## 6690/01: Decision Mathematics D2

| Question number | Scheme | Marks |
|---|---|---|
| **1.** (a) | (By conservation of flow at $B$, $C$ and $D$) | B3, 2ft 1ft 0 |
| | $x = 11$ $y = 5$ $z = 12$ | |
| | ($\sqrt{x} - 6$) ($\sqrt{y} + 7$) | (3) |
| (b) | Flow is 31 | B1 |
| | (max flow = min cut), cut through $AB$, $AC$ and $SD$ | B1 (2) |
| | | **(5 marks)** |

**2.** (a)

| b.v | $x$ | $y$ | $z$ | $r$ | $s$ | Value | Raw ops | Marks |
|---|---|---|---|---|---|---|---|---|
| $z$ | $\frac{1}{2}$ | 0 | 1 | $\frac{1}{4}$ | 0 | 20 | $R_1 \div 4$ | M1 A1 |
| $s$ | 0 | 4 | 0 | $-\frac{1}{2}$ | 1 | 120 | $R_2 - 2R_1$ | M1 A1 ft |
| $P$ | 8 | −8 | 0 | 5 | 0 | 400 | $R_3 + 20R_1$ | A1 ft (5) |

| | | |
|---|---|---|
| (b) | $P + 8x - 8y + 5r = 400$ | B1 ft (1) |
| (c) | Not optimal since there is a negative number in the profit row | B1 ft (1) |
| | | **(7 marks)** |

**3.** (a)

| | $D$ | $E$ | $F$ |
|---|---|---|---|
| $A$ | 20 | 4 | |
| $B$ | | 26 | 6 |
| $C$ | | | 14 |

M1

A1 (2)

| | | |
|---|---|---|
| (b) | $S_A = 0$ $S_B = -1$ $S_c = 7$ | |
| | $D_D = 21$ $D_E = 24$ $D_F = 18$ | M1 |
| | $I_{13} = I_{AF} = 16 - 0 - 18 = -2$ | A1 |
| | $I_{21} = I_{BD} = 18 + 1 - 21 = -2$ | M1 |
| | $I_{31} = I_{CD} = 15 - 7 - 21 = -13$ * | A1 ft |
| | $I_{32} = I_{CE} = 19 - 7 - 24 = -12$ | A1 ft (5) |
| (c) | Eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-)$ $\theta = 14$ | M1 A1 ft |

| | $D$ | $E$ | $F$ |
|---|---|---|---|
| $A$ | 6 | 18 | |
| $B$ | | 12 | 20 |
| $C$ | 14 | | |

Cost £1384

A1 ft A1 (4)

**(11 marks)**

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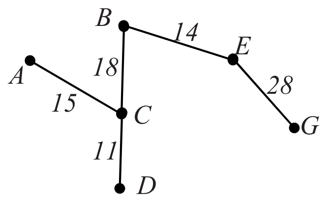
|---|---|---|
| **4.** (a) | Deleting $F$ leaves r.s.t | M1 |
| |  | |
| | r.s.t length = 86 | A1 |
| | So lower bound = 86 + 16 + 19 = 121 | |
| | | M1 A1 (4) |
| | Best $LB$ is 129 by deleting $C$ | B1 ft (1) |
| (b) | Add 33 to $BF$ and $FB$ | B1 |
| | Add 31 to $DE$ and $ED$ | B1 (2) |
| (c) | Tour, visits each vertex, order correct using table of least distances. | M1 A1 |
| | e.g $F\ C\ D\ A\ B\ E\ G\ F$ (actual route $F\ C\ D\ C\ A\ B\ E\ G\ F$) | A1 |
| | Upper bound of 138 km | A1 (4) |
| | | **(11 marks)** |
| **5.** | Let $x_{ij}$ be number of units transported from $i$ to $j$<br>Where $i \in \{W, X, Y\}$ and $j \in \{J, K, L]$<br>Warehouse Supermarket | B1 |
| | Objective minimise "c" = $3x_{WJ} + 6x_{WK} + 3x_{WL} + 5x_{XJ} + 8x_{XK} + 4x_{XL} + 2x_{YJ} + 5x_{YK} + 7x_{YL}$ | B1 |
| | Subject to $x_{WJ} + x_{WK} + x_{WL} = 34$ | M1 A1 |
| | $x_{XJ} + x_{XK} + x_{XL} = 57$ | |
| | $x_{YJ} + x_{YK} + x_{YL} = 25$ | A1 |
| | $x_{WJ} + x_{XJ} + x_{YJ} = 20$ | |
| | $x_{WK} + x_{XK} + x_{YK} = 56$ | |
| | $x_{WL} + x_{XL} + x_{YL} = 40$ | |
| | $x_{ij} \geq 0 \quad \forall\ i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ | B1 |
| | | **(6 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **6.** | | |

| Stage | State | Action | Value | Marks |
|---|---|---|---|---|
| 1 | *H* | *HK* | 18 * | |
| | *I* | *IK* | 19 * | M1 A1 |
| | *J* | *JK* | 21 * | |
| 2 | *F* | **FH** | min(16, 18) = 16 | M1 A1 A1 |
| | | *FI* | min (23, 19) = 19 * | |
| | | *FJ* | min(17, 21) = 17 | |
| | *G* | *GH* | min(20, 18) = 18 | |
| | | *GI* | min(15, 19) = 15 | A1 |
| | | *GJ* | min(28, 21) = 21 * | |
| 3 | *B* | *BG* | min(18, 21) = 18 * | M1 A1 ft |
| | *C* | *CF* | min(25, 19) = 19 * | |
| | | *CG* | min(16, 21) = 16 | |
| | *D* | *DF* | min(22, 19) = 19 * | |
| | | *DG* | min(19, 21) = 19 * | |
| | *E* | *EF* | min(14, 19) = 14 * | |
| 4 | *A* | *AB* | min(24, 18) = 18 | A1 ft |
| | | *AC* | min(25, 19) = 19 * | |
| | | *AD* | min(27, 19) = 19 * | |
| | | *AE* | min(23, 14) = 14 | |

Routes *A C F I K*, or *A D F I K* or *A D G J K* — A1 ft

**(9 marks)**

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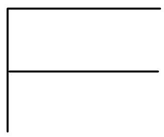
| Question number | Scheme | Marks |
|---|---|---|
| **7.** (a) | To maximise, subtract all entries from $n \geq 30$ | M1 |
| | e.g $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ | |
| |  Minimise uncovered element is 1    So $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ | M1 A2ft 1ft 0 |
| |   | M1 |
| | min. el = 2    min. el = 2 | |
| | $\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$    $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ | A2 ft 1 ft 0 |
| | $A - 2 \quad B - 4 \quad C - 3 \quad D - 1$ | |
| | $A - 3 \quad B - 4 \quad C - 1 \quad D - 2$ | M1 A1 ft (2) |
| (b) | £1160 000 | B2, 1, 0 (2) |
| (c) | Gives other solution | M1 A1 ft (2) |
| | | **(15 marks)** |

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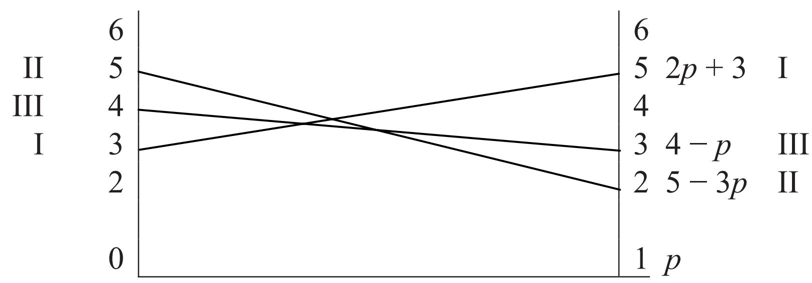
| Question number | Scheme | Marks |
|---|---|---|
| **8.** (a) | (see table below) | M1 A1 |

| | I | II | III | |
|---|---|---|---|---|
| I | 5 | 2 | 3 | Min 2 |
| II | 3 | 5 | 4 | Min 3 ← max |
| | Max 5 | 5 | 4 | |
| | | | min | |

| Question number | Scheme | Marks |
|---|---|---|
| | Since 3 ≠ 4 not stable | A1 (3) |
| (b) | Let $A$ play I with probability $p$ | |
| | Let $A$ play II with probability $(1 - p)$ | |
| | If $B$ plays I $A$'s gain are $5p + 3(1 - p) = 2p + 3$ | |
| | If $B$ plays II $A$'s gain are $2p + 5(1 - p) = 5 - 3p$ | M1 A1 (2) |
| | If $B$ plays III $A$'s gain are $3p + 4(1 - p) = 4 - p$ | |
| | | A 2, 1, 0 (2) |




| Question number | Scheme | Marks |
|---|---|---|
| | Intersection of $2p + 3$ and $4 - p$ $\quad p = \frac{1}{3}$ | M1 A1ft |
| | A should play I of time and II of time; value (to $A$) = 3 | A1 ft A1 ft (2) |
| | | **(15 marks)** |