | 2 | 1 | 7 |
| 3 | 3 | 1 | 2 | 6 |
| 4 | 9 | 7 | 6 | 12 |

Route Matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 2 | 2 | 2 |
| 2 | 1 | 3 | 3 | 3 |
| 3 | 2 | 2 | 2 | 4 |
| 4 | 3 | 3 | 3 | 3 |

**Fig. 3.1**

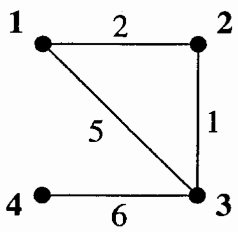


**Fig. 3.2**

**(i)** Draw the complete network of shortest distances. [2]

**(ii)** Explain how to use the route matrix to find the shortest route from vertex 4 to vertex 1 in the original incomplete network. [2]

A new vertex, vertex **5**, is added to the original network. Its distances from vertices to which it is connected are shown in Fig. 3.3.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 5 | – | 3 | – | 1 |

**Fig. 3.3**

**(iii)** Draw the extended network and the complete 5-node network of shortest distances. (You are not required to use an algorithm to find the shortest distances.) [3]

**(iv)** Produce the shortest distance matrix and the route matrix for the extended 5-node network. [3]

**(v)** Apply the nearest neighbour algorithm to your $5 \times 5$ distance matrix, starting at vertex **1**. Give the length of the cycle produced, together with the actual cycle in the original 5-node network. [3]

**(vi)** By deleting vertex **1** and its arcs, and by using Prim's algorithm on the reduced distance matrix, produce a lower bound for the solution to the practical travelling salesperson problem in the original 5-node network. Show clearly your use of the matrix form of Prim's algorithm. [4]

**(vii)** In the original 5-node network find a shortest route starting at vertex **1** and using each of the 6 arcs at least once. Give the length of your route. [3]

PMT



4 Kassi and Theo are discussing how much oil and how much vinegar to use to dress their salad. They agree to use between 5 and 10ml of oil and between 3 and 6ml of vinegar and that the amount of oil should not exceed twice the amount of vinegar.

Theo prefers to have more oil than vinegar. He formulates the following problem to maximise the proportion of oil:

$$\begin{array}{ll} \text{Maximise} & \dfrac{x}{x+y} \\ \text{subject to} & 0 \leqslant x \leqslant 10, \\ & 0 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$

**(i)** Explain why this problem is not an LP. [1]

**(ii)** Use the simplex method to solve the following LP.

$$\begin{array}{ll} \text{Maximise} & x - y \\ \text{subject to} & 0 \leqslant x \leqslant 10, \\ & 0 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$

[7]

**(iii)** Kassi prefers to have more vinegar than oil. She formulates the following LP.

$$\begin{array}{ll} \text{Maximise} & y - x \\ \text{subject to} & 5 \leqslant x \leqslant 10, \\ & 3 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$

Draw separate graphs to show the feasible regions for this problem and for the problem in part **(ii)**. [5]

**(iv)** Explain why the formulation in part **(ii)** produced a solution for Theo's problem, and why it is more difficult to use the simplex method to solve Kassi's problem in part **(iii)**. [2]

**(v)** Produce an initial tableau for using the two-stage simplex method to solve Kassi's problem.

Explain briefly how to proceed. [5]