Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Decision Mathematics 2 (6690/01)

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.	Since maximising, subtract all elements from some $n \geq 30$ and insert large numbers in cells A4 and B2 e.g. $\begin{bmatrix} 21 & 24 & 17 & 100 \\ 16 & 100 & 10 & 17 \\ 22 & 23 & 15 & 22 \\ 16 & 16 & 14 & 16 \end{bmatrix}$	M1 M1
	Reduce rows $\begin{bmatrix} 4 & 7 & 0 & 83 \\ 6 & 90 & 0 & 7 \\ 7 & 8 & 0 & 7 \\ 2 & 2 & 0 & 2 \end{bmatrix}$ then columns $\begin{bmatrix} 2 & 5 & 0 & 81 \\ 4 & 88 & 0 & 5 \\ 5 & 6 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	M1 A1
	$\begin{bmatrix} 0 & 3 & 0 & 79 \\ 2 & 86 & 0 & 3 \\ 3 & 4 & 0 & 3 \\ 0 & 0 & 2 & 0 \end{bmatrix}$	M1 A1ft
	either $\begin{bmatrix} 0* & 0 & 0 & 76 \\ 2 & 83 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 3 & 0* & 5 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0* & 3 & 2 & 79 \\ 0 & 84 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0* & 4 & 0 \end{bmatrix}$ then $\begin{bmatrix} 0* & 2 & 2 & 78 \\ 0 & 83 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0* & 5 & 0 \end{bmatrix}$	M1 A1ft A1
	Two optimal allocations: A 1 1 B 3 4 C 4 3 D 2 2	A1
		10 marks

Notes for Question 1

1M1: Subtracting from some $n \geq 30$, condone up to 2 errors.
2M1: Dealing with the A4 and B2 entries.
3M1: Reducing rows **and then** columns.
1A1: CAO
4M1: Double covered + e; one uncovered – e; and one single covered unchanged. 2 lines needed to 3 lines needed.
2A1ft: follow through on their previous table - no errors
5M1: One double covered + e; one uncovered – e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to optimal table).
3A1ft: Follow through on their previous table - no errors.
4A1: CSO on final table.
5A1: CAO – either one – this mark is dependent on all M marks being awarded.

Special Cases: Minimising (can score a max. of 5)

1M0 2M1 3M1 1A1 4M0 2A0 5M1 3A1ft 4A0 5A0

E.g.

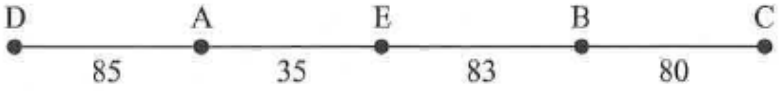
19	16	23	30		3	0	7	14		3	0	5	14
24	30	30	23	rows	1	7	7	0	columns	1	7	5	0
18	17	25	18		1	0	8	1		1	0	6	1
24	24	26	24		0	0	2	0		0	0	0	0

Then either

2	0*	4	14	or	2	0*	4	13
0	7	4	0*		1	8	5	0*
0*	0	5	1		0*	0	5	0
0	1	0*	1		0	1	0*	0

Not dealing with the – (can score a max. of 6)

1M1 2M0 3M1 1A0 4M1 2A1ft 5M1 3A1ft 4A0 5A0

Question Number	Scheme	Marks
2. (a)	A E F B C D A and A E F D B C A 35+75+88+80+108+85 = 471 35+75+88+100+80+130 = 508	M1 A1 A1 A1 **(4)**
(b)		
	RMST weight = 85 + 35 + 83 + 80 = 283 (seconds) Lower bound = 283 + 75 + 88 = 446 (seconds)	M1 A1 A1 (**3**)
(c)	446 ≤ time ≤ 471 [accept 446 < time ≤471]	B3,2,1,0 (**3**)
		10 marks

Notes for Question 2

a1M1: Nearest neighbour either A – E – F – B – C – D – or A – E – F – D – B – C – , condone lack of return to start. Accept 145623 or 156423 across top of table (numbers must be from NN **not** Prim).
a1A1: One route correctly stated, must return to A, accept link back to A.
a2A1: One route length correctly stated. Do not ISW if candidates then go on to double the route length in (a).
a3A1: Second route and its length correctly stated. Do not ISW if candidates then go on to double the route length in (a).
b1M1: Finding RST (maybe implicit) and using the correct two least lengths. Their RST must have only four arcs none of which are incident to F.
b1A1: RMST correct **or** list of arcs **or** 283 **or** 85 + 35 + 83 + 80 seen.
b2A1: CAO 446
c1B1ft: their 471 (must be a cycle) as an upper bound – allow recovery in this part.
c2B1ft: any indication of interval from their 446 (must come from six arcs) to their 471.
c3B1: 446 ≤ time ≤ 471 **or** 446 < time ≤471

Question Number	Scheme	Marks
3. (a)		

b.v	x	y	z	r	s	t	value	θ values
r	5	3	$-\frac{1}{2}$	1	0	0	2500	833.3
s	3	2	1	0	1	0	1650	825
t	$\frac{1}{2}$	-1	2	0	0	1	800	n/a
P	-40	-50	-35	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row ops
r	$\frac{1}{2}$	0	-2	1	$-\frac{3}{2}$	0	25	R1-3R2
y	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	825	R2 ÷ 2
t	2	0	$\frac{5}{2}$	0	$\frac{1}{2}$	1	1625	R3 + R2
P	35	0	-10	0	25	0	41250	R4 + 50R2

M1 A1
B1
M1 A1
(5)

b.v.	x	y	z	r	S	t	value	Row ops
r	$\frac{21}{10}$	0	0	1	$-\frac{11}{10}$	$\frac{4}{5}$	1325	R1 + 2R3
y	$\frac{11}{10}$	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	500	R2 - $\frac{1}{2}$R3
z	$\frac{4}{5}$	0	1	0	$\frac{1}{5}$	$\frac{2}{5}$	650	R3 ÷ $\frac{5}{2}$
P	43	0	0	0	27	4	47750	R4 + 10R3

M1 A1ft
B1
M1 A1
(5)

(b) $P = 47750$ $x = 0$ $y = 500$ $z = 650$ $r = 1325$ $s = t = 0$

B1ft B1
(2)

12 marks

Notes for Question 3

a1M1: Correct pivot located, attempt to divide row. If choosing negative pivot no marks.
a1A1: CAO pivot row correct **including change of b.v.**
a1B1: All row operations CAO – allow if given in terms of old row 2.
a2M1: (ft) The correct row operations used correctly at least once from their pivot, column x, z, s or value 'correct'.
a2A1: CAO on numbers (ignore row operations and b.v.)

a3M1: Their correct pivot located, attempt to divide row. If choosing negative pivot M0M0.
a3A1ft: Pivot row correct on follow through **including change of b.v**.
a2B1: All row operations CAO – allow if given in terms of old row 3.
a4M1: (ft) The correct row operations used correctly at least once from their pivot, column x, s, t or value 'correct'.
a4A1: CAO on numbers (ignore row operations and b.v.)

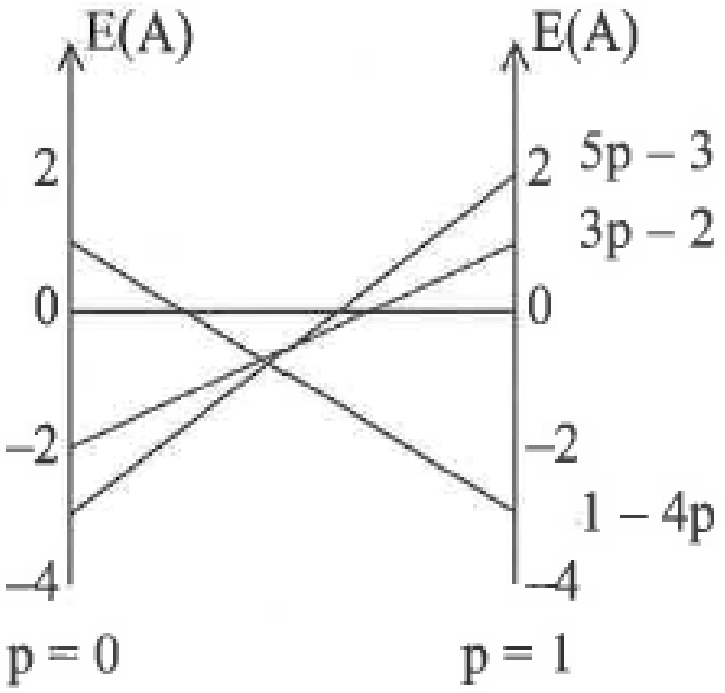
b1B1ft: Their correct values stated for at least P, x, y, z from their 'optimal' iteration. No negatives. Two M marks in (a) must have been awarded.
Allow implicit stating of P e.g. $P+43x+27s+4t = 47750$ with $x, s, t = 0$.
b2B1: CAO For all 7 variables correct and given explicitly.

Question Number	Scheme	Marks
4. (a)	Row mins {-3, -3} Column max {2, 2, 1, 1} Row maximin (-3) ≠ column minmax (1) so not stable	M1 A1 **(2)**
(b)	Column 4 dominates column 2 so delete column 2 **or** if B plays 2 A's expected winnings are $-p + 2(1-p)$ $(= 2-3p)$	B1

	B1	B3	B4
A1	2	1	-3
A2	-3	-2	1

Let A play 1 with probability p and 2 with probability $1-p$ — B1

If B plays 1 A's expected winnings are $2p - 3(1-p) = 5p - 3$
If B plays 3 A's expected winnings are $p - 2(1-p) = 3p - 2$ — M1 A1
If B plays 4 A's expected winnings are $-3p + (1-p) = 1 - 4p$



M1 A1

$5p - 3 = 1 - 4p$ — M1

$p = \frac{4}{9}$ — A1

A should play row 1 with probability $\frac{4}{9}$ and row 2 with probability $\frac{5}{9}$ — A1

(9)

11 marks

Notes for Question 4

a1M1: Finding row minimums and column maximums – condone one error.
a1A1: CAO states -3 $\neq$ 1 (or row (maximin) $\neq$ col (minimax)) and draws the conclusion.
b1B1: CAO Col 4 dominates Col 2 (maybe implied by later working) or correctly stating the expression for A's expected winnings if B plays 2 ($2 - 3p$).
b2B1: Defines p. Allow those who only define that A plays 1 with prob. p – no incorrect statements be generous.
b1M1: Setting up three probability equations, implicit definition of p.
b1A1: CAO (condone incorrect simplification).
b2M1: Either attempt at three lines (correct slant direction and relative intersection with 'axes') or four lines if no earlier domination, accept $p > 1$ or $p < 0$ here. Must be functions of p.
b2A1: CAO $0 \leq p \leq 1$, scaling correct and clear (or 1 line = 1), condone lack of labels. Rulers used.
b3DM1: Finding their correct optimal point, must have three (or four) lines and set up an equation to find $0 \leq p \leq 1$. Dependent on previous M mark. Must have at least three intersection points. Solving all three simultaneous equations and stating incorrect p is M0.
b3A1: CAO (must have scored all marks except b2B1 (define p mark) in this part).
b4A1: CAO

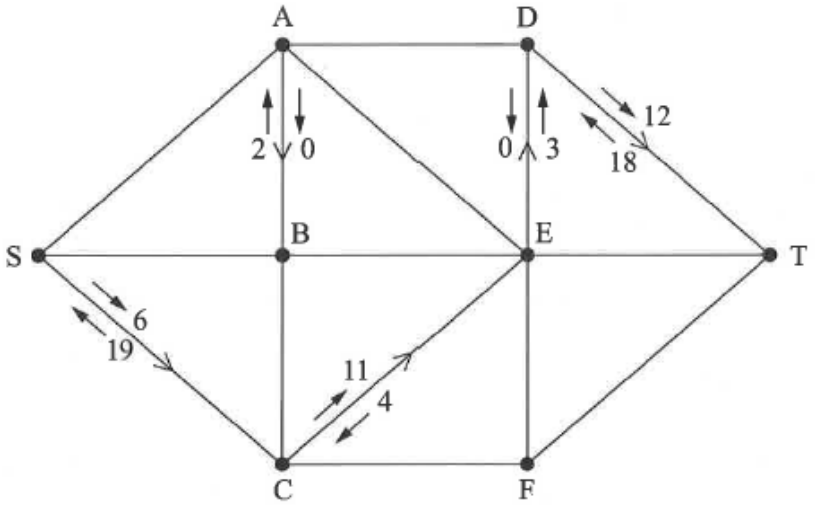
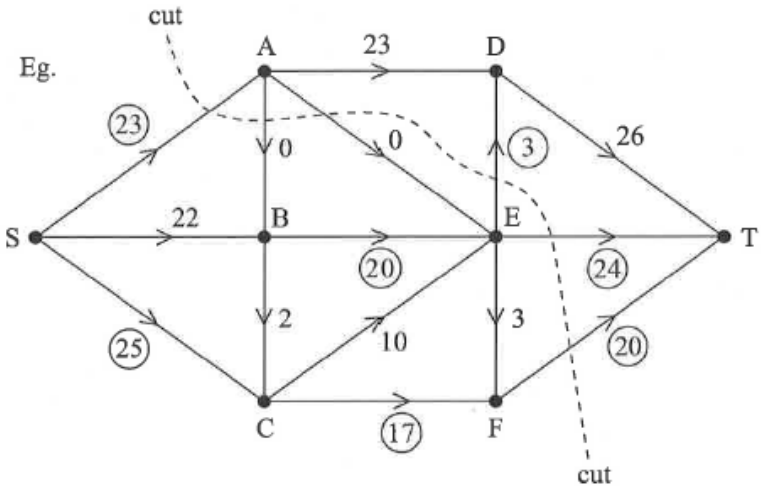
SC1: If column 4 is deleted in (b) candidates can earn a **maximum** of

B0 B1 M1 A0 M1 A0 M1 A0 A1 (max. of 5 out of 9 in part b)

The final A mark is for 'A should play row 1 with prob. 2/3 and row 2 with prob. 1/3.

SC2: If column 1 or 3 is deleted in (b), candidates can earn a **maximum** of

B0 B1 M1 A0 M1 A0 M0 A0 A0 (max. of 3 out of 9 in part b)

Question Number	Scheme	Marks
5.(a)	Initial flow = 62	B1 **(1)**
(b)		M1 A1 **(2)**
(c)	E.g. SCEDT - 3 SCBADT 2 SCEADT 2 SBADT 2 SCEADT – 3 SBCEDT 3 SBCEADT 1 SCEDT 2 SBADT – 2 SCEADT 3 SBADT 2 SCEADT 3	M1 A1 A1 A1 **(4)**
(d)		M1 A1 **(2)**
(e)	The cut through SA, AB, AE, DE, ET and FT has value 70 Value of the flow is 70 so by max flow – min cut theorem flow is maximal	DB1 DB1 **(2)**
		11 marks

Notes for Question 5
a1B1: CAO b1M1: Two numbers on each arc **and** at least two arcs **or** four numbers correct (so correct numbers with the correct arrows). b1A1: CAO do give bod since they might well cross these number out. c1M1: One valid flow augmenting route found and a value stated. c1A1: Flow increased by at least 2. c2A1: A second correct flow route and value correct. c3A1: CSO Flow increased by 8 and no more. d1M1: Consistent flow pattern ≥ 64 (check each node). One number only per arc. No unnumbered arcs. d1A1: CAO, showing flow of 70, must follow from their routes. e1DB1: Must have attempted (d) - at least one number on all but one arc, and either drawn or stated a cut. Cut may be drawn on any diagram. e2DB1: CSO – (d) fully correct (showing a correct flow of 70) and a correct cut. Must refer to max flow-min cut theorem – all four words.

Question Number	Scheme	Marks
6.	Let x_{ij} be the number of washing machines transported from i to j where $i \in \{P,Q,R\}$ and $j \in \{A,B,C,D\}$	B1
	The objective is to minimise $C = 11x_{PA} + 22x_{PB} + 13x_{PC} + 17x_{PD} + 21x_{QA} + 8x_{QB} + 19x_{QC} + 14x_{QD} + 15x_{RA} + 10x_{RB} + 9x_{RC} + 12x_{RD}$	B1 B1
	Subject to $x_{PA} + x_{PB} + x_{PC} + x_{PD} = 25$ or $\sum x_{Pj} = 25$	M1
	$x_{QA} + x_{QB} + x_{QC} + x_{QD} = 27$ or $\sum x_{Qj} = 27$	A1
	$x_{RA} + x_{RB} + x_{RC} + x_{RD} = 28$ or $\sum x_{Rj} = 28$ $x_{PA} + x_{QA} + x_{RA} = 18$ or $\sum x_{iA} = 18$ $x_{PB} + x_{QB} + x_{RB} = 16$ or $\sum x_{iB} = 16$ $x_{PC} + x_{QC} + x_{RC} = 20$ or $\sum x_{iC} = 20$ $x_{PD} + x_{QD} + x_{RD} = 26$ or $\sum x_{iD} = 26$	A1
	$x_{ij} \geq 0$	A1
		7 marks

Notes for Question 6

1B1: Variables defined correctly – withhold this mark if definition of x_{ij} or the elements of sets i and j are inconsistent with their later use in the objective function and constraints. Penalise poor variable choice, (AP etc.) here.
2B1: Minimise + an attempt at an objective with at least 5 correct terms.
3B1: Objective function correct (minimised not required for this mark).
1M1: At least 3 'correct' constraints listed with **unit** coefficients (accept = or any inequality for the M mark) – rhs values must be correct.
1A1: At least 3 correct constraints (accept consistent use of = or $\leq$ on at least 3).
2A1: At least 6 correct constraints (accept consistent use of = or $\leq$ on at least 6).
3A1: All 8 constraints correct (first seven constraints consistently either = or $\leq$ but final constraint must be ≥ 0).

Question Number	Scheme	Marks
7.	E.g.	

Stage	State	Action	Dest	Value	
July	2	1	0	1000 + 2000	= 3000*
(3)	1	2	0	500 + 2000	= 2500*
	0	3	0	2000	= 2000*
June	2	2	0	1000 + 2000	+ 2000 = 5000*
(4)		3	1	1000 + 2000	+ 2500 = 5500
		4	2	1000 + 2000 + 1000 + 3000 = 7000	
	1	3	0	500 + 2000	+ 2000 = 4500*
		4	1	500 + 2000 +1000	+ 2500 = 6000
	0	4	0	2000 + 1000 + 2000 = 5000*	
May	2	0	0	1000	+ 5000 = 6000*
(2)		1	1	1000 + 2000	+ 4500 = 7500
		2	2	1000 + 2000	+ 5000 = 8000
	1	1	0	500 + 2000	+ 5000 = 7500
		2	1	500 + 2000	+ 4500 = 7000*
		3	2	500 + 2000	+ 5000 = 7500
	0	2	0	2000	+ 5000 = 7000
		3	1	2000	+ 4500 = 6500*
		4	2	2000 + 1000 + 5000 = 8000	
April	2	2	0	1000 + 2000	+ 6500 = 9500*
(4)		3	1	1000 + 2000	+ 7000 =10000
		4	2	1000 + 2000 + 1000 + 6000 =10000	
	1	3	0	500 + 2000	+ 6500 = 9000*
		4	1	500 + 2000 + 1000 + 7000 =10500	
	0	4	0	2000 + 1000 + 6500 = 9500*	
March	0	3	0	2000	+ 9500 =11500*
(3)		4	1	2000 + 1000 + 9000 =12000	

Month	March	April	May	June	July
Number made	3	4	3	3	3

Total cost: £11500

1M1 1A1 **(2)**

2M1 2A1ft 3A1 **(3)**

3M1 4A1ft 5A1 **(3)**

4M1 6A1 **(2)**

5M1 7A1 **(2)**

1B1

2B1 **(2)**

14 marks

Notes for Question 7

ALL M marks - Must bring earlier optimal results into calculations. Ignore extra rows. Must have right 'ingredients' (– storage costs, overheads, additional space costs) at least once per stage.

Penalise lack of * only once per question.

1M1: First stage completed. 3 rows, something in each cell.
1A1: CAO condone missing * here. No extra rows.
2M1: Second stage completed with 3 states and at least 6 rows. Bod if something in each cell.
2A1ft: Any 2 states correct. Ft for their * values or the correct * values.
3A1: CAO All 3 states correct. No missing/extra rows.
3M1: 3rd stage completed with 3 states and at least 9 rows. Bod if something in each cell.
4A1ft: Any state correct. Ft on their * values or the correct * values.
5A1: CAO All 3 states correct. No missing/extra rows.
4M1: 4th stage completed with 3 states and at least 6 rows. Bod if something in each cell.
6A1: CAO All 3 states correct. No missing/extra rows.
5M1: 5th stage completed with at least 2 rows. Bod if something in each cell.
7A1: CAO Final, state correct. No missing/extra rows.
1B1: CAO Must have earned all previous M marks.
2B1: CAO Must have earned all previous M marks.

Alt correct solution – adding the storage costs at start of each month

Stage	State	Action	Dest	Value	
July	2	1	0	2000 = 2000*	1M1
(3)	1	2	0	2000 = 2000*	1A1
	0	3	0	2000 = 2000*	
June	2	2	0	2000 + 2000 = 4000*	2M1
(4)		3	1	500 + 2000 + 2000 = 4500	
		4	2	1000 + 2000 + 1000 + 2000 = 6000	
	1	3	0	2000 + 2000 = 4000*	2A1ft
		4	1	500 + 2000 + 1000 + 2000 = 5500	
	0	4	0	2000 + 1000 + 2000 = 5000*	3A1
May	2	0	0	5000 = 5000*	3M1
(2)		1	1	500 + 2000 + 4000 = 6500	
		2	2	1000 + 2000 + 4000 = 7000	
	1	1	0	2000 + 5000 = 7000	
		2	1	500 + 2000 + 4000 = 6500*	4A1ft
		3	2	1000 + 2000 + 4000 = 7000	
	0	2	0	2000 + 5000 = 7000	
		3	1	500 + 2000 + 4000 = 6500*	5A1
		4	2	1000 + 2000 + 1000 + 4000 = 8000	
April	2	2	0	2000 + 6500 = 8500*	4M1
(4)		3	1	500 + 2000 + 6500 = 9000	
		4	2	1000 + 2000 + 1000 + 5000 = 9000	
	1	3	0	2000 + 6500 = 8500*	6A1
		4	1	500 + 2000 + 1000 + 6500 = 10000	
	0	4	0	2000 + 1000 + 6500 = 9500*	
March	0	3	0	2000 + 9500 = 11500*	5M1
(3)		4	1	500 + 2000 + 1000 + 8500 = 12000	7A1

Month	March	April	May	June	July	
Number made	3	4	3	3	3	1B1

Total cost: £11500 2B1