Mark Scheme 4772
June 2005

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

M marks can be implied by correct answers.

A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

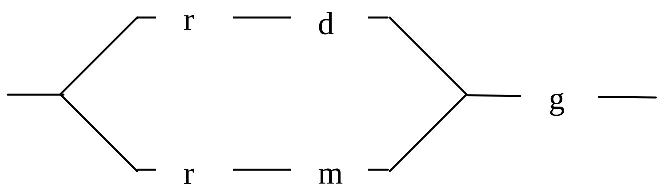
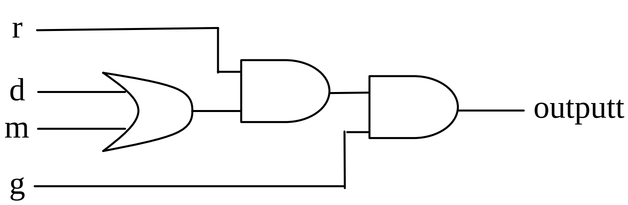
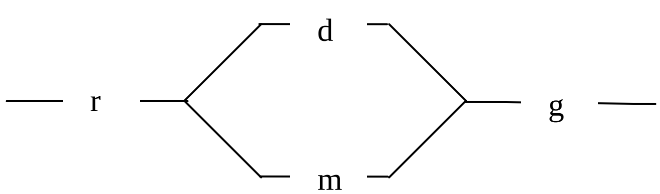
B marks are for specific results or statements, and are independent of method.

√ marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of –1, –2 or –3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

SC special case

1.

(a) (i) If sidelights and headlights are on, and if the foglights are switched on.	B1 B1
(ii) $\sim(\sim s \vee \sim h) \wedge f$	M1 A1
(iii) (truth table below) Accept t/table showing $s \wedge h = \sim(\sim s \vee \sim h)$	M1 8 rows A1 $s \wedge h \wedge f$ A1 $\sim(\sim s \vee \sim h) \wedge f$ alternative: B1 comment re $\wedge f$ M1 4 lines A1
(b) (i) 	M1 A1
(ii) 	M1 A1 "or" A1 first "and" A1 second "and"
(iii) $r \wedge (d \vee m) \wedge g = (r \wedge (d \vee m)) \wedge g$ by associativity $= ((r \wedge d) \vee (r \wedge m)) \wedge g$ by distributivity	M1 distributive law A1 handling brackets (law names not needed)
(iv) 	B1

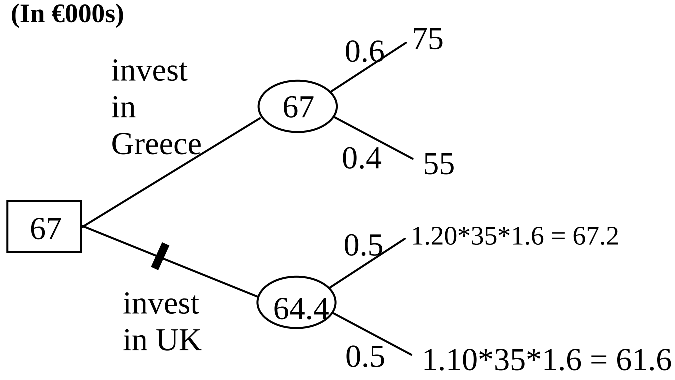
Truth table for (a)(iii):

(s	$\wedge$	h)	$\wedge$	f	$\Leftrightarrow$	~	(~	s	$\vee$	~	h)	$\wedge$	f
0	0	0	0	0	1	0	1	0	1	1	0	0	0
0	0	0	0	1	1	0	1	0	1	1	0	0	1
0	0	1	0	0	1	0	1	0	1	0	1	0	0
0	0	1	0	1	1	0	1	0	1	0	1	0	1
1	0	0	0	0	1	0	0	1	1	1	0	0	0
1	0	0	0	1	1	0	0	1	1	1	0	0	1
1	1	1	0	0	1	1	0	1	0	0	1	0	0

2.

(i)

(In €000s)

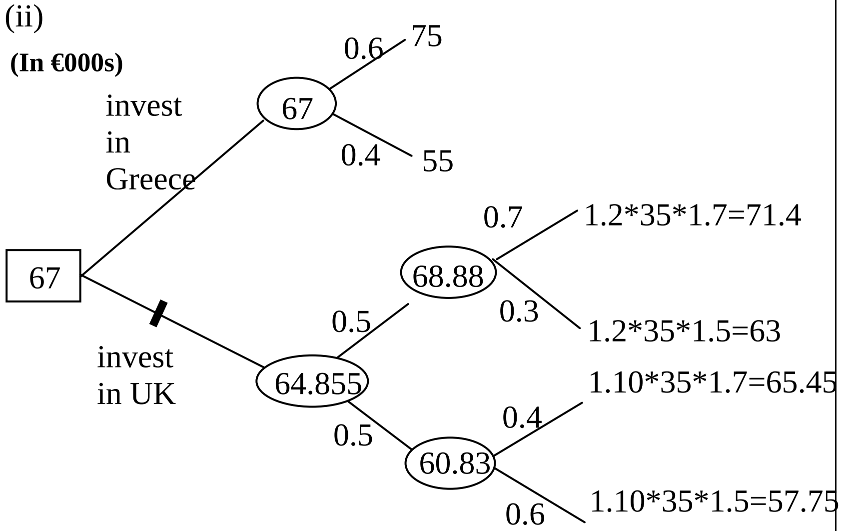


M1
A1 chance nodes
A1 choice node

B1√ invest in Greece

(ii)

(In €000s)

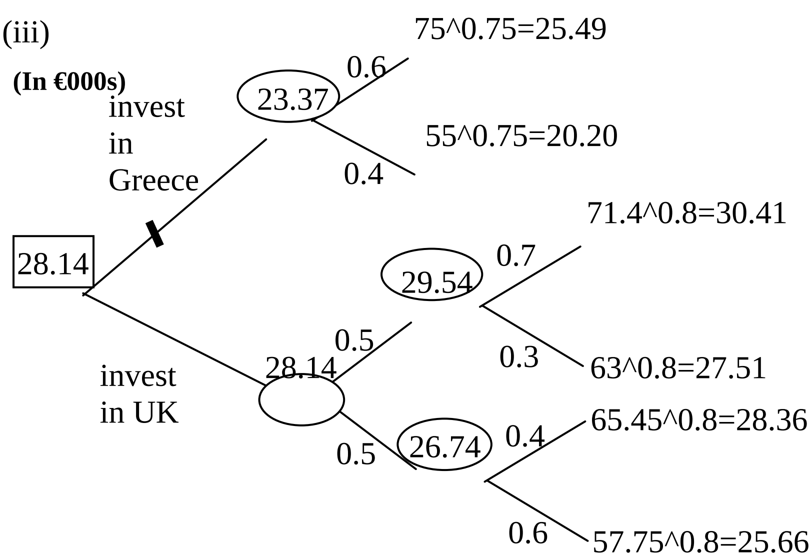


M1 new chance nodes
A1 64.855 or .86 or .85

B1 invest in Greece

(iii)

(In €000s)



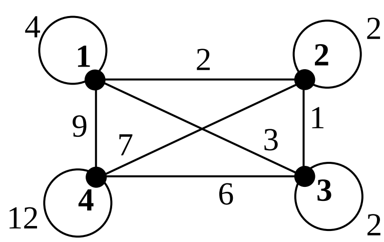
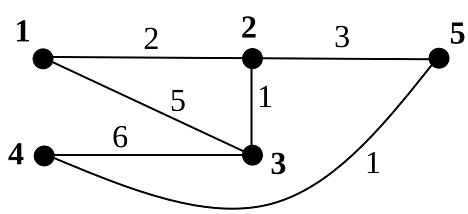
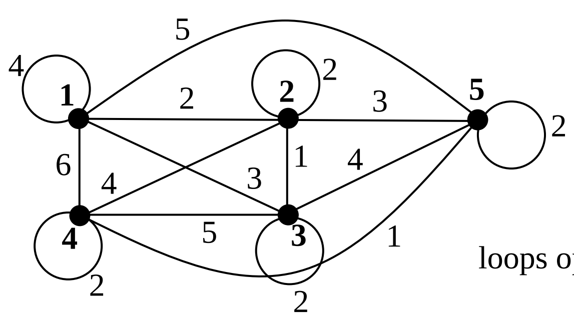
M1 utilities
A1 23.37 and 28.14

B1 invest in UK

2 (cont)

(iv) Require $\frac{1.2+1.1}{2} \times 35 \times x = 67$, giving x = 1.665	M1 A1 cao
(v) Require $\frac{(1.2 \times 35 \times y)^{0.8} + (1.1 \times 35 \times y)^{0.8}}{2} = 23.37$. Trying y = 1.277: $(1.2 \times 35 \times 1.277)^{0.8} = 24.185$ $(1.1 \times 35 \times 1.277)^{0.8} = 22.559$ (24.185+22.559)/2 = 23.37	M1 cash M1 house A1 one bracket evaluated correctly A1

3.

(i)  loops optional	M1 A1
(ii) First vertex en route is 3. First vertex en route from 3 to 1 is 2. First vertex en route from 2 to 1 is 1.	M1 A1
(iii) 	B1
 loops optional	M1 A1

(iv)

	1	**2**	**3**	**4**	**5**
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

	1	**2**	**3**	**4**	**5**
1	2	2	2	2	2
2	1	3	3	5	5
3	2	2	2	2	2
4	5	5	5	5	5
5	2	2	2	4	4

B1 distance matrix

M1 route matrix
A1 cao

(v) 1 2 3 5 4 1
14
1 2 3 2 5 4 5 2 1

M1
A1
A1

(vi)

		1	2	4	3
	1	**2**	**3**	**4**	**5**
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

M1 Prim on matrix
A1

Lower bound is 5 + 2 + 3 = 10

B1 B1

(vii) e.g.
1 2 5 4 <u>3 2 3</u> 1
19

M1 A1 cao
B1

4.

(i) The objective is nonlinear. — B1

(ii)

P	x	y	S1	S2	S3	RHS
1	–1	1	0	0	0	0
0	1	0	1	0	0	10
0	0	1	0	1	0	6
0	1	–2	0	0	1	0
1	0	–1	0	0	1	0
0	0	2	1	0	–1	10
0	0	1	0	1	0	6
0	1	–2	0	0	1	0
1	0	0	1/2	0	1/2	5
0	0	1	1/2	0	–1/2	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

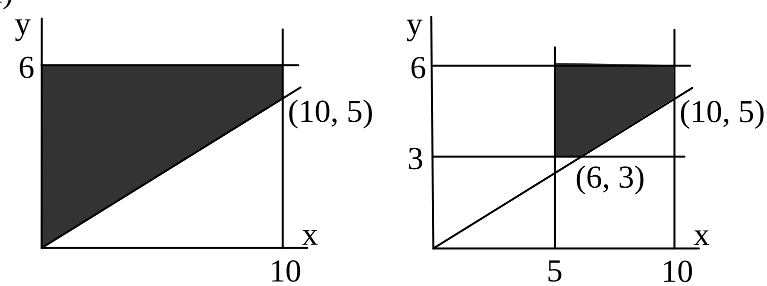
M1 tableau
A1

M1 pivot choice
A1 pivot

M1 pivot choice
A1 pivot

10 ml of oil and 5 ml of vinegar — B1

(iii)



B1 x ≤ 10 and y ≤ 6
B1 5 ≤ x and 3 ≤ y

B1 proportion line

B1 region 1
B1 region 2

(iv) Omitted constraints non-active — B1
(0, 0) not in feasible region. — B1

(v)

C	P	x	y	s1	s2	s3	s4	s5	a1	a2	RHS
1	0	1	1	0	–1	0	–1	0	0	0	8
0	1	1	–1	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	10
0	0	1	0	0	–1	0	0	0	1	0	5
0	0	0	1	0	0	1	0	0	0	0	6
0	0	0	1	0	0	0	–1	0	0	1	3
0	0	1	–2	0	0	0	0	1	0	0	0

B1 > constraints

B1 artificial columns

B1 new objective

Minimise C, hopefully to zero. — B1
Thereafter delete C row and a1/a2 columns, and proceed as usual. — B1

Mark Scheme 4772
June 2006

1.

(i)

~ (	~	T	⇒	~	S)	⇔	~	T	∧	S
0	1	0	1	1	0	1	1	0	0	0
1	1	0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	1	0	1	0	0
0	0	1	1	0	1	1	0	1	0	1

M1 4 lines
A1 T and S
A1 ~T (twice) and ~S
A1 ⇒
A1 ∧
A1 ~on LHS
M1
A1 result

(ii)

A	⇒	B	⇔	~	A	∨	B
0	1	0	1	1	0	1	0
0	1	1	1	1	0	1	1
1	0	0	1	0	1	0	0
1	1	1	1	0	1	1	1

or a correct verbal argument

M1
A1

$$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow \sim(T \vee \sim S) \Leftrightarrow \sim T \wedge S$$

M1 Boolean
A1 applying result
A1 correct negating

(iii) Joanna will not try and will succeed

B1 not try
B1 and
B1 succeed

2.

(i)

	1	2	3	4
1	∞	2	6	4
2	2	∞	3	1
3	6	3	∞	1
4	4	1	1	∞

	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	2	3	4
1	∞	2	6	4
2	2	4	3	1
3	6	3	12	1
4	4	1	1	8

	1	2	3	4
1	1	2	3	4
2	1	1	3	4
3	1	2	1	4
4	1	2	3	1

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1 no change

	1	2	3	4
1	4	2	4	3
2	2	2	2	1
3	4	2	2	1
4	3	1	1	2

	1	2	3	4
1	2	2	(2)	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	2

A1 circled element
A1 rest

(ii) distance = 4 (row 1, col 3 of dist matrix)
route = 1, 2, 4, 3 (1 – r1c3 – r2c3 – r4c3 of route matrix)

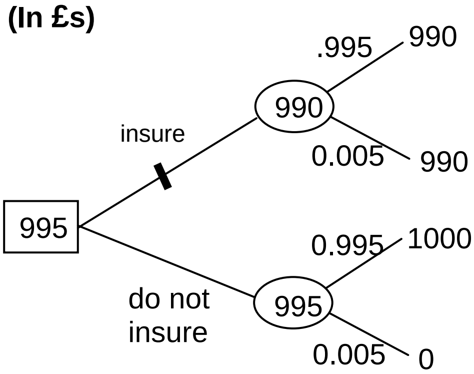
M1 A1
M1 A1

(iii) **1**, **2**, **4**, **3**, **1**
1, **2**, **4**, **3**, **4**, **2**, **1**
8

B1
M1 A1
B1

3.

(i) **(In £s)**



M1 pay–offs
A1

M1 chance nodes
A1

M1 decision node
A1

(ii) Do not insure.
Pay no more than £5 for it.

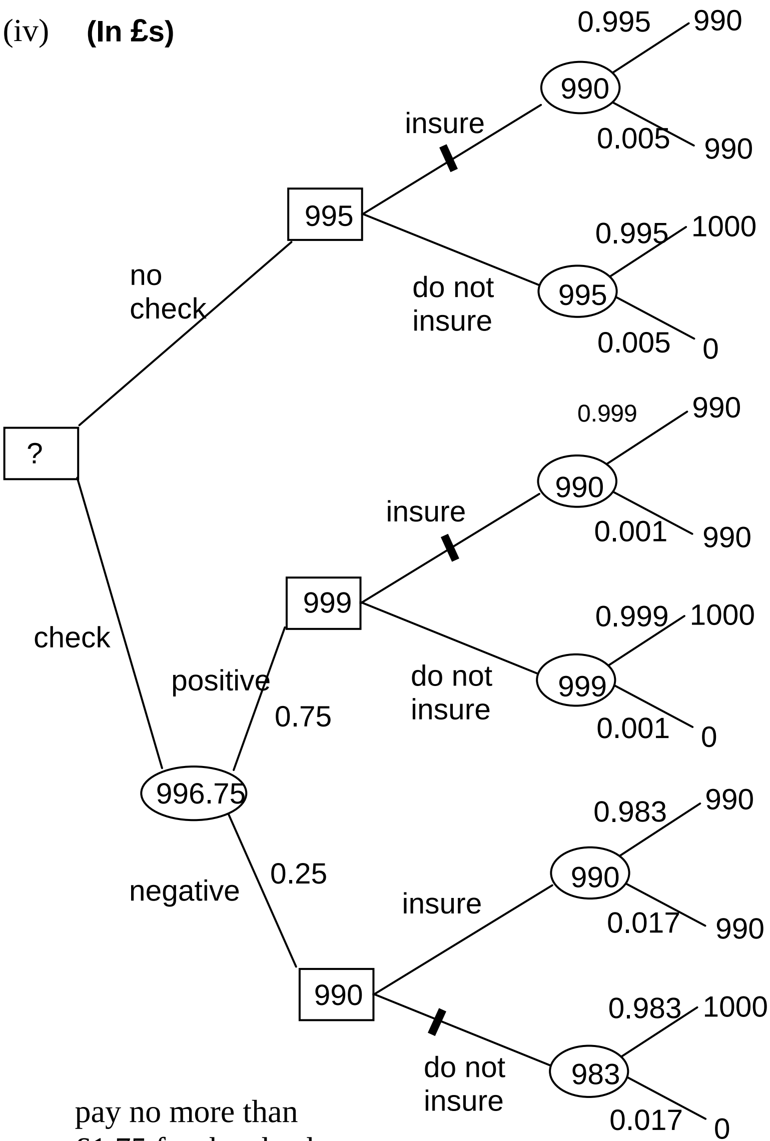
B1
B1

(iii) Yes $\left(\left(\sqrt[3]{990}\times(0.995+0.005)\right) \text{ v } \left(0.995\times\sqrt[3]{1000}\right)\right)$

$\sqrt[3]{1000-x}=9.95$ giving x = £14.93

B1
M1 A1

(iv) **(In £s)**



pay no more than
£1.75 for the check

M1 check/no check
A1

M1 positive/negative
A1

M1 insure/not insure
A1

M1 go/no go
A1

B1

4. (i) a is the number of aardvarks, etc. — B1
First inequality models the furry material constraint — M1
Second inequality models the woolly material constraint — A1
Third inequality models the glass eyes constraint

That would model a "pairs of glass eyes" constraint. — B1

(ii) The problem is an IP, so the number of eyes used will be integer anyway. — B1

(iii) e.g.

P	a	b	c	s1	s2	s3	RHS	Marks
1	−3	–5	–2	0	0	0	0	
0	0.5	1	1	1	0	0	11	M1
0	2	1.5	1	0	1	0	24	A1
0	2	2	2	0	0	1	30	
1	–0.5	0	3	5	0	0	55	M1 pivot choice
0	0.5	1	1	1	0	0	11	A1 pivot
0	1.25	0	-0.5	–1.5	1	0	7.5	
0	1	0	0	–2	0	1	8	
1	0	0	2.8	4.4	0.4	0	58	M1 pivot choice
0	0	1	1.2	1.6	–0.4	0	8	A1 pivot
0	1	0	–0.4	–1.2	0.8	0	6	
0	0	0	0 4	0 8	0 8	1	2	

Make 6 aardvarks and 8 bears giving £58 profit. — B1 B1
2 eyes are left over. — B1

(iv)

P	a	b	c	s 1	s 2	s 3	su 4	a	RHS
1	−3	–5	–(2+M)	0	0	0	M	0	–2M
0	0.5	1	1	1	0	0	0	0	11
0	2	1.5	1	0	1	0	0	0	24
0	2	2	2	0	0	1	0	0	30
0	0	0	1	0	0	0	–1	1	2

B1 new constraint
M1 objective
A1

or

C	P	a	b	c	s 1	s 2	s 3	su 4	a	RHS
1	0	0	0	1	0	0	0	–1	0	2
0	1	−3	–5	–2	0	0	0	0	0	0
0	0	0.5	1	1	1	0	0	0	0	11
0	0	2	1.5	1	0	1	0	0	0	24
0	0	2	2	2	0	0	1	0	0	30
0	0	0	0	1	0	0	0	–1	1	2

(v) $8\times0.5 + 2\times1 + 5\times1 = 11$
$8\times2 + 2\times1.5 + 5\times1 = 24$ — B1
$8\times2 + 2\times2 + 5\times2 = 30$

$3\times8 + 5\times2 + 2\times5 = 44$ but $3\times6 + 5\times6 + 2\times2 = 52$ — B1
$1\,m^2$ of woolly material and 2 eyes left. — B1

Mark Scheme 4772
June 2007

1.

(a)(i) He should salute it.
Since all objects which don't move are painted any unpainted object must move, and anything that moves must be saluted.

B1

M1 A1

(ii) We do not know.
We do not know about painted objects. Some will have been painted because they do not move, but there may be some objects which move which are painted. We do not know whether this object moves or not.

B1

M1 A1

(b)

((m	⇒	s)	∧	(~	m	⇒	p))	∧	~	p	⇒	s
1	1	1	**1**	0	1	1	1	0	**0**	1	**1**	1
1	1	1	**1**	0	1	1	0	1	**1**	0	**1**	1
1	0	0	**0**	0	1	1	1	0	**0**	1	**1**	0
1	0	0	**0**	0	1	1	0	0	**1**	0	**1**	0
0	1	1	**1**	1	0	1	1	0	**0**	1	**1**	1
0	1	1	**0**	1	0	0	0	0	**1**	0	**1**	1
0	1	0	**1**	1	0	1	1	0	**0**	1	**1**	0
0	1	0	**0**	1	0	0	0	0	**1**	0	**1**	0

M1 8 rows
A1 m⇒s
A1 ~m⇒p
A1 first ∧
A1 second ∧
A1 result

(c) $\left(\left(m \Rightarrow s\right) \wedge \left(\sim m \Rightarrow p\right)\right) \wedge \sim p$

$\Leftrightarrow \left(\sim p \wedge \left(\sim m \Rightarrow p\right)\right) \wedge \left(m \Rightarrow s\right)$

$\Leftrightarrow \left(\sim p \wedge \left(\sim p \Rightarrow m\right)\right) \wedge \left(m \Rightarrow s\right)$ (contrapositive)

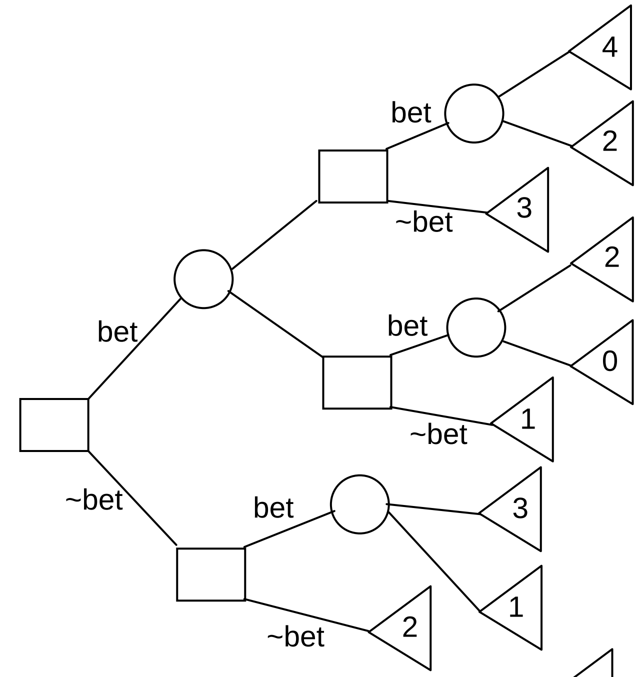
$\Rightarrow m \wedge \left(m \Rightarrow s\right)$ (modus ponens)

$\Rightarrow s$ (modus ponens)

M1
A1 reordering
A1 contrapositive
A1 modus ponens

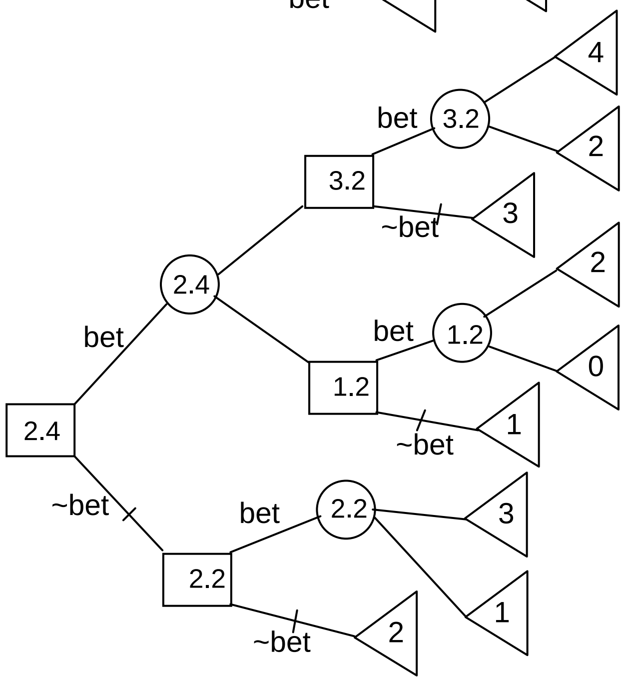
2.

(i)



Mark	Comment
M1	
A1	first D box
A1	D box on ~bet branch
A1	P box on bet branch
A1	D boxes following P box
A1	remaining P boxes
M1	outcomes
A1	

(ii)(A)

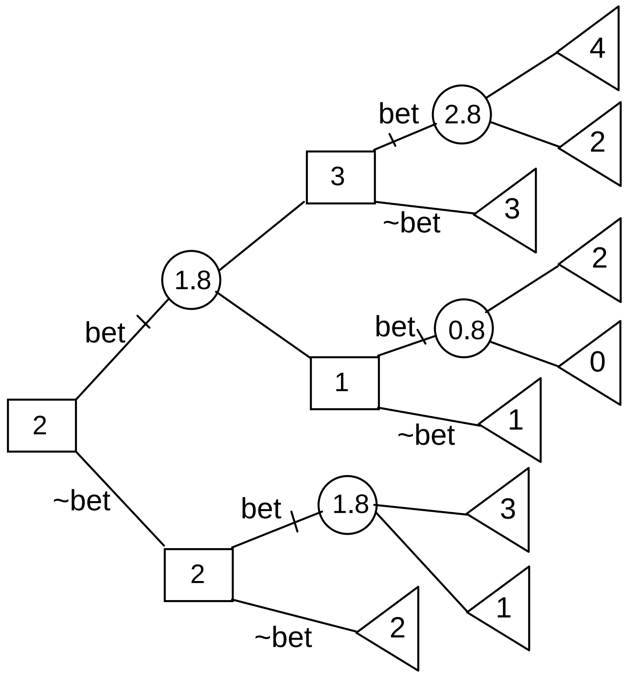


Mark	Comment
M1	
A1	

EMV = 2.4 by betting and betting again

Mark	Comment
B1	course of action

2(cont).

(ii)(B) 	A1
EMV = 2 by not betting	B1 course of action
(iii) $2^{0.5}\times 0.4 = 0.566 < 1$, but $2^{1.5}\times 0.4 = 1.131 > 1$	M1 A1 A1

3.

(i)

	1	**2**	**3**	**4**
1	6	3	6	5
2	3	4	3	2
3	6	3	2	1
4	5	2	1	2

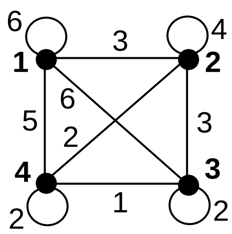
	1	**2**	**3**	**4**
1	2	2	2	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	3

M1 distances
A2 6 changes
(–1 each error)
M1 a correct update
A1 1 to 3 route (2)
A2 rest
(–1 each error)

(ii) Distance from row 1 col 3 of distance matrix (6)
Route from row 1 col 3 of route matrix (2), then from row 2 col 3 (4), then from row 4 col 3 (3). So 1 2 4 3.

B1 B1
B1
B1

(iii)

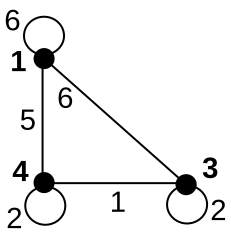


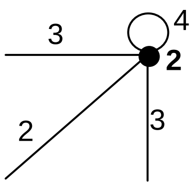
B1 whether or not loops included

(iv) **1 2 4 3 1**
length = 12
1 2 4 3 4 2 1

B1
B1
B1

(v)





MST has length 6, so lower bound = 6 + 2 + 3 = 11

M1
A1 MST
A1 add back

(vi) TSP length is either 11 or 12

B1 11 to 12
B1 either 11 or 12

4.

(i)

P	x	y	s_1	s_2	RHS
1	–1	–1	0	0	0
0	2	1	1	0	1250
0	2	–1	0	1	0
1	1	0	1	0	1250
0	2	1	1	0	1250
0	4	0	1	1	1250

M1 initial tableau
A1

M1 pivot
A2 (–1 each error)

1250 m^2 of paving and no decking — B1 interpretation

(ii) 2-phase

A	P	x	y	s_1	s_2	s_3	a	RHS
1	0	1	0	0	0	–1	0	200
0	1	1	0	1	0	0	0	1250
0	0	2	1	1	0	0	0	1250
0	0	4	0	1	1	0	0	1250
0	0	1	0	0	0	–1	1	200
1	0	0	0	0	0	0	–1	0
0	1	0	0	1	0	1	–1	1050
0	0	0	1	1	0	2	–2	850
0	0	0	0	1	1	4	–4	450
0	0	1	0	0	0	–1	1	200

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

Big-M alternative

P	*x*	*y*	s_1	s_2	s_3	*a*	*RHS*
1	*1–M*	*0*	*1*	*0*	*M*	*0*	*1250–2M*
0	*2*	*1*	*1*	*0*	*0*	*0*	*1250*
0	*4*	*0*	*1*	*1*	*0*	*0*	*1250*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*
1	*0*	*0*	*1*	*0*	*1*	*M–1*	*1050*
0	*0*	*1*	*1*	*0*	*2*	*–2*	*850*
0	*0*	*0*	*1*	*1*	*4*	*–4*	*450*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

850 m^2 of paving and 200 m^2 of decking. — A1 interpretation

(iii)

C	x	y	s_1	s_2	s_3	s_4	RHS
1	0	0	1.25	0	1.75	0	1212.5
0	0	1	1	0	2	0	850
0	0	0	1	1	4	0	450
0	1	0	0	0	–1	0	200
0	0	0	1	0	1	1	50
1	0	0	–0.5	0	0	–1.75	1125
0	0	1	–1	0	0	–2	750
0	0	0	–3	1	0	–4	250
0	1	0	1	0	0	1	250
0	0	0	1	0	1	1	50

B1 new objective

B1 new constraint

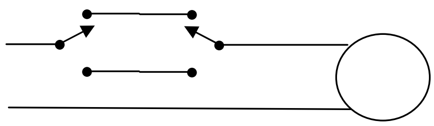
M1
A1

750 m^2 of paving and 250 m^2 of decking at an annual cost of £1125

A1 interpretation

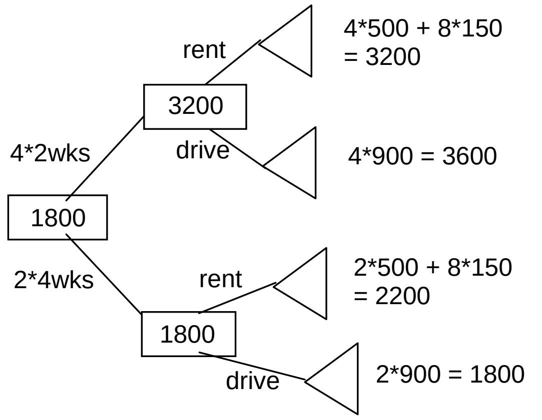
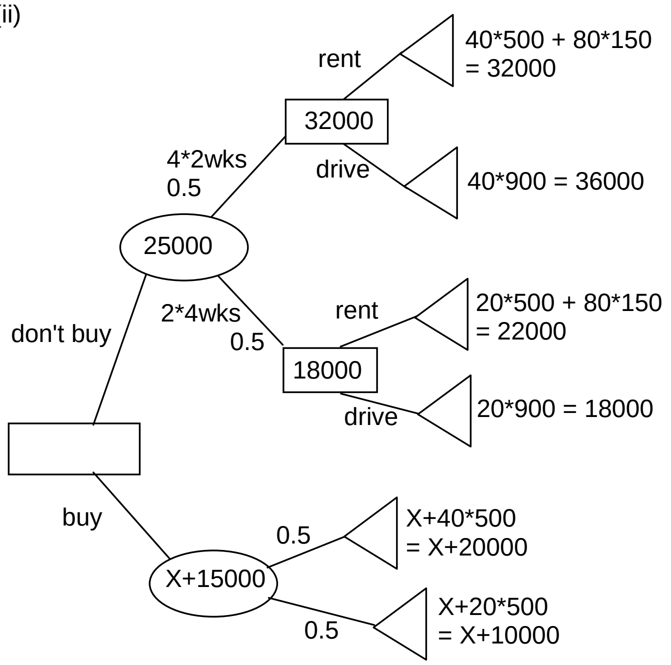
4772 Decision Mathematics 2

1.

(a)(i) "not fail" → "succeed" "disagree less" → "agree more"	B1 B1
(ii) e.g. "I don't entirely agree with you".	M1 same meaning A1 simpler
(b) e.g. 	M1 2 switches + light in a circuit A4 one for each correct setting
(c) see table below	M1 4 lines A1 a's and b's A1 negations A1 level 1 and's A1 level 1 or's A1 level 2 A1 result

((a	$\wedge$	b)	$\vee$	(~a	$\wedge$	~b))	$\Leftrightarrow$	((~a	$\vee$	b)	$\wedge$	(a	$\vee$	~b))
1	1	1	**1**	0	0	0	**1**	0	1	1	**1**	1	1	0
1	0	0	**0**	0	0	1	**1**	0	0	0	**0**	1	1	1
0	0	1	**0**	1	0	0	**1**	1	1	1	**0**	0	0	0
0	0	0	**1**	1	1	1	**1**	1	1	0	**1**	0	1	1

2.

(i) e.g. (Decisions could be in other order.)	
	M1 4*2/2*4 M1 rent/drive A1 M1 costs A1
Drive down for 2 lots of 4 weeks	B1 advice
(ii)	
	B1 buy/don't buy M1 don't buy analysis A1 costings M1 chance node A1 buy analysis M1 buy costings A1 B1
Jane could save money if she spent less than £10000 on a car	B1
(iii) EMV – expected monetary value – probabilistically weighted cash values Utility measure is an alternative.	B1

3.

(a) (i)

	1	**2**	**3**	**4**
1	∞	14	11	24
2	14	∞	15	∞
3	11	15	∞	12
4	24	∞	12	∞

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	**2**	**3**	**4**
1	∞	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

	1	**2**	**3**	**4**
1	28	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	**2**	**3**	**4**
1	2	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

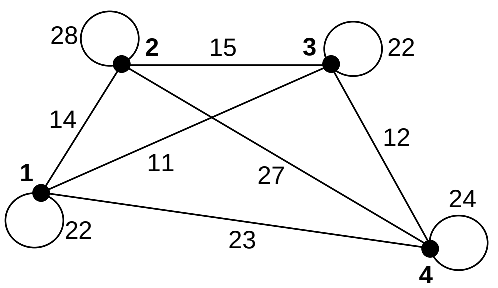
	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

A1

	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3



B1 loops
B1 rest

(ii) **1 3 4 2 1**
64
⇒ **1 3 4 3 2 1**

M1 A1
B1
B1

(iii) 27 + 11 + 14 = 52 TSP solution has length between 52 and 64	M1 A1 M1 A1
(b) e.g. **1 3 1 2 3 4 1** length = 87 One repeated arc → Eulerian	M1 A1 A1 B1

4.

(i) Let a be the number of tonnes of A produced …	M1 A1
Max a+b+c	B1
st 3a+2b+5c<60	B1
5a+6b+2c<50	B1

(ii) e.g.

P	a	b	c	s_1	s_2	RHS
1	–1	–1	–1	0	0	0
0	3	2	5	1	0	60
0	5	6	2	0	1	50
1	–0.4	–0.6	0	0.2	0	12
0	0.6	0.4	1	0.2	0	12
0	3.8	5.2	0	–0.4	1	26
1	>0	0	0	>0	>0	15
0		0	1			10
0	19/26	1	0	–2/26	5/26	5

M1 initial tableau
A1

M1 pivot
A1

M1
A1

Make 5 tonnes of B and 10 tonnes of C — B1 interpretation

(iii) & (iv) e.g.

A	P	a	b	c	s_1	s_2	s_3	art	RHS
1	0	1	0	0	0	0	–1	0	8
0	1	–1	–1	–1	0	0	0	0	0
0	0	3	2	5	1	0	0	0	60
0	0	5	6	2	0	1	0	0	50
0	0	1	0	0	0	0	–1	1	8
1	0	0	0	0	0	0	0	–1	0
0	1	0	–1	–1	0	0	–1	1	8
0	0	0	2	5	1	0	3	–3	36
0	0	0	6	2	0	1	5	–5	10
0	0	1	0	0	0	0	–1	1	8
	1	0	2	0	0	0.5	1.5		13
	0	0	–13	0	1	–2.5	–4.5		11
	0	0	3	1	0	0.5	2.5		5
	0	1	0	0	0	0	–1		8

B1 new constraint
M1 surplus +
A1 artificial
B1 new objective

M1
A1

B1

Make 8 tonnes of A and 5 tonnes of C — B1 interpretation

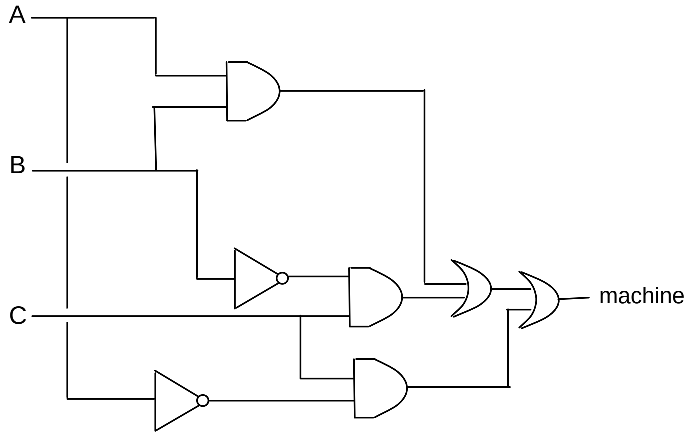
4772 Decision Mathematics 2

Question 1.

(a) e.g.
"It is easy to overestimate the effect that your contribution will make."

M1 remove double negatives
A1 same meaning

(b) e.g.



M1 combinatorial
A1 "ands"
A1 negations
A1 "ors"
A3 one for each alternative

(c) e.g.

((a	∧	b)	∨	(~a	∧	c))	∨	(~b	∧	c)
1	1	1	**1**	0	0	1	1	0	0	1
1	1	1	**1**	0	0	0	1	0	0	0
1	0	0	**0**	0	0	1	1	1	1	1
1	0	0	**0**	0	0	0	0	1	0	0
0	0	1	**1**	1	1	1	1	0	0	1
0	0	1	**0**	1	0	0	0	0	0	0
0	0	0	**1**	1	1	1	1	1	1	1
0	0	0	**0**	1	0	0	0	1	0	0

M1 8 lines
A1 a, b, c
A1 negations
A1 "and"s
A1 "or"s

~	((~a	∧	~c)	∨	(~b	∧	~c))
1	0	0	0	**0**	0	0	0
1	0	0	1	**0**	0	0	1
1	0	0	0	**0**	1	0	0
0	0	0	1	**1**	1	1	1
1	1	0	0	**0**	0	0	0
0	1	1	1	**1**	0	0	1
1	1	0	0	**0**	1	0	0
0	1	1	1	**1**	1	1	1

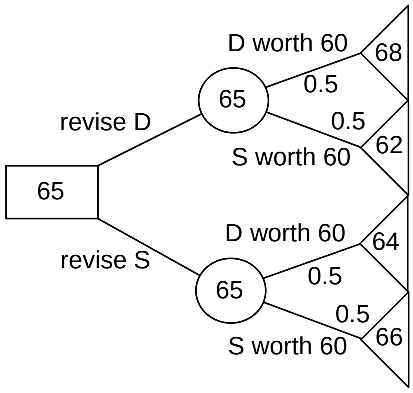
M1
A1

Question 2.

(i)

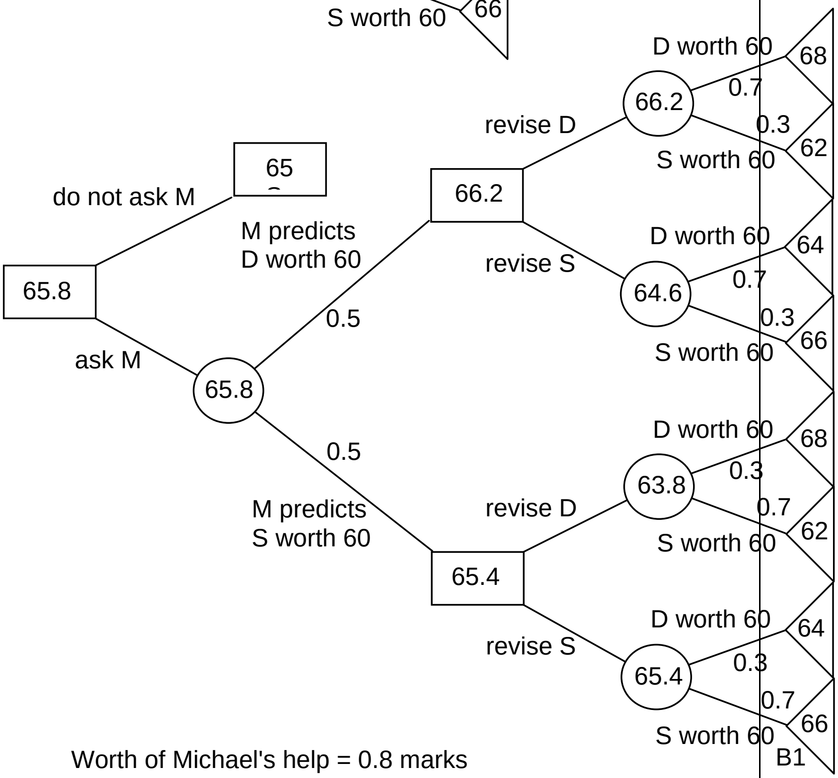
revised	60marks	score	Marks
D	D	48+20 = 68	M1 A1
D	S	32+30 = 62	A1
S	D	36+28 = 64	A1
S	S	24+42 = 66	A1

(ii)



M1 chance node
A1
A1

M1 decision node
A1



M1 chances
A1

M1decisions
A1 revise
A1 ask/don't

Worth of Michael's help = 0.8 marks B1

Question 3.

(i) a is the number of acres of land put to crop A, etc $a + b \le 20$ is equivalent to $a + b \le c + d$ Given that $a + b + c + d \le 40$, the maximisation will ensure that $a + b + c + d = 40$ (and it's easier to solve using simplex).	B1 B1 B1

(ii)

P	a	b	c	d	s1	s2	RHS
1	–50	–40	–40	–30	0	0	0
0	1	1	0	0	1	0	20
0	1	1	1	1	0	1	40
1	0	10	–40	–30	50	0	1000
0	1	1	0	0	1	0	20
0	0	0	1	1	–1	1	20
1	0	10	0	10	10	40	1800
0	1	1	0	0	1	0	20
0	0	0	1	1	–1	1	20

Marks: M1, A1, A1, A1 (initial tableau); M1 A1 (second tableau); M1 A1 (third tableau)

20 acres to A and 20 acres to C, giving profit of £1800 — B1 B1

(iii) Max $50a + 40b + 40c + 30d$

st $a + b \le 20$

$a + b + c + d \le 40$

$a + b + c + d \ge 40$ — B1

A	P	a	b	c	d	s1	s2	sur	art	R
1	0	1	1	1	1	0	0	–1	0	40
0	1	–50	–40	–40	–30	0	0	0	0	0
0	0	1	1	0	0	1	0	0	0	20
0	0	1	1	1	1	0	1	0	0	40
0	0	1	1	1	1	0	0	–1	1	40

B1 new obj
B1 surplus
B1 artificial
B1 3 constraints

Minimise A (to zero) then drop A row and art column and continue normally — B1 B1

OR — **OR**

P	a	b	c	d	s1	s2	sur	art	R
1	–50 –M	–40 –M	–40 –M	–30 –M	0	0	M	0	–40M
0	1	1	0	0	1	0	0	0	20
0	1	1	1	1	0	1	0	0	40
0	1	1	1	1	0	0	–1	1	40

M1
A1
B1 surplus
B1 artificial

Proceed as per simplex, regarding M as a large fixed number. — B1 B1

Question 4.

(a) (i),(ii) and (iii)

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	∞	20	5	23		**2**	1	2	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	∞	16		**4**	1	2	3	4	5
5	15	23	∞	16	∞		**5**	1	2	3	4	5

M1 distance
A1 1 to 5 etc
A1 rest

B1 route

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	30	16		**4**	1	2	3	1	5
5	15	23	∞	16	30		**5**	1	2	3	4	1

Not part of the question

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

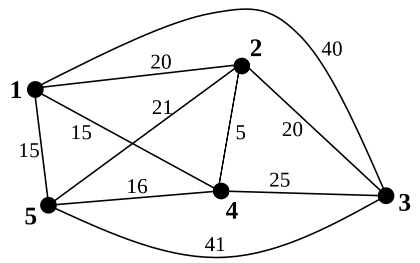
M1
A1 10 changed dists

M1 2's in r3 of route
A1 rest of route

	1	**2**	**3**	**4**	**5**			**1**	**2**	**3**	**4**	**5**
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

Shortest distance from **3** to **1** is 40
(1st row and 3rd column of distance matrix)

B1
B1

	Shortest route is **3 2 4 1** **3** followed by route matrix (3,1) = **2** followed by route matrix (2,1) = **4** followed by route matrix (4,1) = **1**	B1 M1 A1
(iv)		M1 A1
(v)	**2** (5) **4** (15) **1** (15) **5** (41) **3** (20) **2** Total length = 96	B1 B1
	2 4 1 5 (4 2) 3 2	M1 A1
	Finds a (hopefully short) route visiting every vertex and returning to the start, **or**, upper bound to the TSP	B1

1.

(a)(i) $\sim c \Rightarrow e$	B1	
(ii) $(c \Rightarrow \sim e) \Leftrightarrow (\sim c \Rightarrow e)$ 1 0 01 0 01 1 1 or 0 1 10 0 10 0 0	M1 A1 M1 A1	line of a TT both propositions 1 or both 0 an "$\Rightarrow$" correct all OK
(b)(i) Circuit is $\sim x \vee y$. This is $x \Rightarrow y$. 10 1 0 0 1 0 10 1 1 0 1 1 01 0 0 1 0 0 01 1 1 1 1 1	B1 B4	
(ii) $(\sim p \wedge \sim q) \Rightarrow r$	M1 A1	implication noted
(iii) $(\sim p \wedge \sim q) \Rightarrow r$ is equivalent to $\sim r \Rightarrow \sim(\sim p \wedge \sim q)$	B1	
But we have $\sim r$, so we have $\sim(\sim p \wedge \sim q)$.	B1	
$\sim(\sim p \wedge \sim q)$ is equivalent to $p \vee q$	B1	
But we have $\sim q$, so therefore p.	B1	

2.

(i) Distances longer — B1

(ii)

	1	2	3	4	5	6
1	∞	15	∞	∞	7	8
2	15	∞	6	2	6	∞
3	∞	6	∞	3	∞	∞
4	∞	2	3	∞	10	17
5	7	6	∞	10	∞	8
6	8	∞	∞	17	8	∞

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	1	2	3	4	5	6
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	5	6
6	1	2	3	4	5	6

not part of answer

	1	2	3	4	5	6
1	∞	15	∞	∞	7	8
2	15	30	6	2	6	23
3	∞	6	∞	3	∞	∞
4	∞	2	3	∞	10	17
5	7	6	∞	10	14	8
6	8	23	∞	17	8	16

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	1	1	3	4	5	1
3	1	2	3	4	5	6
4	1	2	3	4	5	6
5	1	2	3	4	1	6
6	1	1	3	4	5	1

not part of answer

	1	2	3	4	5	6
1	30	15	21	17	7	8
2	15	30	6	2	6	23
3	21	6	12	3	12	29
4	17	2	3	4	8	17
5	7	6	12	8	12	8
6	8	23	29	17	8	16

	1	2	3	4	5	6
1	2	2	2	2	5	6
2	1	1	3	4	5	1
3	2	2	2	4	2	2
4	2	2	3	2	2	6
5	1	2	2	2	2	6
6	1	1	1	4	5	1

M1 30 in top left
A1 times
A1 6 to 3 route = 1
A1 rest of route

	1	2	3	4	5	6
1	30	15	21	17	7	8
2	15	12	6	2	6	23
3	21	6	12	3	12	29
4	17	2	3	4	8	17
5	7	6	12	8	12	8
6	8	23	29	17	8	16

	1	2	3	4	5	6
1	2	2	2	2	5	6
2	1	3	3	4	5	1
3	2	2	2	4	2	2
4	2	2	3	2	2	6
5	1	2	2	2	2	6
6	1	1	1	4	5	1

not part of answer

	1	2	3	4	5	6
1	30	15	20	17	7	8
2	15	4	5	2	6	19
3	20	5	6	3	11	20
4	17	2	3	4	8	17
5	7	6	11	8	12	8
6	8	19	20	17	8	16

	1	2	3	4	5	6
1	2	2	2	2	5	6
2	1	4	4	4	5	4
3	4	4	4	4	4	4
4	2	2	3	2	2	6
5	1	2	2	2	2	6
6	1	4	4	4	5	1

not part of answer

	1	2	3	4	5	6
1	14	13	18	15	7	8
2	13	4	5	2	6	14
3	18	5	6	3	11	19
4	15	2	3	4	8	16
5	7	6	11	8	12	8
6	8	14	19	16	8	16

	1	2	3	4	5	6
1	5	5	5	5	5	6
2	5	4	4	4	5	5
3	4	4	4	4	4	4
4	2	2	3	2	2	2
5	1	2	2	2	2	6
6	1	5	5	5	5	1

not part of answer

	1	2	3	4	5	6
1	14	13	18	15	7	8
2	13	4	5	2	6	14
3	18	5	6	3	11	19
4	15	2	3	4	8	16
5	7	6	11	8	12	8
6	8	14	19	16	8	16

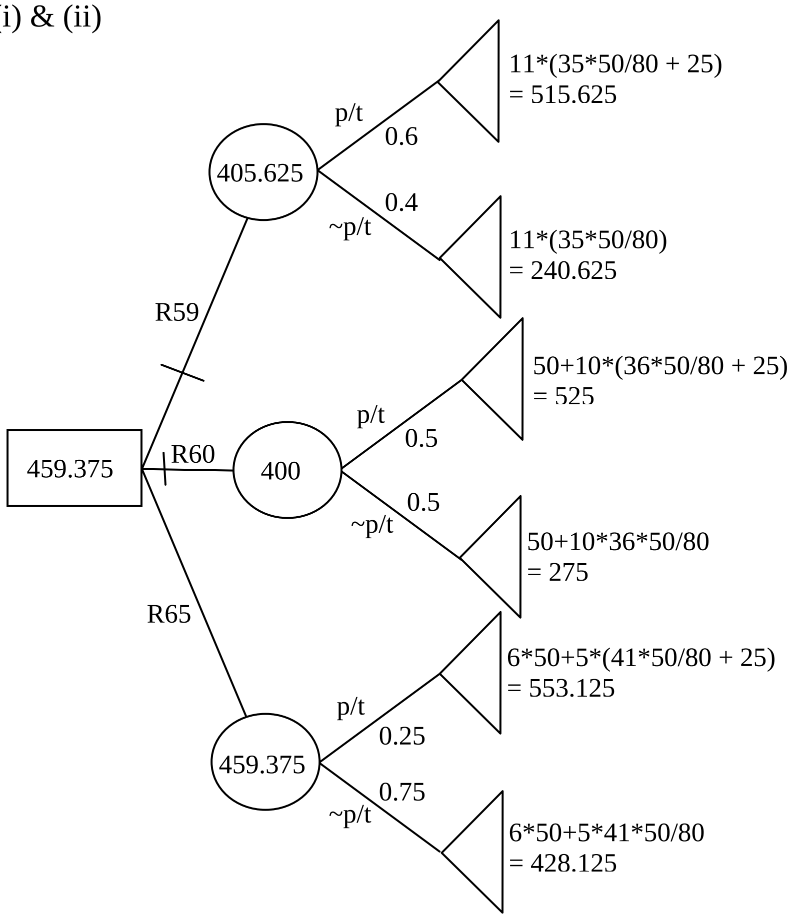
	1	2	3	4	5	6
1	5	5	5	5	5	6
2	5	4	4	4	5	5
3	4	4	4	4	4	4
4	2	2	3	2	2	2
5	1	2	2	2	2	6
6	1	5	5	5	5	1

not part of answer

(iii) cont It has found all shortest times and corresponding routes. Shortest time from x to y is in x row and y column of time matrix. For route look in x row and y column of route matrix. This gives first vertex “en route”. Repeat, looking in row corresponding to the current “en route” vertex and the y column, until the “en route” vertex is y. Shortest time from 3 to 6 is 19. Corresponding route is 3 to 4 to 2 to 5 to 6.	B1 B1 B1 B1 B1
(iv) On time matrix – 1(7)5(6)2(2)4(3)3(19)6(8)1 so 45 From route matrix – 1 5 2 4 3 4 2 5 6 1	B1 B1
(v) Lower bound = 7 + 8 + 19 = 34	M1 A1 7 + 8 A1 19
(vi) 82 + 8 = 90 minutes	B1

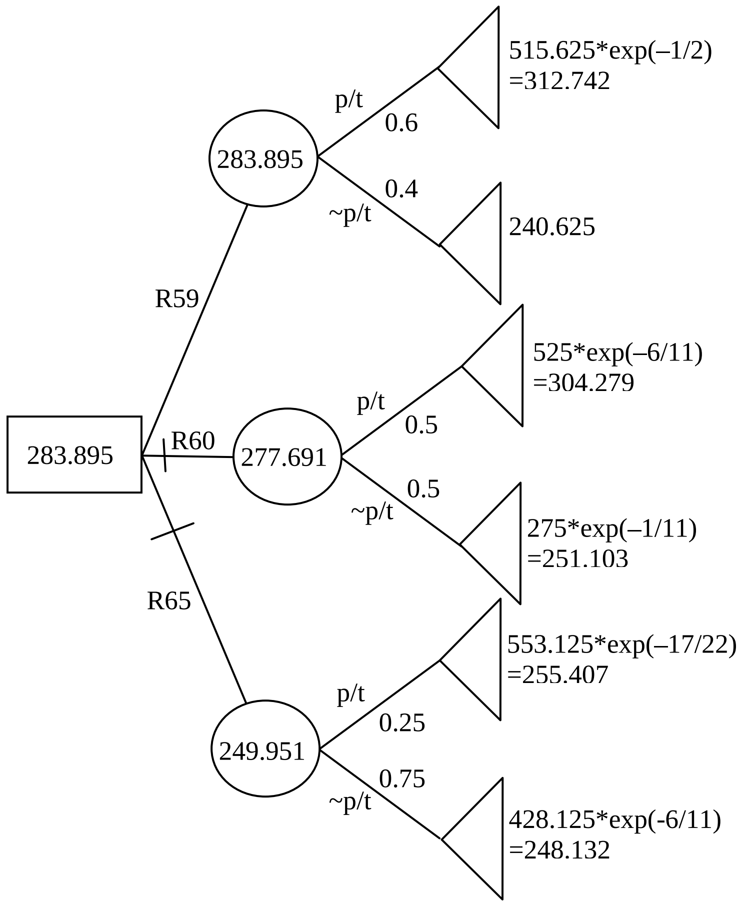
3.

(i) & (ii)



Retire at 65, EMV = 459375

M1	3-way split
A1	choice node
M1	2-way splits
A1	chance nodes
B1	pension calculations
M1	(income + pension) × time
A1	retire at 59
A1	retire at 60
A1	retire at 65
M1	EMV's
A1√	
M1	choice
A1√	

(iii) p's (in order) $\frac{1}{2}$ 0 $\frac{6}{11}$ $\frac{1}{11}$ $\frac{17}{22}$(given) $\frac{6}{11}$	M1 A1
	M1 final utilites A1 cao M1 expecteds A1 √
Retire at 59.	B1 choice

4.

(i) Max 180x + 90y + 110z — B1

st 2x + 5y + 3z ≤ 30 — B1

4x + y + 2z ≤ 24 — B1

(ii)

P	x	y	z	s1	s2	RHS
1	–180	–90	–110	0	0	0
0	2	5	3	1	0	30
0	4	1	2	0	1	24
1	0	–45	–20	0	45	1080
0	0	4.5	2	1	–0.5	18
0	1	0.25	0.5	0	0.25	6
1	0	0	0	10	40	1260
0	0	1	4/9	2/9	–1/9	4
0	1	0	7/18	–1/18	5/18	5

M1 initial tableau
A1

M1 first iteration
A1

M1 second iteration
A1

Identification of basic variables + values — B1 B1

(iii) Over two weeks (x = 3 and z = 18) — B1

(iv) Degeneracy (technical term not required) – objective planes are parallel to boundary line.

B1 same obj value
B1 line of solutions

(v)

A	P	x	y	z	s1	s2	s3	s4	a	RHS
1	0	1	1	0	0	0	0	–1	0	7
0	1	–180	–90	–110	0	0	0	0	0	0
0	0	2	5	3	1	0	0	0	0	30
0	0	4	1	2	0	1	0	0	0	24
0	0	1	1	0	0	0	1	0	0	7
0	0	1	1	0	0	0	0	–1	1	7

B1 = → ≤ + ≥
B1 ≤ row
B1 ≥ row
B1 new objective
B1 minimise A

or

P	x	y	z	s1	s2	s3	s4	a	RHS
1	–M–180	–M–90	–110	0	0		M	0	–7M
0	2	5	3	1	0	0	0	0	30
0	4	1	2	0	1	0	0	0	24
0	1	1	0	0	0	1	0	0	7
0	1	1	0	0	0	0	–1	1	7

B1 = → ≤ + ≥
B1 ≤ row
B1 ≥ row
B1 new objective
B1 maximise P

(vi) Point is on the line – gives £1260 profit — B1 (either)

4772, June 2011, Markscheme

1.

(a) To not discontinue, i.e. to continue. "Will the minister not …" is a form of words in which the negation is not intended.	B1 to continue B1 double negation B1 understanding ~~ B1 language confusion	
(b)(i) [(A∧B) ∨ (A∧C)] ∧ [D ∨ (E∧~F)] ⇔[(A∨(A∧C)) ∧ (B∨(A∧C))] ∧ [(D∨E) ∧ (D∨~F)] ⇔(A∨(A∧C)) ∧ (B∨(A∧C)) ∧ [(D∨E) ∧ (D∨~F)] ⇔A ∧ [(B∨A) ∧ (B∨C)] ∧ (D∨E) ∧(D∨~F) ⇔ A ∧(B∨A) ∧ (B∨C) ∧ (D∨E) ∧(D∨~F) ⇔ A ∧ (B∨C) ∧ (D∨E) ∧(D∨~F)	M1 uses distributive rule A1 distributive rule A1 "distributive rule" M1 uses associative rule A1 associative rule A1 "associative rule" B1 rest correct	 ft ft SC if refers to "absorption" then +1
or A ∧ (B∨C) ∧ (D∨E) ∧(D∨~F) ⇔[A ∧ (B∨C)] ∧ (D∨E) ∧ (D∨~F) ⇔[A ∧ (B∨C)] ∧ [(D∨E) ∧ (D∨~F)] ⇔[(A∧B) ∨ (A∧C)] ∧ [(D∨E) ∧ (D∨~F)] ⇔[(A∧B) ∨ (A∧C)] ∧ [D ∨ (E∧~F)]	M1 uses distributive rule A1 distributive rule A1 "distributive rule" M1 uses associative rule A1 associative rule A1 "associative rule" B1 rest correct	 ft ft
(ii) Out, LBW! Either first square bracket and second square bracket, or all 4 conditions are satisfied	B1 "out" B1 using either test	
(iii) Can't have D and E both true at the same time. Logic still valid. Logic not concerned with consistency of input, only whether out or not.	B1 B1 B1	

2.

(i)

	1	**2**	**3**	**4**
1	∞	15	7	2
2	15	∞	2	2
3	7	2	∞	10
4	2	2	10	∞

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

B1 time matrix
B1 route matrix

	1	**2**	**3**	**4**
1	∞	15	7	2
2	15	30	2	2
3	7	2	14	9
4	2	2	9	4

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	1	3	4
3	1	2	1	1
4	1	2	1	1

M1 replacing an ∞ by a correct value
A1

	1	**2**	**3**	**4**
1	30	15	7	2
2	15	30	2	2
3	7	2	4	4
4	2	2	4	4

	1	**2**	**3**	**4**
1	2	2	3	4
2	1	1	3	4
3	1	2	2	2
4	1	2	2	1

A1 ft

	1	**2**	**3**	**4**
1	14	9	7	2
2	9	4	2	2
3	7	2	4	4
4	2	2	4	4

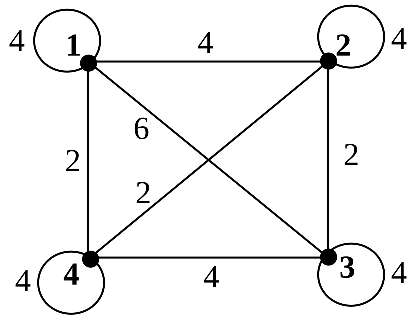
	1	**2**	**3**	**4**
1	3	3	3	4
2	3	3	3	4
3	1	2	2	2
4	1	2	2	1

A1 ft

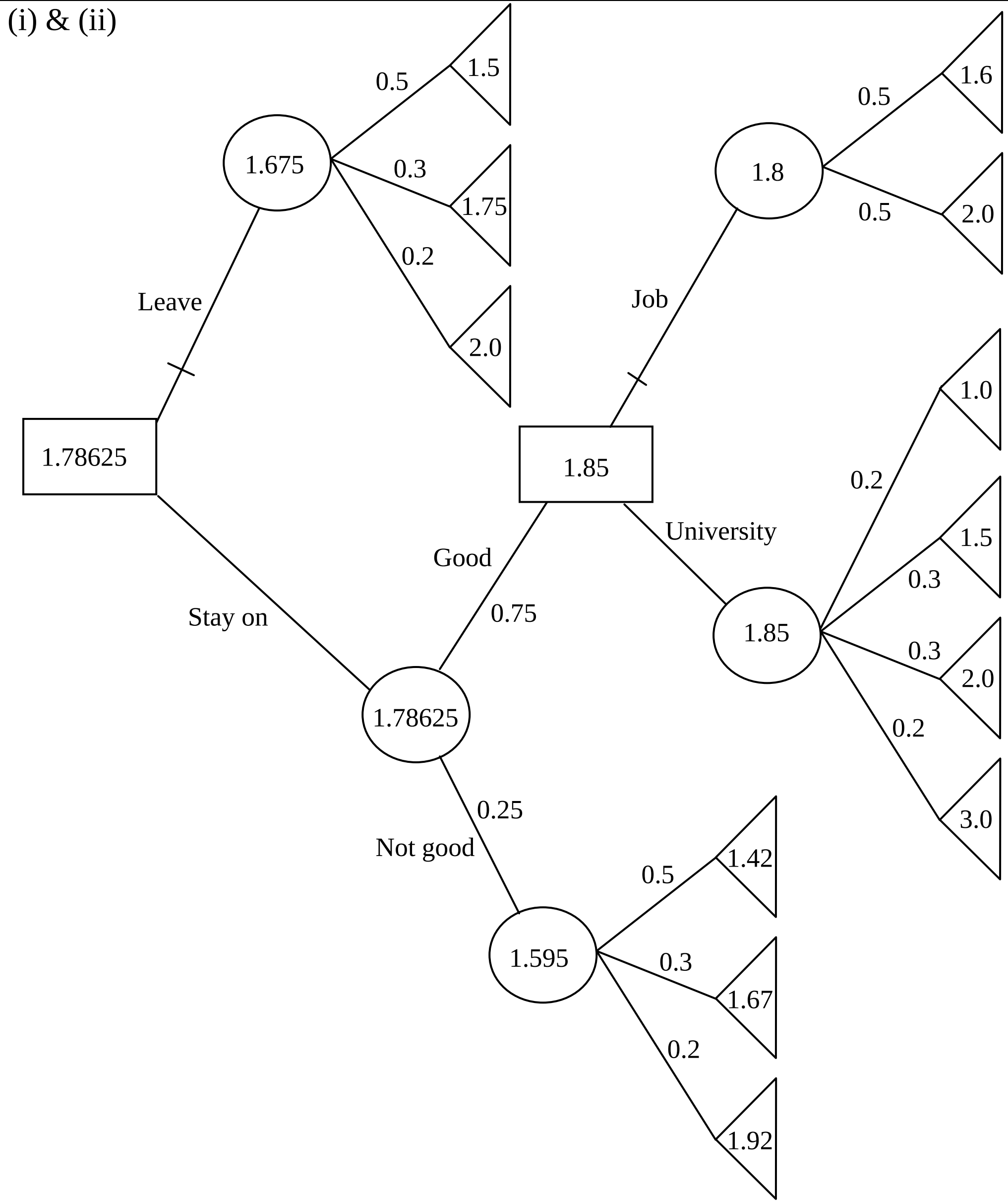
	1	**2**	**3**	**4**
1	4	4	6	2
2	4	4	2	2
3	6	2	4	4
4	2	2	4	4

	1	**2**	**3**	**4**
1	4	4	4	4
2	4	3	3	4
3	2	2	2	2
4	1	2	2	1

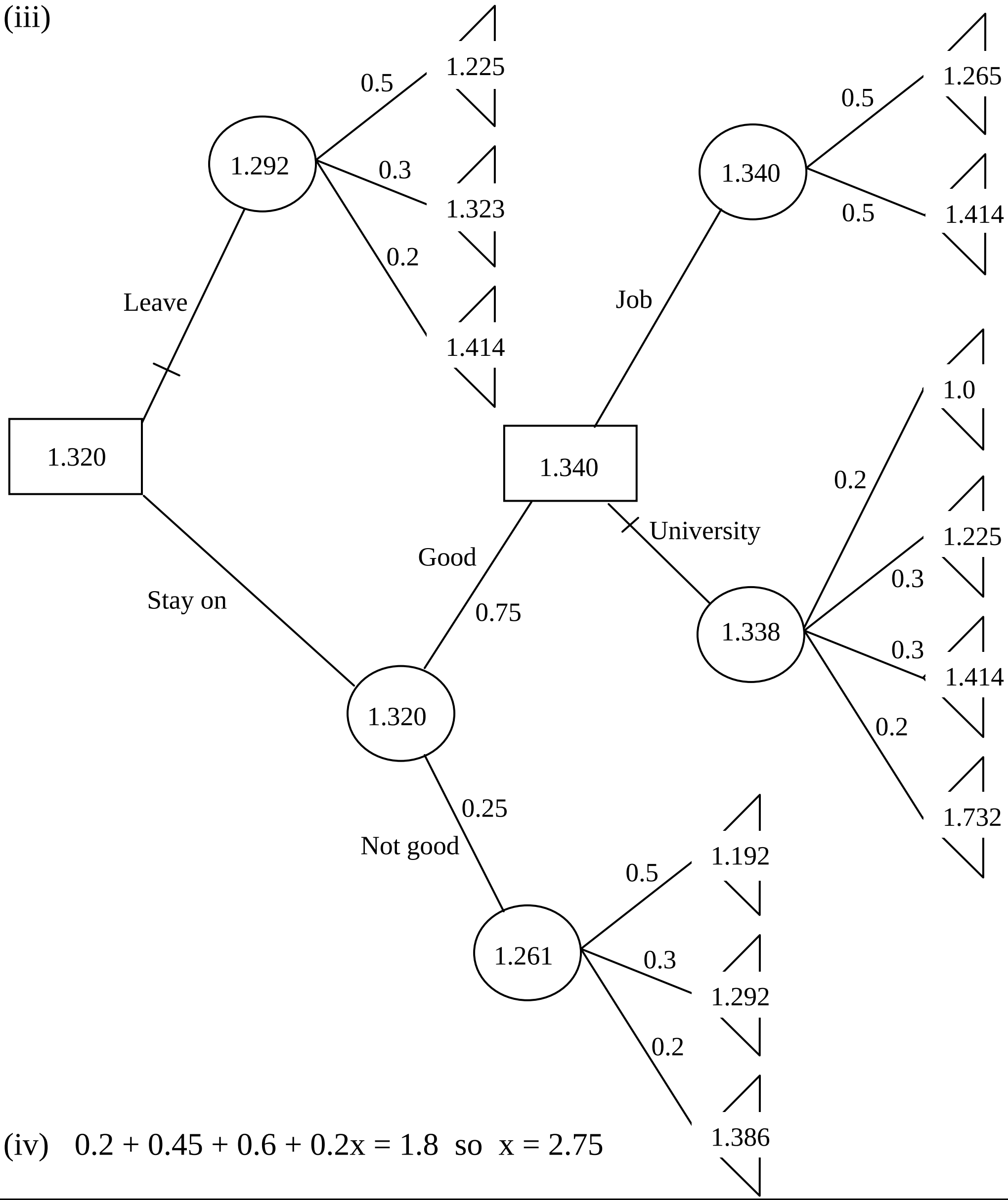
A1 entries other than row 3 col 1 of route matrix ... ft
B1 row 3 col 1 of route matrix ... cao

(ii) 	B1 ft	
(iii) Upper – nearest neighbour – e.g. 2+2+2+6 = 12	M1 nearest neighbour A1	mention of nearest neighbour or a nearest neighbour computation allow 2+2+2+7=13 etc for working in original network
Lower – e.g. "delete" **1**, and compute (2+2)+2+4 = 10	M1 delete a vertex A1 rest of computation	needs to be consistent with above
(iv) e.g. if the requirement is for part loads, and deliver to one department en route to another, then might save time. e.g. if the requirement is for part whole loads then might not be relevant.	B1 B1	answer should be valid and refer to the specific situation of the DAA
(v) A directed network.	B1	

3.

(i) & (ii)		
	B1 stay on/leave B1 leave B1 good A/not good B1 not good B1 job/uni B1 job B1 uni B1 leave computation B1 job computation B1 uni computation B1 good comp B1 not good comp B1 good/not good B1 stay on for 1.78625	decision node chance node with 3 branches chance node chance node with 3 branches decision node chance node with 2 branches chance node with 4 branches cao cao cao ft cao ft cao

3 (cont)

(iii) 	M1 utilities of outcomes A1 cao M1 computing backwards A1 ft	
(iv) 0.2 + 0.45 + 0.6 + 0.2x = 1.8 so x = 2.75	M1 A1 cao	equation with 0.2 x or division by 0.2

4.

(i) Definition of variables Max 5x + 9y + 15z st x + 2y + 4z ≤ 60 15x + 25y + 40z ≤ 700	B1 B1 objective B1 constraints	needs to say "number of"
(ii) (tableau below)	M1 initial tableau A1 ft M1 first iteration A1 ft M1 second iteration A1 ft	two slack variables identifying correct pivot identifying correct pivot
Identification of basic variables (y and z) + values (inc objective)	B1 ft B1 ft	

P	x	y	z	s1	s2	RHS
1	–5	–9	–15	0	0	0
0	1	2	4	1	0	60
0	15	25	40	0	1	700
1	–5/4	–3/2	0	15/4	0	225
0	1/4	1/2	1	1/4	0	15
0	5	5	0	–10	1	100
1	1/4	0	0	3/4	3/10	255
0	–1/4	0	1	5/4	–1/10	5
0	–1	1	0	–2	1/5	20

(iii)

A	P	x	y	z	s1	s2	s3	a	RHS
1	0	1	0	0	0	0	–1	0	5
0	1	1/4	0	0	3/4	3/10	0	0	255
0	0	–1/4	0	1	5/4	–1/10	0	0	5
0	0	1	1	0	–2	1/5	0	0	20
0	0	1	0	0	0	0	–1	1	5
1	0	0	0	0	0	0	0	–1	0
0	1	0	0	0	3/4	3/10	1/4	–1/4	253.75
0	0	0	0	1	5/4	–1/10	–1/4	1/4	6.25
0	0	0	1	0	–2	1/5	1	–1	15
0	0	1	0	0	0	0	–1	1	5

B1 ≥ row
B1 new objective

M1 pivot
A1 objectives cao
A1 constraints cao for basic variables

If from scratch, then M1 for first pivot, A1 for final objective row(s) and A1 for final constraint rows.

or

P	x	y	z	s1	s2	s3	RHS
1	–M+1/4	0	0	3/4	3/10	M	255–5M
0	–1/4	0	1	5/4	–1/10	0	5
0	1	1	0	–2	1/5	0	20
0	1	0	0	0	0	–1	5
1	0	0	0	3/4	3/10	1/4	253.75
0	0	0	1	5/4	–1/10	–1/4	6.25
0	0	1	0	–2	1/5	1	15
0	1	0	0	0	0	–1	5

or (same scheme)

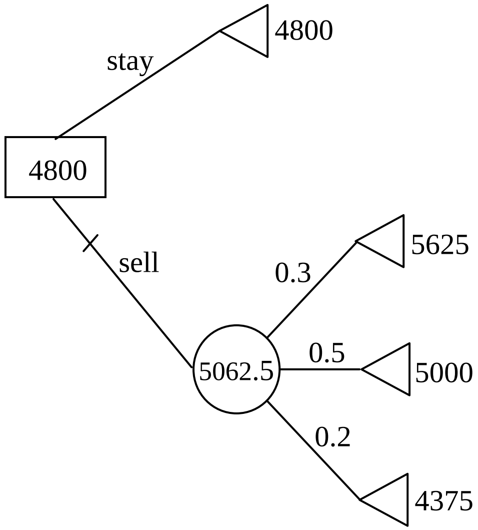
B1 ≥ row
B1 new objective

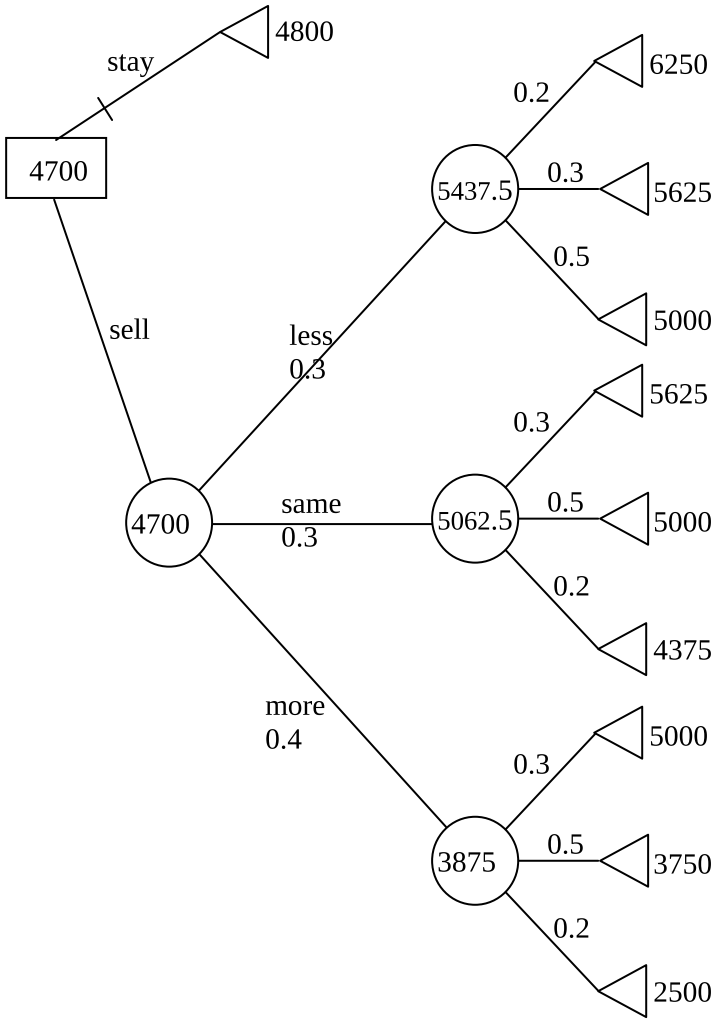
M1 pivot
A1 objective cao
A1 constraints cao for basic variables

(iv) 5, 15 and 6 at £250000 — B1

(v) 8, 12 and 7 is feasible and gives £253000 — B1 B1
IP solution need not be "near" to LP solution — B1

Question			Answer	Marks	Guidance	
1	(a)	(i)	She should not tick the first box. She should tick the second box.	B1 B1 **[2]**		
1	(a)	(ii)	She should tick both boxes.	B1B1 **[2]**		
1	(a)	(iii)	eg To tick neither box would be contradictory, confirming that it is original, but having reason to believe that it is not.	M1 A1 **[2]**	looking at "neither" case, or equivalent.	
1	(b)		eg I – lunch in Italy F – foggy T – top lift not working $((F \vee T) \Rightarrow I) \Leftrightarrow (\sim F \Rightarrow \sim I)$ (ignore $\sim F \Rightarrow I$ if included) 0 1 1 1 1 0 10 0 01	M1 A1 A1 A1 A1 A1 **[6]**	identification of propositions Angus's statement Chloe's statement equivalence 0/1s for Angus and Chloe 0 for equivalence	SC B1 for examining not foggy and lift not working
1	(c)		$(X \vee \sim Y) \Rightarrow Z$ $\sim Z \Rightarrow \sim(X \vee \sim Y)$ contrapositive $\sim Z \Rightarrow \sim X \wedge Y$ De Morgan $\sim Z$ given $\sim X \wedge Y$ Y	M1A1 B1 B1 **[4]**	deducing Y from $\sim X \wedge Y$	

Question			Answer	Marks	Guidance	
2	**(i)**			B1 M1 A1 M1 A1 B1 **[6]**	decision node chance node 3-split rent costs (a correct value) –1 each error decision	
2	**(ii)**		$0.3\times\sqrt{5625}+0.5\times\sqrt{5000}+0.2\times\sqrt{4375}=71.08$ $\sqrt{4800}=69.28$, so no change	M1A1 A1 **[3]**		

Question		Answer	Marks	Guidance	
	(iii)		M1 A1 M1 A1 M1 A1 B1 **[7]**	new chance node 3-split "less" (a correct value) "more" (a correct value) 4700	 (follow through)

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>3</td><td>(i)</td><td>(a)</td><td>
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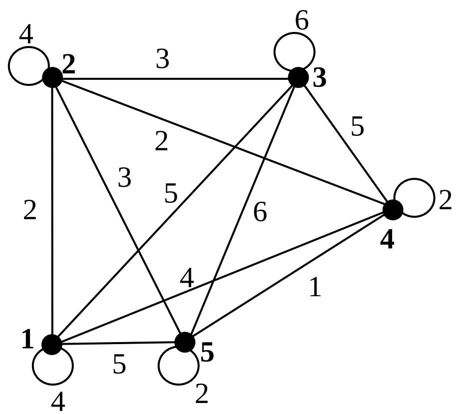
</table>
</td><td>M1
A2

M1
A2

[6]</td><td>(–1 each error)

(–1 each error)</td><td></td></tr>
<tr><td>3</td><td>(i)</td><td>(b)</td><td>$3 \to 2 \to 4 \to 5$</td><td>B1
[1]</td><td></td><td></td></tr>
<tr><td>3</td><td>(i)</td><td>(c)</td><td>
</td><td>M1
A1

[2]</td><td>complete</td><td></td></tr>
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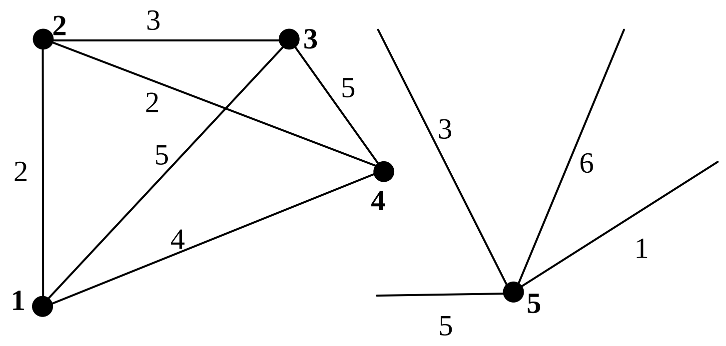
Question			Answer	Marks	Guidance	
3	(ii)		 Lower bound = (2 + 2 + 3) + (1 + 3) = 11	M1 A1 A1 **[3]**	delete vertex **5** plus arcs (2 + 2 + 3) 1 + 3	
3	(iii)		**1** → **2** → **4** → **5** → **3** → **1** of total length 16	M1A1B1 **[3]**	M1 for **1** → **2** → **4** → **5**	
3	(iv)		**1** → **2** → **4** → **5** → (**4** → **2**) → **3** → (**2**) → **1**	M1A1 **[2]**	SC 1 id seen elsewhere	
3	(v)		eg **1** → **2** → **3** → **2** → **4** → **3** → **5** → **4** → **5** → **1** Length = 32	M1A1 B1 **[3]**	2→3 or 5→4 repeated for M1	
4	(i)		Let x be the number of maths books produced … Line 1 ⇔ max $6x + 3y + 7z$ (10 − 4 = 6 etc.) Line 2 ⇔ $2x + 1.5y + 2.5z \le 10000$ (printing time) Line 3 ⇔ $x + 0.5y + 1.5z \le 7500$ (packing time) Line 4 ⇔ $300x + 200y + 400z \le 2000000$ (storage space)	B1 B1 B1 B1 B1 **[5]**	variable defs.“number of” objective constraints	

Question		Answer	Marks	Guidance
4	(ii)	(see tableau below)		

P	*x*	*y*	*z*	s1	s2	s3	RHS	Marks	Guidance
1	–6	–3	–7	0	0	0	0		
0	2	1.5	2.5	1	0	0	10000		
0	1	0.5	1.5	0	1	0	7500		
0	300	200	400	0	0	1	2000000	B1	pivot (pivot on *x* OK)
1	–0.4	1.2	0	2.8	0	0	28000		
0	0.8	0.6	1	0.4	0	0	4000	M1	a correct row or column
0	–0.2	–0.4	0	–0.6	1	0	1500	A1	
0	–20	–40	0	–160	0	1	400000	B1	pivot
1	0	1.5	0.5	3	0	0	30000		
0	1	0.75	1.25	0.5	0	0	5000	M1	a correct row or column
0	0	–0.25	0.25	–0.5	1	0	2500	A1	
0	0	–25	25	–150	0	1	500000		

Answer	Marks
Produce 5000 maths books –	B1
at a profit of £30000.	B1
(2500 packing minutes spare and 0.5 m^3 storage space spare.)	
	[8]

Question		Answer	Marks	Guidance
4	(iii)	£1.50 and 50p respectively.	B1B1 **[2]**	

Question		Answer	Marks	Guidance
4	**(iv)**	(tableau below)	B1	new objective
			B1	surplus + artificial
			B1	new constraint
		Minimise Q until 0 (if feasible).	B1	
		Then drop Q and a and proceed to optimum.	B1	
			[5]	

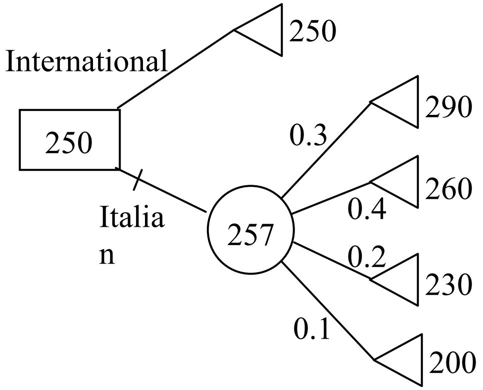
Q	*P*	*x*	*y*	*z*	s1	s2	s3	s4	a	RHS
1	0	0	1	0	0	0	0	–1	0	1000
0	1	–6	–3	–7	0	0	0	0	0	0
0	0	2	1.5	2.5	1	0	0	0	0	10000
0	0	1	0.5	1.5	0	1	0	0	0	7500
0	0	300	200	400	0	0	1	0	0	2000000
0	0	0	1	0	0	0	0	–1	1	1000

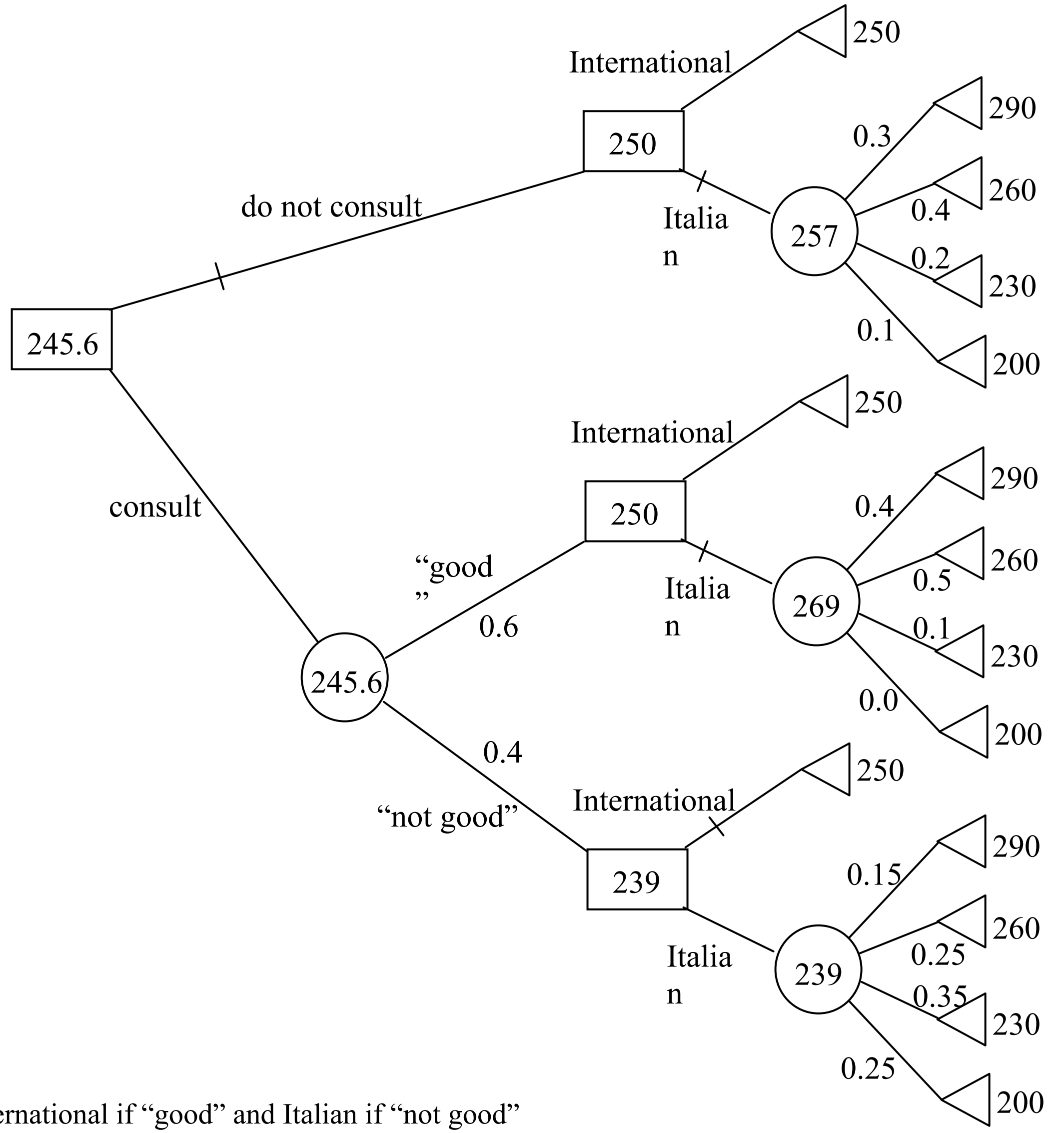
Question			Answer	Marks	Guidance
1	(a)	(i)	Wrong ... only refers to one pair of nodes	B1 B1	
1	(a)	(ii)	Right	B1	
1	(a)	(iii)	Wrong ... only refers to one pair of arcs	B1 B1	
1	(a)	(iv)	Wrong ... arguably general in reference to nodes, but refers to route rather than arc	B1 B1 **[7]**	
1	(b)	(i)	(table below)	M1 A3	8 combinations –1 each error

1	2	3	light
up	up	up	on
up	up	down	on
up	down	up	on
up	down	down	off
down	up	up	off
down	up	down	off
down	down	up	on
down	down	down	on

Question			Answer	Marks	Guidance
1	(b)	(ii)	No. e.g. if 1 and 2 both up, then 3 has no effect.	B1 **[5]**	Needs valid counter-example
1	(c)		(table below)	M1 A3 **[4]**	8 rows –1 each error

(A	∧	(B	∨	C))	∨	~	(~	A	∨	(B	∧	C))
0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	1	1	0	0	1	0	1	0	0	1
0	0	1	1	0	0	0	1	0	1	1	0	0
0	0	1	1	1	0	0	1	0	1	1	1	1
1	0	0	0	0	1	1	0	1	0	0	0	0
1	1	0	1	1	1	1	0	1	0	0	0	1
1	1	1	1	0	1	1	0	1	0	1	0	0
1	1	1	1	1	1	0	0	1	1	1	1	1

Question			Answer	Marks	Guidance
2	(i)			B1 M1 A1 M1 A1	decision node chance node 4 possibilities costs (90, 60, 30, 0 OK) cao 257
			Buy international and pass.	B1 **[6]**	

Question			Answer	Marks	Guidance
2	(ii)			B1 B1 B1 B1 B1 B1 B1 B1	new decision node "do not consult" branch "consult" chance node EMV at chance node cao EMV at "good" decision node cao 269 at chance node cao EMV "not good" decision node cao 239 at chance node cao
			Consult Buy international if "good" and Italian if "not good"	B1 B1 **[10]**	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>3</td><td>(i)</td><td>(A)</td><td>
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<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><th>1</th><td>4</td><td>5</td><td>8</td><td>2</td><td>3</td></tr>
<tr><th>2</th><td>5</td><td>6</td><td>7</td><td>3</td><td>8</td></tr>
<tr><th>3</th><td>8</td><td>7</td><td>12</td><td>6</td><td>9</td></tr>
<tr><th>4</th><td>2</td><td>3</td><td>6</td><td>4</td><td>5</td></tr>
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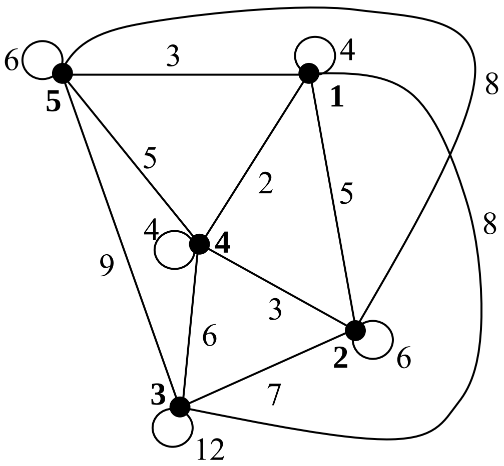
<tr><th>5</th><td>1</td><td>1</td><td>3</td><td>1</td><td>1</td></tr>
</table>
</td><td>M1
A2

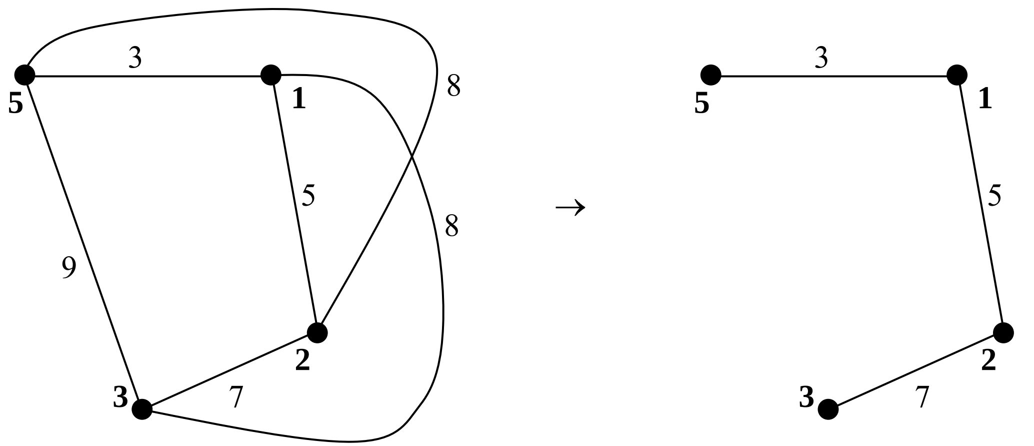
M1
A2

[6]</td><td>distances 1→1 and 1→2
rest OK

route 5→2
rest OK</td></tr>
<tr><td>3</td><td>(i)</td><td>(B)</td><td>5 → 1 → 4 → 2</td><td>B1
[1]</td><td>cao</td></tr>
<tr><td>3</td><td>(i)</td><td>(C)</td><td>
</td><td>M1
A1

[2]</td><td>complete, inc loops
cao</td></tr>
<tr><td>3</td><td>(ii)</td><td></td><td>4 → (2) → 1 → (3) → 5 → (8) → 2 → (7) → 3 → (6) → 4

Length = 26</td><td>M1
A1
B1
[3]</td><td>4 → 1 → 5
complete, inc return to 4
cao</td></tr>
<tr><td>3</td><td>(iii)</td><td></td><td>4 → 1 → 5 → (1 → 4) → 2 → 3 → 4</td><td>B1
[1]</td><td></td></tr>
<tr><td>3</td><td>(iv)</td><td></td><td>Starting at 1, 2 or 5 gives an HC of length 24.</td><td>B1
[1]</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance
3	(v)		 lower bound = 15 + 2 + 3 = 20	M1 A1 B1 **[3]**	3-arc connector 15 + 2 + 3
3	(vi)		odd vertices are **1**, **2**, **3**, **5** Pairings (**1**,**2**) and (**3**,**5**) ... 5+9 = 14 (**1**,**3**) and (**2**,**5**) ... 8+8 = 16 (**1**,**5**) and (**2**,**3**) ... 3+7 = 10 So min length = 43 + 3 + 7 = 53 eg. route ... **1 5 1 2 3 2 4 3 5 4 1**	M1 A1 B1 **[3]**	must have indication of pairing odd vertices cao cao

Question			Answer	Marks	Guidance
4	(i)		materials $15b + 6c + 2f \le 100$ time $4b + 2c + \frac{1}{2}f \le 30$	B1 B1 **[2]**	cao cao
4	(ii)		see tableau (ii) below	B1 B1 **[2]**	objective ... cao rest ... cao
4	(iii)		see tableaux (iii) below Non-integer solution ($3\frac{1}{3}$ bowls and $8\frac{1}{3}$ candle holders) using all of budget and all available time, giving income of £225	B1 M1 A1 B1 M1 A1 B1 B1 **[8]**	pivot first iteration cao pivot second iteration cao solution ft resources and income cao
4	(iv)		e.g. see tableaux (iv) below Make 15 candleholders. Same income as before, but £10 materials remain (and integer solution this time).	M1 A1 A1 **[3]**	Might miss out "b" col. Any valid approach using simplex solution ft comment cao

4 (ii)

I	b	c	f	s1	s2	RHS
1	–30	–15	–3	0	0	0
0	15	6	2	1	0	100
0	4	2	$\frac{1}{2}$	0	1	30

4 (iii)

1	0	–3	1	2	0	200
0	1	$\frac{2}{5}$	$\frac{2}{15}$	$\frac{1}{15}$	0	$\frac{20}{3}$
0	0	$\frac{2}{5}$	$-\frac{1}{30}$	$-\frac{4}{15}$	1	$\frac{10}{3}$
1	0	0	$\frac{3}{4}$	0	$\frac{15}{2}$	225
0	1	0	$\frac{1}{6}$	$\frac{1}{3}$	–1	$\frac{10}{3}$
0	0	1	$-\frac{1}{12}$	$-\frac{2}{3}$	$\frac{5}{2}$	$\frac{25}{3}$

4 (iv)

I	b	c	f	s1	s2	RHS
1	0	–15	–3	0	0	0
0	15	6	2	1	0	100
0	4	2	$\frac{1}{2}$	0	1	30
1	30	0	$\frac{3}{4}$	0	$\frac{15}{2}$	225
0	3	0	$\frac{1}{2}$	1	–3	10
0	2	1	$\frac{1}{4}$	0	$\frac{1}{2}$	15

Question			Answer	Marks	Guidance
4	(v)		two-phase (tableau below)	B1 B1 B1	new objective bowls ≤ 4 bowls ≥ 4
			OR big-M (tableau below)	or B1 B1 B1	objective bowls ≤ 4 bowls ≥ 4
			Special case ... Candidates may ignore the instruction and set up an ordinary simplex with b excluded and with reduced resources of £40 and 14 hours.	SC2	–1 each error
				[3]	
4	(vi)		4 bowls, 6 candle holders and 2 key fobs. (Uses all of the budget. Leaves an hour to spare. Gives an income of £216.)	B1 **[1]**	
4	(vii)		There might be another solution with less income, but even less expenditure.	B1 **[1]**	

two-phase

A	I	b	c	f	s1	s2	s3	s4	a	RHS
1	0	1	0	0	0	0	0	–1	0	4
0	1	–30	–15	–3	0	0	0	0	0	0
0	0	15	6	2	1	0	0	0	0	100
0	0	4	2	$\frac{1}{2}$	0	1	0	0	0	30
0	0	1	0	0	0	0	1	0	0	4
0	0	1	0	0	0	0	0	–1	1	4

big-M

I	b	c	f	s1	s2	s3	s4	RHS
1	–30–M	–15	–3	0	0	0	M	–4M
0	15	6	2	1	0	0	0	100
0	4	2	$\frac{1}{2}$	0	1	0	0	30
0	1	0	0	0	0	1	0	4
0	1	0	0	0	0	0	–1	4