OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

Wednesday **25 MAY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Registered Charity 1066969
[Turn over

1 The switching circuit in Fig. 1.1 shows switches, s for a car's sidelights, h for its dipped headlights and f for its high-intensity rear foglights. It also shows the three sets of lights.

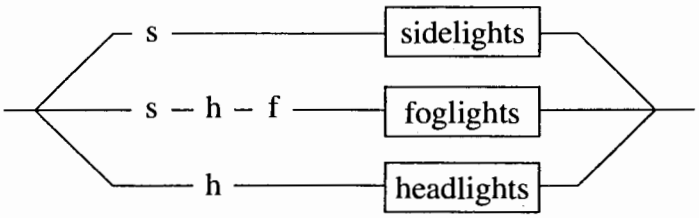


Fig. 1.1

(Note: s and h are each "ganged" switches. A ganged switch consists of two connected switches sharing a single switch control, so that both are either on or off together.)

(a) **(i)** Describe in words the conditions under which the foglights will come on. [2]

Fig. 1.2 shows a combinatorial circuit.

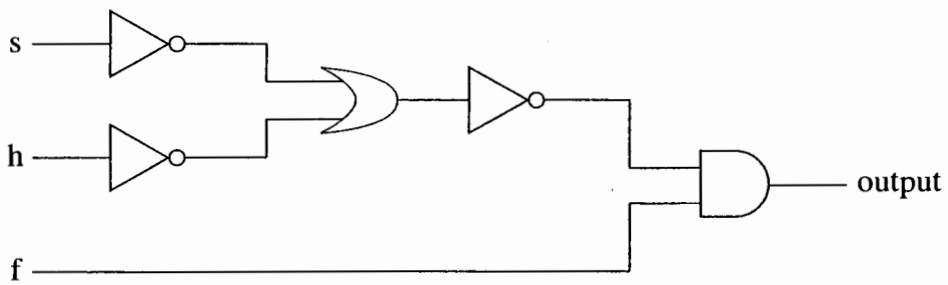


Fig. 1.2

(ii) Write the output in terms of a Boolean expression involving s, h and f. [2]

(iii) Use a truth table to prove that $s \wedge h \wedge f = \sim(\sim s \vee \sim h) \wedge f$. [3]

(b) A car's first gear can be engaged (g) if either both the road speed is low (r) and the clutch is depressed (d), or if both the road speed is low (r) and the engine speed is the correct multiple of the road speed (m).

(i) Draw a switching circuit to represent the conditions under which first gear can be engaged. Use two ganged switches to represent r, and single switches to represent each of d, m and g. [2]

(ii) Draw a combinatorial circuit to represent the Boolean expression $r \wedge (d \vee m) \wedge g$. [4]

(iii) Use Boolean algebra to prove that $r \wedge (d \vee m) \wedge g = ((r \wedge d) \vee (r \wedge m)) \wedge g$. [2]

(iv) Draw another switching circuit to represent the conditions under which first gear can be selected, but without using a ganged switch. [1]

2 Karl is considering investing in a villa in Greece. It will cost him 56 000 euros (€ 56 000). His alternative is to invest his money, £35 000, in the United Kingdom.

He is concerned with what will happen over the next 5 years. He estimates that there is a 60% chance that a house currently worth € 56000 will appreciate to be worth € 75 000 in that time, but that there is a 40% chance that it will be worth only € 55 000.

If he invests in the United Kingdom then there is a 50% chance that there will be 20% growth over the 5 years, and a 50% chance that there will be 10% growth.

(i) Given that £1 is worth € 1.60, draw a decision tree for Karl, and advise him what to do, using the EMV of his investment (in thousands of euros) as his criterion. [4]

In fact the £/€ exchange rate is not fixed. It is estimated that at the end of 5 years, if there has been 20% growth in the UK then there is a 70% chance that the exchange rate will stand at 1.70 euros per pound, and a 30% chance that it will be 1.50. If growth has been 10% then there is a 40% chance that the exchange rate will stand at 1.70 and a 60% chance that it will be 1.50.

(ii) Produce a revised decision tree incorporating this information, and give appropriate advice. [3]

A financial analyst asks Karl a number of questions to determine his utility function. He estimates that for x in cash (in thousands of euros) Karl's utility is $x^{0.8}$, and that for y in property (in thousands of euros), Karl's utility is $y^{0.75}$.

(iii) Repeat your computations from part **(ii)** using utility instead of the EMV of his investment. Does this change your advice? [3]

(iv) Using EMVs, find the exchange rate (number of euros per pound) which will make Karl indifferent between investing in the UK and investing in a villa in Greece. [2]

(v) Show that, using Karl's utility function, the exchange rate would have to drop to 1.277 euros per pound to make Karl indifferent between investing in the UK and investing in a villa in Greece. [4]

[Turn over

3 The distance and route matrices shown in Fig. 3.1 are the result of applying Floyd's algorithm to the incomplete network on 4 vertices shown in Fig. 3.2.

Distance Matrix

	1	2	3	4
1	4	2	3	9
2	2	2	1	7
3	3	1	2	6
4	9	7	6	12

Route Matrix

	1	2	3	4
1	2	2	2	2
2	1	3	3	3
3	2	2	2	4
4	3	3	3	3

Fig. 3.1

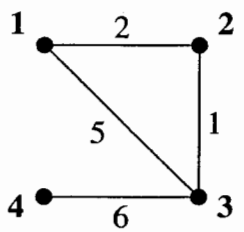


Fig. 3.2

(i) Draw the complete network of shortest distances. [2]

(ii) Explain how to use the route matrix to find the shortest route from vertex 4 to vertex 1 in the original incomplete network. [2]

A new vertex, vertex **5**, is added to the original network. Its distances from vertices to which it is connected are shown in Fig. 3.3.

	1	2	3	4
5	–	3	–	1

Fig. 3.3

(iii) Draw the extended network and the complete 5-node network of shortest distances. (You are not required to use an algorithm to find the shortest distances.) [3]

(iv) Produce the shortest distance matrix and the route matrix for the extended 5-node network. [3]

(v) Apply the nearest neighbour algorithm to your 5×5 distance matrix, starting at vertex **1**. Give the length of the cycle produced, together with the actual cycle in the original 5-node network. [3]

(vi) By deleting vertex **1** and its arcs, and by using Prim's algorithm on the reduced distance matrix, produce a lower bound for the solution to the practical travelling salesperson problem in the original 5-node network. Show clearly your use of the matrix form of Prim's algorithm. [4]

(vii) In the original 5-node network find a shortest route starting at vertex **1** and using each of the 6 arcs at least once. Give the length of your route. [3]

4 Kassi and Theo are discussing how much oil and how much vinegar to use to dress their salad. They agree to use between 5 and 10ml of oil and between 3 and 6ml of vinegar and that the amount of oil should not exceed twice the amount of vinegar.

Theo prefers to have more oil than vinegar. He formulates the following problem to maximise the proportion of oil:

$$\begin{array}{ll} \text{Maximise} & \dfrac{x}{x+y} \\ \text{subject to} & 0 \leqslant x \leqslant 10, \\ & 0 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$

(i) Explain why this problem is not an LP. [1]

(ii) Use the simplex method to solve the following LP.

$$\begin{array}{ll} \text{Maximise} & x - y \\ \text{subject to} & 0 \leqslant x \leqslant 10, \\ & 0 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$ [7]

(iii) Kassi prefers to have more vinegar than oil. She formulates the following LP.

$$\begin{array}{ll} \text{Maximise} & y - x \\ \text{subject to} & 5 \leqslant x \leqslant 10, \\ & 3 \leqslant y \leqslant 6, \\ & x - 2y \leqslant 0. \end{array}$$

Draw separate graphs to show the feasible regions for this problem and for the problem in part **(ii)**. [5]

(iv) Explain why the formulation in part **(ii)** produced a solution for Theo's problem, and why it is more difficult to use the simplex method to solve Kassi's problem in part **(iii)**. [2]

(v) Produce an initial tableau for using the two-stage simplex method to solve Kassi's problem.

Explain briefly how to proceed. [5]

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Question **2**.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages and an insert.

Registered Charity 1066969
[Turn over

1 **(i)** Use a truth table to prove $\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S)$. [8]

(ii) Prove that $(A \Rightarrow B) \Leftrightarrow (\sim A \vee B)$ and hence use Boolean algebra to prove that

$$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S).$$ [5]

(iii) A teacher wrote on a report "It is not the case that if Joanna doesn't try then she won't succeed." He meant to say that if Joanna were to try then she would have a chance of success. By letting T be "Joanna will try" and S be "Joanna will succeed", find the real meaning of what the teacher wrote. [3]

2 **Answer this question on the insert provided.**

Fig. 2 shows a network in which the weights on the arcs represent distances.

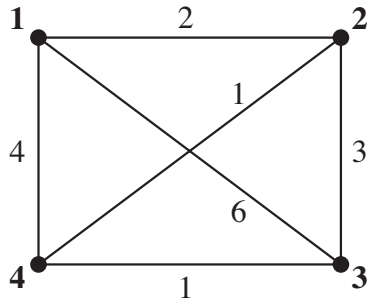


Fig. 2

(i) Apply Floyd's algorithm on the insert provided to find the complete network of shortest distances. [8]

(ii) Show how to use your final matrices to find the shortest route from vertex **1** to vertex **3**, together with the length of that route. [4]

(iii) Use the nearest neighbour algorithm, starting at vertex **1**, to find a Hamilton cycle in the complete network of shortest distances.

Give the corresponding cycle in the original network, together with its length. [4]

3 Emma has won a holiday worth £1000. She is wondering whether or not to take out an insurance policy which will pay out £1000 if she should fall ill and be unable to go on the holiday. The insurance company tells her that this happens to 1 in 200 people. The insurance policy costs £10. Thus Emma's monetary value if she buys the insurance and does not fall ill is £990.

(i) Draw a decision tree for Emma's problem. Use the EMV criterion in your calculations. [6]

(ii) Interpret your tree and say what the maximum cost of the insurance would have to be for Emma to consider buying it if she uses the EMV criterion. [2]

Suppose that Emma's utility function is given by $utility = \sqrt[3]{monetary\ value}$.

(iii) Using expected utility as the criterion, should Emma purchase the insurance?

Under this criterion what is the cost at which she will be indifferent to buying or not buying it? [3]

Emma could pay for a blood pressure check to help her to make her decision. Statistics show that 75% of checks are positive, and that when a check is positive the chance of missing a holiday through ill heath is 0.001. However, when a check is negative the chance of cancellation through ill health is 0.017.

(iv) Draw a decision tree to help Emma decide whether or not to pay for the check. Use EMV, not expected utility, in your calculations and assume that the insurance policy costs £10.

What is the maximum amount that she should pay for the blood pressure check? [9]

[Question 4 is printed overleaf.]

4 The "Cuddly Friends Company" produces soft toys. For one day's production run it has available 11 m^2 of furry material, 24 m^2 of woolly material and 30 glass eyes. It has three soft toys which it can produce:

The "Cuddly Aardvark", each of which requires 0.5 m^2 of furry material, 2 m^2 of woolly material and two eyes. Each sells at a profit of £3.

The "Cuddly Bear", each of which requires 1 m^2 of furry material, 1.5 m^2 of woolly material and two eyes. Each sells at a profit of £5.

The "Cuddly Cat", each of which requires 1 m^2 of furry material, 1 m^2 of woolly material and two eyes. Each sells at a profit of £2.

An analyst formulates the following LP to find the production plan which maximises profit.

$$\begin{aligned} \text{Maximise} \quad & 3a + 5b + 2c \\ \text{subject to} \quad & 0.5a + b + c \leqslant 11, \\ & 2a + 1.5b + c \leqslant 24, \\ & 2a + 2b + 2c \leqslant 30. \end{aligned}$$

(i) Explain how this formulation models the problem, and say why the analyst has not simplified the last inequality to $a + b + c \leqslant 15$. [4]

(ii) The final constraint is different from the others in that the resource is integer valued. Explain why that does not impose an additional difficulty for this problem. [1]

(iii) Solve this problem using the simplex algorithm.

Interpret your solution and say what resources are left over. [9]

On a particular day an order is received for two Cuddly Cats, and the extra constraint $c \geqslant 2$ is added to the formulation.

(iv) Set up an initial simplex tableau to deal with the modified problem using either the big-M approach or two-phase simplex. Do not perform any iterations on your tableau. [3]

(v) Show that the solution given by $a = 8$, $b = 2$ and $c = 5$ uses all of the resources, but that $a = 6$, $b = 6$ and $c = 2$ gives more profit.

What resources are left over from the latter solution? [3]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4772/01**

MATHEMATICS (MEI)

Decision Mathematics 2

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

[Turn over

1 **(a)** A joke has it that army recruits used to be instructed: "If it moves, salute it. If it doesn't move, paint it."

Assume that this instruction has been carried out completely in the local universe, so that everything that doesn't move has been painted.

(i) A recruit encounters something which is not painted. What should he do, and why? [3]

(ii) A recruit encounters something which is painted. Do we know what he or she should do? Justify your answer. [3]

(b) Use a truth table to prove $(((m \Rightarrow s) \wedge (\sim m \Rightarrow p)) \wedge \sim p) \Rightarrow s$. [6]

(c) You are given the following two rules.

1 $(a \Rightarrow b) \Leftrightarrow (\sim b \Rightarrow \sim a)$

2 $(x \wedge (x \Rightarrow y)) \Rightarrow y$

Use Boolean algebra to prove that $(((m \Rightarrow s) \wedge (\sim m \Rightarrow p)) \wedge \sim p) \Rightarrow s$. [4]

2 Bill is at a horse race meeting. He has £2 left with two races to go. He only ever bets £1 at a time. For each race he chooses a horse and then decides whether or not to bet on it. In both races Bill's horse is offered at "evens". This means that, if Bill bets £1 and the horse wins, then Bill will receive back his £1 plus £1 winnings. If Bill's horse does not win then Bill will lose his £1.

(i) Draw a decision tree to model this situation. Show Bill's payoffs on your tree, i.e. how much money Bill finishes with under each possible outcome. [8]

Assume that in each race the probability of Bill's horse winning is the same, and that it has value p.

(ii) Find Bill's EMV when

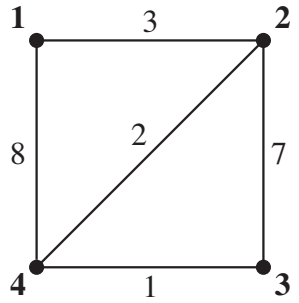
(*A*) $p = 0.6$,

(*B*) $p = 0.4$.

Give his best course of action in each case. [5]

(iii) Suppose that Bill uses the utility function $utility = (money)^x$, to decide whether or not to bet £1 on one race. Show that, with $p = 0.4$, Bill will not bet if $x = 0.5$, but will bet if $x = 1.5$. [3]

3 Floyd's algorithm is applied to the following network:



At the end of the third iteration of the algorithm the distance and route matrices are as follows:

	1	**2**	**3**	**4**
1	6	3	10	5
2	3	6	7	2
3	10	7	14	1
4	5	2	1	2

	1	**2**	**3**	**4**
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	3

(i) Perform the fourth (final) iteration of the algorithm. [7]

(ii) Explain how to use the final matrices to find the shortest distance and the shortest route from vertex **1** to vertex **3**, and give the distance and route. [4]

(iii) Draw the complete network of shortest distances. [1]

(iv) Apply the nearest neighbour algorithm, starting at vertex **1**, to your complete network of shortest distances. Give the Hamilton cycle it produces, its length, and the corresponding route through the original network. [3]

(v) By considering vertex **2** and its arcs, construct a lower bound for the length of the solution to the travelling salesperson problem in the original network. [3]

(vi) Explain what you can deduce from your answers to parts **(iv)** and **(v)**. [2]

[Question 4 is printed overleaf.]

4 Noel is designing a hotel patio. It will consist of decking and paving.

Decking costs £4 per m^2 and paving costs £2 per m^2. He has a budget of £2500.

Noel prefers paving to decking, and he wants the area given to paving to be at least twice that given to decking.

He wants to have as large a patio as possible.

Noel's problem is formulated as the following LP.

Let x be the number of m^2 of decking.

Let y be the number of m^2 of paving.

$$\begin{aligned} \text{Maximise} \quad & P = x + y \\ \text{subject to} \quad & 2x + y \leqslant 1250 \\ & 2x - y \leqslant 0 \\ & x \geqslant 0 \\ & y \geqslant 0 \end{aligned}$$

(i) Use the simplex algorithm to solve this LP. Pivot first on the positive element in the y column. [6]

Noel would like to have at least 200 m^2 of decking.

(ii) Add a line corresponding to this constraint to your solution tableau from part **(i)**, and modify the resulting table either for two-stage simplex or the big-M method. Hence solve the problem. [9]

Noel finally decides that he will minimise the annual cost of maintenance, which is given by £$(0.75x + 1.25y)$, subject to the additional constraint that there is at least 1000 m^2 of patio.

(iii) Starting from your solution to part **(ii)**, use simplex to solve this problem. [5]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4772/01**

MATHEMATICS (MEI)

Decision Mathematics 2

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

  [Turn over

1 **(a)** The Plain English Society presents an annual "Foot in Mouth" award for "a truly baffling comment". In 2004 it was presented to Boris Johnson MP for a comment on the 12th December 2003 edition of "Have I Got News For You":

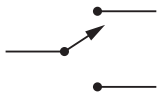
"I could not fail to disagree with you less."

(i) Explain why this can be rewritten as:

"I could succeed in agreeing with you more." [2]

(ii) Rewrite the comment more simply in your own words without changing its meaning. [2]

(b) Two switches are to be wired between a mains electricity supply and a light so that when the state of either switch is changed the state of the light changes (i.e. from off to on, or from on to off). Draw a switching circuit to achieve this.

The switches are both 2-way switches, thus:  [5]

(c) Construct a truth table to show the following.

$[(a \wedge b) \vee ((\sim a) \wedge (\sim b))] \Leftrightarrow [((\sim a) \vee b) \wedge (a \vee (\sim b))]$ [7]

2 Jane has a house on a Mediterranean island. She spends eight weeks a year there, either visiting twice for four weeks each trip or four times for two weeks each trip. Jane is wondering whether it is best for her to fly out and rent a car, or to drive out.
Flights cost £500 return and car rental costs £150 per week.
Driving out costs £900 for ferries, road tolls, fuel and overnight expenses.

(i) Draw a decision tree to model this situation. Advise Jane on the cheapest option. [6]

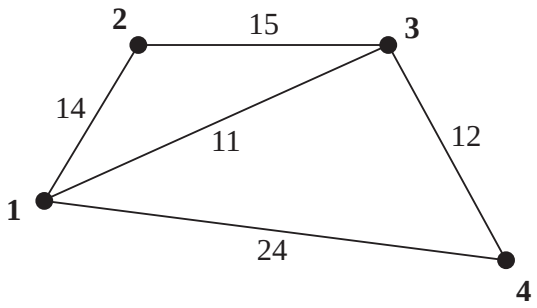
As an alternative Jane considers buying a car to keep at the house. This is a long-term alternative, and she decides to cost it over 10 years. She has to cost the purchase of the car and her flights, and compare this with the other two options.

In her costing exercise she decides that she will not be tied to two trips per year nor to four trips per year, but to model this as a random process in which she is equally likely to do either.

(ii) Draw a decision tree to model this situation. Advise Jane on how much she could spend on a car using the EMV criterion. [8]

(iii) Explain what is meant by "the EMV criterion" and state an alternative approach. [2]

3 The weights on the network represent distances.



(a) (i) Apply Floyd's algorithm to the network to find the complete network of shortest distances, showing that the final matrices are as follows.

	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

Draw the complete network of shortest distances. [8]

(ii) Starting at vertex **1**, apply the nearest neighbour algorithm to the complete network of shortest distances to find a Hamilton cycle. Give the length of your cycle and interpret it in the original network. [4]

(iii) By temporarily deleting vertex **1** and its connecting arcs from the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson's Problem in that network. Say what this implies in the original network. [4]

(b) Solve the route inspection problem in the **original** network, and say how you can be sure that your solution is optimal. [4]

[Question 4 is printed overleaf]

4 A factory's output includes three products. To manufacture a tonne of product A, 3 tonnes of water are needed. Product B needs 2 tonnes of water per tonne produced, and product C needs 5 tonnes of water per tonne produced.

Product A uses 5 hours of labour per tonne produced, product B uses 6 hours and product C uses 2 hours.

There are 60 tonnes of water and 50 hours of labour available for tomorrow's production.

(i) Formulate a linear programming problem to maximise production within the given constraints. [5]

(ii) Use the simplex algorithm to solve your LP, **pivoting first on your column relating to product C**. [7]

(iii) An extra constraint is imposed by a contract to supply at least 8 tonnes of A. Use either two stage simplex or the big M method to solve this revised problem. [8]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Decision Mathematics 2

4772

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Answer Booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes

OCE/T67864

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** The following was said in a charity appeal on Radio 4 in October 2006.

"It is hard to underestimate the effect that your contribution will make."

Rewrite the comment more simply in your own words without changing its meaning. **[2]**

(b) A machine has three components, A, B and C, each of which is either active or inactive.

- The machine is active if A and B are both active.
- The machine is active if A is inactive and C is active.
- The machine is active if B is inactive and C is active.
- Otherwise the machine is inactive.

The states (active or inactive) of the components and the machine are to be modelled by a combinatorial circuit in which "active" is represented by "true" and "inactive" is represented by "false".

Draw such a circuit. **[7]**

(c) Construct a truth table to show the following.

$$\left[\left(\left(a \wedge b\right) \vee \left(\left(\sim a\right) \wedge c\right)\right) \vee \left(\left(\sim b\right) \wedge c\right)\right] \Leftrightarrow \sim \left[\left(\left(\sim a\right) \wedge \left(\sim c\right)\right) \vee \left(\left(\sim b\right) \wedge \left(\sim c\right)\right)\right] \quad \textbf{[7]}$$

2 Zoe is preparing for a Decision Maths test on two topics, Decision Analysis (D) and Simplex (S). She has to decide whether to devote her final revision session to D or to S.

There will be two questions in the test, one on D and one on S. One will be worth 60 marks and the other will be worth 40 marks. Historically there is a 50% chance of each possibility.

Zoe is better at D than at S. If her final revision session is on D then she would expect to score 80% of the D marks and 50% of the S marks. If her final session is on S then she would expect to score 70% of the S marks and 60% of the D marks.

(i) Compute Zoe's expected mark under each of the four possible circumstances, i.e. Zoe revising D and the D question being worth 60 marks, etc. **[5]**

(ii) Draw a decision tree for Zoe. **[5]**

Michael claims some expertise in forecasting which question will be worth 60 marks. When he forecasts that it will be the D question which is worth 60, then there is a 70% chance that the D question will be worth 60. Similarly, when he forecasts that it will be the S question which is worth 60, then there is a 70% chance that the S question will be worth 60. He is equally likely to forecast that the D or the S question will be worth 60.

(iii) Draw a decision tree to find the worth to Zoe of Michael's advice. **[6]**

3 A farmer has 40 acres of land. Four crops, A, B, C and D are available.
Crop A will return a profit of £50 per acre. Crop B will return a profit of £40 per acre.
Crop C will return a profit of £40 per acre. Crop D will return a profit of £30 per acre.
The total number of acres used for crops A and B must not be greater than the total number used for crops C and D.

The farmer formulates this problem as:

Maximise $50a + 40b + 40c + 30d$,

subject to $a + b \leqslant 20$,
$a + b + c + d \leqslant 40$.

(i) Explain what the variables a, b, c and d represent.
Explain how the first inequality was obtained.
Explain why expressing the constraint on the total area of land as an inequality will lead to a solution in which all of the land is used. **[3]**

(ii) Solve the problem using the simplex algorithm. **[10]**

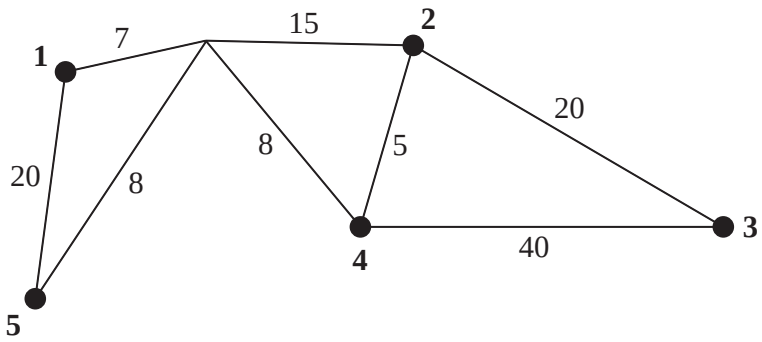
Suppose now that the farmer had formulated the problem as:

Maximise $50a + 40b + 40c + 30d$,

subject to $a + b \leqslant 20$,
$a + b + c + d = 40$.

(iii) Show how to adapt this problem for solution either by the two-stage simplex method or the big-M method. In either case you should show the initial tableau and describe what has to be done next. You should not attempt to solve the problem. **[7]**

4 The diagram shows routes connecting five cities. Lengths are in km.



(i) Produce the initial matrices for an application of Floyd's algorithm to find the complete network of shortest distances between the five cities. **[4]**

The following are the distance and route matrices after the third iteration of Floyd's algorithm.

	1	**2**	**3**	**4**	**5**
1	44	22	42	15	15
2	22	44	20	5	23
3	42	20	40	25	43
4	15	5	25	10	16
5	15	23	43	16	30

	1	**2**	**3**	**4**	**5**
1	2	2	2	4	5
2	1	1	3	4	5
3	2	2	2	2	2
4	1	2	2	2	5
5	1	2	2	4	1

(ii) Perform the fourth iteration. **[4]**

There are no changes on the fifth iteration, so your answer to part **(ii)** should give the complete network of shortest distances.

(iii) Use your matrices to find the shortest distance and route from vertex **3** to vertex **1**, and explain how you do it. **[5]**

(iv) Draw the complete network of shortest distances, not including the loops. **[2]**

(v) Apply the nearest neighbour algorithm to your network in part **(iv)**, starting at vertex **2**. Give the length of the Hamilton cycle that is produced.

Interpret the Hamilton cycle in terms of the original diagram and state what the algorithm has achieved. **[5]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Decision Mathematics 2

4772

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

OCE/23399

4772

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Mickey ate the last of the cheese. Minnie was put out at this. Mickey's defence was "There wasn't enough left not to eat it all".

Let "c" represent "there is enough cheese for two" and "e" represent "one person can eat all of the cheese".

(i) Which of the following best captures Mickey's argument?

$c \Rightarrow e$ $c \Rightarrow \sim e$ $\sim c \Rightarrow e$ $\sim c \Rightarrow \sim e$ **[1]**

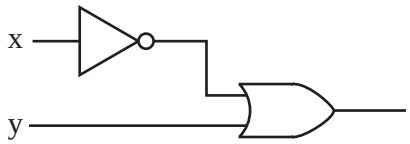
In the ensuing argument Minnie concedes that if there's a lot left then one should not eat it all, but argues that this is not an excuse for Mickey having eaten it all when there was not a lot left.

(ii) Prove that Minnie is right by writing down a line of a truth table which shows that

$(c \Rightarrow \sim e) \Leftrightarrow (\sim c \Rightarrow e)$

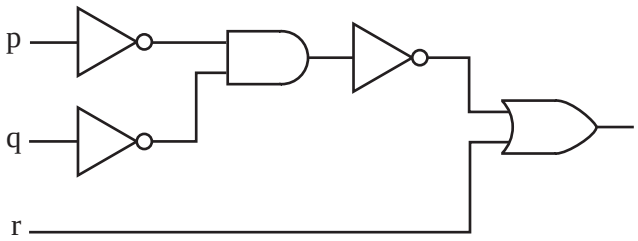
is false.
(You may produce the whole table if you wish, but you need to indicate a specific line of the table.) **[4]**

(b) **(i)** Show that the following combinatorial circuit is modelling an implication statement. Say what that statement is, and prove that the circuit and the statement are equivalent.



[5]

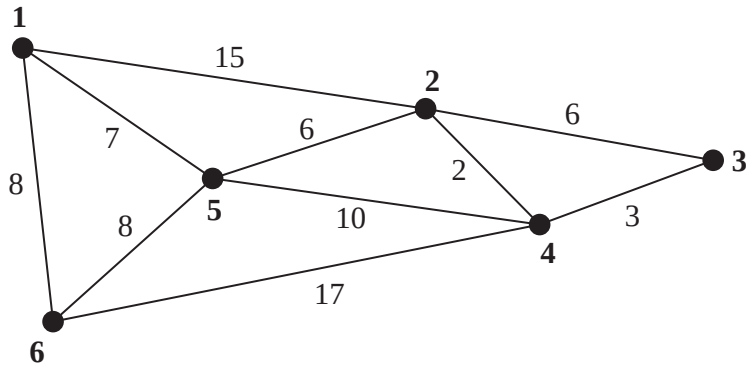
(ii) Express the following combinatorial circuit as an equivalent implication statement.



[2]

(iii) Explain why $(\sim p \wedge \sim q) \Rightarrow r$, together with $\sim r$ and $\sim q$, give p. **[4]**

2 The network is a representation of a show garden. The weights on the arcs give the **times** in minutes to walk between the six features represented by the vertices, where paths exist.



(i) Why might it be that the time taken to walk from vertex **2** to vertex **3** via vertex **4** is less than the time taken by the direct route, i.e. the route from **2** to **3** which does not pass through any other vertices? **[1]**

The matrices shown below are the results of the first iteration of Floyd's algorithm when applied to the network.

	1	**2**	**3**	**4**	**5**	**6**
1	∞	15	∞	∞	7	8
2	15	30	6	2	6	23
3	∞	6	∞	3	∞	∞
4	∞	2	3	∞	10	17
5	7	6	∞	10	14	8
6	8	23	∞	17	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	1	1	3	4	5	1
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	1	6
6	1	1	3	4	5	1

(ii) Complete the second iteration of Floyd's algorithm. **[4]**

5

The matrices below are the final matrices resulting from Floyd's algorithm.

	1	**2**	**3**	**4**	**5**	**6**
1	14	13	18	15	7	8
2	13	4	5	2	6	14
3	18	5	6	3	11	19
4	15	2	3	4	8	16
5	7	6	11	8	12	8
6	8	14	19	16	8	16

	1	**2**	**3**	**4**	**5**	**6**
1	5	5	5	5	5	6
2	5	4	4	4	5	5
3	4	4	4	4	4	4
4	2	2	3	2	2	2
5	1	2	2	2	2	6
6	1	5	5	5	5	1

(iii) Explain what the algorithm has achieved.

Show how to find the shortest time and the quickest route from vertex **3** to vertex **6**.

Give the shortest time and the quickest route from vertex **3** to vertex **6**. **[5]**

A visitor to the garden wishes to visit all six features, starting from the feature represented by vertex **1**.

(iv) Use the final matrices to find an upper bound for the minimum time for which the visitor must walk, and give a route through the garden corresponding to this. **[2]**

(v) By deleting vertex **1** and its arcs construct a lower bound for the time for which the visitor must walk. You may construct a minimum connector for the reduced network without using an algorithm. **[3]**

(vi) Given that the sum of the times taken to walk the paths is 82 minutes, find the minimum time that could be taken by a member of staff to start at vertex **1**, walk along every path, and return to vertex **1**. **[1]**

3 It is Ken's 59th birthday, and he is considering whether or not to retire early. He can retire and take his pension now, when he reaches 60, or when he reaches 65.

Ken's pension is computed by taking the number of years for which he has worked, multiplying by his final salary, and dividing by 80. He has currently worked for 35 years. He is at the top of his grade and is earning £50 000 per annum. (Ignore any changes which might occur due to inflation or pay rises.)

Ken estimates that, at age 59, he has a 0.6 probability of getting a part-time (50%) contract on his current grade; at age 60 the probability will be 0.5; at age 65 the probability will be 0.25. (His part-time earnings will not affect his pension.)

Ken intends to retire completely when he reaches 70.

(i) Draw up a decision tree showing Ken's options. **[4]**

(ii) Find the EMV of Ken's gross income (before tax and other stoppages) from age 59 to age 70 in each scenario, and indicate the course of action which will maximise his EMV. **[9]**

Ken values his time and decides to apply a utility function to his gross incomes to reflect this. In each 11-year scenario he computes his utility as (gross income $\times 3^{-p}$) where p is the proportion of working time for which he is working. Thus, in the scenario in which he retires at 65 and succeeds in securing a part-time contract thereafter, $p = \dfrac{6 + 2.5}{11} = \dfrac{17}{22}$.

(iii) Find Ken's expected utilities and indicate the course of action which will maximise his expected utility. **[7]**

4 A craft workshop produces three products, xylophones, yodellers and zithers. The times taken to make them and the total time available per week are shown in the table. Also shown are the costs and the total weekly capital available.

	xylophones	yodellers	zithers	resource availability
time (hours)	2	5	3	30
cost (£00s)	4	1	2	24

Profits are £180 per xylophone, £90 per yodeller and £110 per zither.

(i) Formulate a linear programming problem to find the weekly production plan which maximises profit within the resource constraints. **[3]**

(ii) Use the simplex algorithm to solve the problem, pivoting first on the column of your tableau containing the variable which represents the number of xylophones produced. Explain how your final tableau shows that the workshop should produce 5 xylophones and 4 yodellers. **[8]**

If, when applying the simplex algorithm, the first pivot is on the column containing the variable which represents the number of zithers produced, then the final solution produced is for the workshop to produce 1.5 xylophones and 9 zithers per week.

(iii) How can this production plan be implemented? **[1]**

(iv) Explain how the simplex algorithm can lead to different solutions. **[2]**

To satisfy demand an extra constraint has to be added to the problem – that the number of xylophones plus the number of yodellers produced per week must total exactly 7.

(v) Show how to adapt this problem for solution either by the two-stage simplex method or the big-M method. In either case you should show the initial tableau and describe what has to be done next. You are not required to apply your method. **[4]**

(vi) The simplex solution to the revised problem is for the workshop to produce $4\frac{1}{15}$ xylophones, $2\frac{14}{15}$ yodellers and $2\frac{2}{5}$ zithers. Ignoring the practicalities explain how this solution relates to the two solutions referred to in part **(iv)**. **[2]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

4772

Decision Mathematics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4772
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 23 June 2011
Morning

Duration: 1 hour 30 minutes

OCE/18045

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

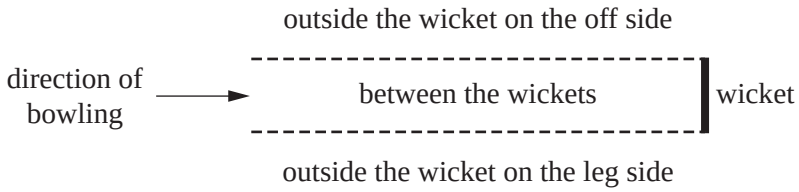
- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 **(a)** Heard in Parliament: “Will the minister not now discontinue her proposal to ban the protest?”

The minister replied “Yes I will.”

To what had the minister committed herself logically, and why might that not have been her intention? **[4]**

(b) In a cricket tournament an umpire might be required to decide whether or not a batsman is out ‘lbw’, ie ‘leg before wicket’. The lbw law for the tournament refers to parts of the cricket pitch as shown in the diagram (assuming a right-handed batsman):



The umpire has to make a number of judgements:

A Would the ball have hit the wicket?

B Did the ball hit the batsman, or part of his equipment other than the bat, without hitting the bat?

C Did the ball hit the batsman, or part of his equipment other than the bat, before hitting the bat?

D Was the part of the batsman or his equipment which was hit by the ball, between the wickets when it was hit?

E Was the part of the batsman or his equipment which was hit by the ball, outside of the wicket on the off side when it was hit?

F Was the batsman attempting to play a stroke?

The law can be interpreted as saying that the batsman is out lbw if $[(A \wedge B) \vee (A \wedge C)] \wedge [D \vee (E \wedge \sim F)]$.

The tournament’s umpiring manual, in attempting to simplify the law, states that the batsman is out lbw if $A \wedge (B \vee C) \wedge (D \vee E) \wedge (D \vee \sim F)$.

For an lbw decision this requires 4 conditions each to be true.

(i) Use the rules of Boolean algebra to show that the manual’s rule is logically equivalent to the law as stated above, naming the rules used at each step. **[7]**

A trainee umpire, using the manual, considers each condition in turn and judges that the following are true: A; B; E; D.

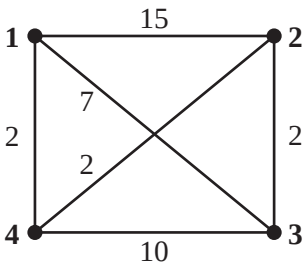
(ii) What is her decision and why? **[2]**

(iii) What is odd about her judgement, and does this make the logic invalid? **[3]**

2 A government has just created a new ministry, the Ministry of Administrative Affairs. The ministry is to have four departments:

the Administration
the Bureaucracy
the Certification Service
the Duplication Section.

Each of these departments is to be established in a separate office on one of four existing sites. The diagram shows the direct journey times in minutes between these four sites.



(i) Use Floyd's algorithm to find the shortest journey times between the four office sites. **[8]**

(ii) Draw a network showing your shortest times. **[1]**

(iii) Use appropriate algorithms to find upper and lower bounds for the optimum solution to the Travelling Salesperson Problem in the original network, briefly explaining the steps taken. **[4]**

(iv) A van is to be organised to deliver bundles of paperwork between the departments. Why might the optimum solution to the TSP be useful in planning this, and why might it not be? **[2]**

(v) Journeys to locations **2** and **3**, from locations **1** and **4**, are subject to a congestion charge which is equivalent in costing terms to 15 minutes of journey time. What sort of network would be needed to model this? **[1]**

3 Magnus has been researching career possibilities. He has just completed his GCSEs, and could leave school and get a good job. He estimates, discounted at today's values and given a 49 year working life, that there is a 50% chance of such a job giving him lifetime earnings of £1.5m, a 30% chance of £1.75m, and a 20% chance of £2m.

Alternatively Magnus can stay on at school and take A levels. He estimates that, if he does so, there is a 75% chance that he will achieve good results. If he does not achieve good results then he will still be able to take the same job as earlier, but he will have lost two years of his lifetime earnings. This will give a 50% chance of lifetime earnings of £1.42m, a 30% chance of £1.67m and a 20% chance of £1.92m.

If Magnus achieves good A level results then he could take a better job, which should give him discounted lifetime earnings of £1.6m with 50% probability or £2m with 50% probability. Alternatively he could go to university. This would cost Magnus another 3 years of lifetime earnings and would not guarantee him a well-paid career, since graduates sometimes choose to follow less well-paid vocations. His research shows him that graduates can expect discounted lifetime earnings of £1m with 20% probability, £1.5m with 30% probability, £2m with 30% probability, and £3m with 20% probability.

(i) Draw up a decision tree showing Magnus's options. **[7]**

(ii) Using the EMV criterion, find Magnus's best course of action, and give its value. **[7]**

Magnus has read that money isn't everything, and that one way to reflect this is to use a utility function and then compare expected utilities. He decides to investigate the outcome of using a function in which utility is defined to be the square root of value.

(iii) Using the expected utility criterion, find Magnus's best course of action, and give its utility. **[4]**

(iv) The possibility of high earnings (£3m) swings Magnus's decision towards a university education. Find what value instead of £3m would make him indifferent to choosing a university education under the EMV criterion. (Do not change the probabilities.) **[2]**

4 A small alpine hotel is planned. Permission has been obtained for no more than 60 beds, and these can be accommodated in rooms containing one, two or four beds.

The total floor areas needed are $15\,m^2$ for a one-bed room, $25\,m^2$ for a two-bed room and $40\,m^2$ for a four-bed room. The total floor area of the bedrooms must not exceed $700\,m^2$.

Marginal profit contributions per annum, in thousands of euros, are estimated to be 5 for a one-bed room, 9 for a two-bed room and 15 for a four-bed room.

(i) Formulate a linear programming problem to find the mix of rooms which will maximise the profit contribution within the two constraints. **[3]**

(ii) Use the simplex algorithm to solve the problem, and interpret your solution. **[8]**

It is decided that, for marketing reasons, at least 5 one-bed rooms must be provided.

(iii) Solve this modified problem using either the two-stage simplex method or the big-M method. You may wish to adapt your final tableau from part **(ii)** to produce an initial tableau, but you are not required to do so. **[5]**

(iv) The simplex solution to the revised problem is to provide 5 one-bed rooms, 15 two-bed rooms and 6.25 four-bed rooms, giving a profit contribution of €253 750. Interpret this solution in terms of the real world problem. **[1]**

(v) Compare the following solution to your answer to part **(iv)**: 8 one-bed rooms, 12 two-bed rooms and 7 four-bed rooms. Explain your findings. **[3]**

OCR
RECOGNISING ACHIEVEMENT

Friday 22 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4772 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4772
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** When marking coursework, a teacher has to complete a form which includes the following:

In your opinion is this the original work of the pupil? (tick as appropriate)	
I have no reason to believe that it is not	☐
I cannot confirm that it is	☐

(i) The teacher suspects that a pupil has copied work from the internet. For each box, state whether the teacher should tick the box or not. **[2]**

(ii) The teacher has no suspicions about the work of another pupil, and has no information about how the work was produced. Which boxes should she tick? **[2]**

(iii) Explain why the teacher must always tick at least one box. **[2]**

(b) Angus, the ski instructor, says that the class will have to have lunch in Italy tomorrow if it is foggy or if the top ski lift is not working. On the next morning Chloe, one of Angus's students, says that it is not foggy, so they can have lunch in Switzerland.

Produce a line of a truth table which shows that Chloe's deduction is incorrect. You may produce a complete truth table if you wish, but you must indicate a row which shows that Chloe's deduction is incorrect. **[6]**

(c) You are given that the following two statements are true.

$$(X \vee \sim Y) \Rightarrow Z$$
$$\sim Z$$

Use Boolean algebra to show that Y is true. **[4]**

2 Adrian is considering selling his house and renting a flat.

Adrian still owes £150 000 on his house. He has a mortgage for this, for which he has to pay £4800 annual interest. If he sells he will pay off the £150 000 and invest the remainder of the proceeds at an interest rate of 2.5% per annum. He will use the interest to help to pay his rent.

His estate agent estimates that there is a 30% chance that the house will sell for £225 000, a 50% chance that it will sell for £250 000, and a 20% chance that it will sell for £275 000.

A flat will cost him £7500 per annum to rent.

(i) Draw a decision tree to help Adrian to decide whether to keep his house, or to sell it and rent a flat. Compare the EMVs of Adrian's annual outgoings, and ignore the costs of selling. **[6]**

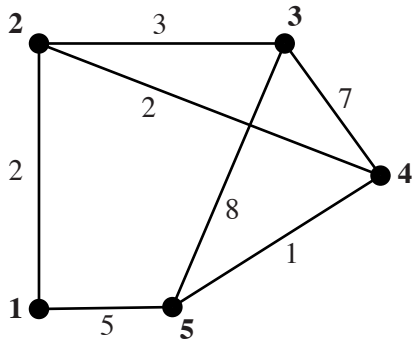
(ii) Would the analysis point to a different course of action if Adrian were to use a square root utility function, instead of EMVs? **[3]**

Adrian's circumstances change so that he has to decide now whether to sell or not in one year's time. Economic conditions might then be less favourable for the housing market, the same, or more favourable, these occurring with probabilities 0.3, 0.3 and 0.4 respectively. The possible selling prices and their probabilities are shown in the table.

Economic conditions and probabilities		Selling prices (£) and probabilities					
less favourable	0.3	200 000	0.2	225 000	0.3	250 000	0.5
unchanged	0.3	225 000	0.3	250 000	0.5	275 000	0.2
more favourable	0.4	250 000	0.3	300 000	0.5	350 000	0.2

(iii) Draw a decision tree to help Adrian to decide what to do. Compare the EMVs of Adrian's annual outgoings. Assume that he will still owe £150 000 in one year's time, and that the cost of renting and interest rates do not change. **[7]**

3 The weights on the network represent distances.



(i) The answer book shows the initial tables and the results of iterations 1, 2, 3 and 5 when Floyd's algorithm is applied to the network.

(*A*) Complete the two tables for iteration 4. **[6]**

(*B*) Use the final route table to give the shortest route from vertex **3** to vertex **5**. **[1]**

(*C*) Use the final distance table to produce a complete network with weights representing the shortest distances between vertices. **[2]**

(ii) Using the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson Problem by deleting vertex **5** and its arcs, and by finding the length of a minimum connector for the remainder. (You may find the minimum connector by inspection.) **[3]**

(iii) Use the nearest neighbour algorithm, starting at vertex **1**, to produce a Hamilton cycle in the complete network. Give the length of your cycle. **[3]**

(iv) Interpret your Hamilton cycle in part **(iii)** in terms of the original network. **[2]**

(v) Give a walk of minimum length which traverses every arc on the original network at least once, and which returns to the start. Give the length of your walk. **[3]**

4 A publisher is considering producing three books over the next week: a mathematics book, a novel and a biography. The mathematics book will sell at £10 and costs £4 to produce. The novel will sell at £5 and costs £2 to produce. The biography will sell at £12 and costs £5 to produce. The publisher wants to maximise profit, and is confident that all books will be sold.

There are constraints on production. Each copy of the mathematics book needs 2 minutes of printing time, 1 minute of packing time, and 300 cm^3 of temporary storage space.

Each copy of the novel needs 1.5 minutes of printing time, 0.5 minutes of packing time, and 200 cm^3 of temporary storage space.

Each copy of the biography needs 2.5 minutes of printing time, 1.5 minutes of packing time, and 400 cm^3 of temporary storage space.

There are 10 000 minutes of printing time available on several printing presses, 7500 minutes of packing time, and 2 m^3 of temporary storage space.

(i) Explain how the following initial feasible tableau models this problem. **[5]**

P	x	y	z	$s1$	$s2$	$s3$	RHS
1	–6	–3	–7	0	0	0	0
0	2	1.5	2.5	1	0	0	10 000
0	1	0.5	1.5	0	1	0	7500
0	300	200	400	0	0	1	2 000 000

(ii) Use the simplex algorithm to solve your LP, and interpret your solution. **[8]**

(iii) The optimal solution involves producing just one of the three books. By how much would the price of each of the other books have to be increased to make them worth producing? **[2]**

There is a marketing requirement to provide at least 1000 copies of the novel.

(iv) Show how to incorporate this constraint into the initial tableau ready for an application of the two-stage simplex method.

Briefly describe how to use the modified tableau to solve the problem. You are NOT required to perform the iterations. **[5]**

BLANK PAGE

BLANK PAGE

OCR
RECOGNISING ACHIEVEMENT

Friday 21 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4772/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4772/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** A graph is simple if it contains neither loops nor multiple arcs, ie none of the following:  or  …

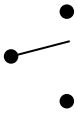
In an examination question, students were asked to describe in words when a graph is simple. Mark the following responses as right or wrong, giving reasons for your decisions if you mark them wrong.

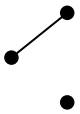
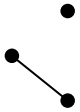
(i) A graph is simple if there are no loops and if two nodes are connected by a single arc.

(ii) A graph is simple if there are no loops and no two nodes are connected by more than one arc.

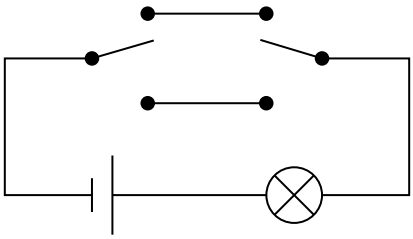
(iii) A graph is simple if there are no loops and two arcs do not have the same ends.

(iv) A graph is simple if there are no loops and there is at most one route from one node to another. **[7]**

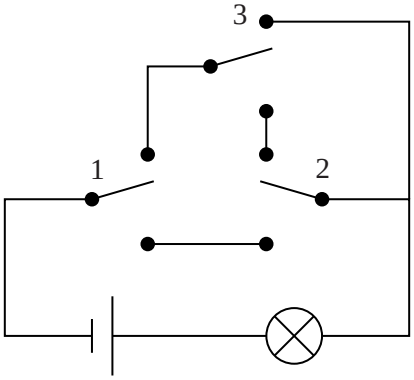
(b) The following picture represents a two-way switch  .

It can either be in the up state  or in the down state  .

Two switches can be used to construct a circuit in which changing the state of either switch changes the state of a lamp.



Georgios tries to connect together three two-way switches so that changing the state of any switch changes the state of the lamp. His circuit is shown below. The switches have been labelled 1, 2 and 3.



(i) List the possible combination of switch states and determine whether the lamp is on or off for each of them.

(ii) Say whether or not Georgios has achieved his objective, justifying your answer. **[5]**

(c) Use a truth table to show that $(A \wedge (B \vee C)) \vee \sim(\sim A \vee (B \wedge C)) \Leftrightarrow A$. **[4]**

2 Graham skis each year in an Italian resort which shares a ski area with a Swiss resort. He can buy an Italian lift pass, or an international lift pass which gives him access to Switzerland as well as to Italy. For his 6-day holiday the Italian pass costs €200 and the international pass costs €250. If he buys an Italian pass then he can still visit Switzerland by purchasing day supplements at €30 per day.

If the weather is good during his holiday, then Graham visits Switzerland three times. If the weather is moderate he goes twice. If poor he goes once. If the weather is windy then the lifts are closed, and he is not able to go at all.

In his years of skiing at the resort he has had good weather on 30% of his visits, moderate weather on 40%, poor weather on 20% and windy weather on 10% of his visits.

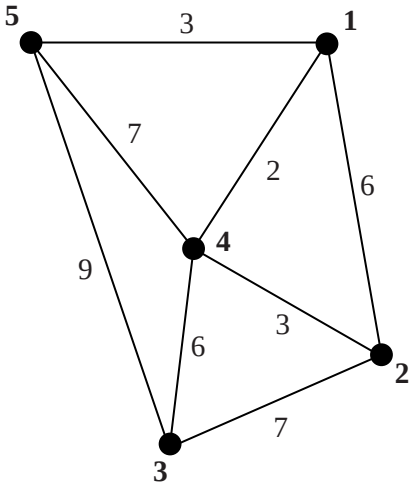
(i) Draw a decision tree to help Graham decide whether to buy an Italian lift pass or an international lift pass. Give the action he should take to minimize the EMV of his costs. **[6]**

When he arrives at the resort, and before he buys his lift pass, he finds that he has internet access to a local weather forecast, and to records of the past performance of the forecast. The 6-day forecast is limited to "good"/"not good", and the records show the actual weather proportions following those forecasts. It also shows that 60% of historical forecasts have been "good" and 40% "not good".

Actual / Forecast	good	moderate	poor	windy	proportion of forecasts
good	0.4	0.5	0.1	0.0	0.6
not good	0.15	0.25	0.35	0.25	0.4

(ii) Draw a decision tree to help Graham decide the worth of consulting the forecast before buying his lift pass. Give the actions he should take to minimize the EMV of his costs. **[10]**

3 Five towns, **1**, **2**, **3**, **4** and **5**, are connected by direct routes as shown. The arc weights represent distances.



(i) The printed answer book shows the initial tables and the results of iterations 1, 2, 3 and 5 when Floyd's algorithm is applied to the network.

(*A*) Complete the two tables for iteration 4. **[6]**

(*B*) Use the final route table to give the shortest route from vertex **5** to vertex **2**. **[1]**

(*C*) Use the final distance table to produce a complete network with weights representing the shortest distances between vertices. **[2]**

(ii) Use the nearest neighbour algorithm, starting at vertex **4**, to produce a Hamilton cycle in the complete network. Give the length of your cycle. **[3]**

(iii) Interpret your Hamilton cycle from part **(ii)** in terms of towns actually visited. **[1]**

(iv) Find an improved Hamilton cycle by applying the nearest neighbour algorithm starting from one of the other vertices. **[1]**

(v) Using the complete network of shortest distances (excluding loops), find a lower bound for the solution to the Travelling Salesperson Problem by deleting vertex **4** and its arcs, and by finding the length of a minimum connector for the remainder. (You may find the minimum connector by inspection.) **[3]**

(vi) Given that the sum of the road lengths in the original network is 43, give a walk of minimum length which traverses every arc on the original network at least once, and which returns to the start. Show your methodology. Give the length of your walk. **[3]**

4 Colin has a hobby from which he makes a small income. He makes bowls, candle holders and key fobs.

The materials he uses include wood, metal parts, polish and sandpaper. They cost, on average, £15 per bowl, £6 per candle holder and £2 per key fob. Colin has a monthly budget of £100 for materials.

Colin spends no more than 30 hours per month on manufacturing these objects. Each bowl takes 4 hours, each candle holder takes 2 hours and each key fob takes half an hour.

(i) Let b be the number of bowls Colin makes in a month, c the number of candle holders and f the number of key fobs. Write out, in terms of these variables, two constraints corresponding to the limit on monthly expenditure on materials, and to the limit on Colin's time. **[2]**

Colin sells the objects at craft fairs. He charges £30 for a bowl, £15 for a candle holder and £3 for a key fob.

(ii) Set up an initial simplex tableau for the problem of maximising Colin's monthly income subject to your constraints from part **(i)**, assuming that he sells all that he produces. **[2]**

(iii) Use the simplex algorithm to solve your LP, and interpret the solution from the simplex algorithm. **[8]**

Over a spell of several months Colin finds it difficult to sell bowls so he stops making them.

(iv) Modify and solve your LP, using simplex, to find how many candle holders and how many key fobs he should make, and interpret your solution. **[3]**

At the next craft fair Colin takes an order for 4 bowls. He promises to make exactly 4 bowls in the next month.

(v) Set up this modified problem either as an application of two-stage simplex, or as an application of the big-M method. You are not required to solve the problem. **[3]**

The solution now is for Colin to produce 4 bowls, $6\frac{2}{3}$ candle holders and no key fobs.

(vi) What is Colin's best integer solution to the problem? **[1]**

(vii) Your answer to part **(vi)** is not necessarily the integer solution giving the maximum profit for Colin. Explain why. **[1]**

BLANK PAGE

BLANK PAGE