D2 JUNE 13
INT

Write your answers in the D2 answer book for this paper.

1. Four workers, Chris (C), James (J), Katie (K) and Nicky (N), are to be allocated to four tasks, 1, 2, 3 and 4. Each worker is to be allocated to one task and each task must be allocated to one worker.

The profit, in pounds, resulting from allocating each worker to each task, is shown in the table below. The profit is to be maximised.

	1	2	3	4
Chris	127	116	111	113
James	225	208	205	208
Katie	130	113	112	114
Nicky	228	212	203	210

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit. You must make your method clear and show the table after each stage.

(8)

(b) State which worker should be allocated to each task and the resulting total profit made.

(2)

(Total 10 marks)

	1	2	3	4
C	103	114	119	117
J	5	22	25	22
K	100	117	128	116
N	2	28	27	20

max so subtract each from 230

	1	2	3	4	
C	0	11	16	14	-103
J	0	17	20	17	-5
K	0	17	28	16	-100
N	0	26	25	18	-2

Reduce Rows

	1	2	3	4
C	0	0	0	0
J	0	6	4	3
K	0	6	12	2*
N	0	15	9	4
	x	-11	-16	-14

Reduce Columns

2 lines ⇒ not optimal

	1	2	3	4
C	2	0	0	0
J	0	4	2	1*
K	0	4	10	0
N	0	13	7	2

3 lines ⇒ not optimal

	1	2	3	4
C	3	0	0	0
J	0	3	1*	0
K	1	4	10	0
N	0	12	6	1

	1	2	3	4
C	4	0	0	1
J	0	2	0	0
K	1	3	9	0
N	0	11	5	1

(b)

Worker	Task
Chris	2 (116)
James	3 (205)
Katie	4 (114)
Nicky	1 (228)

Maximum total profit: £ 663

2. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

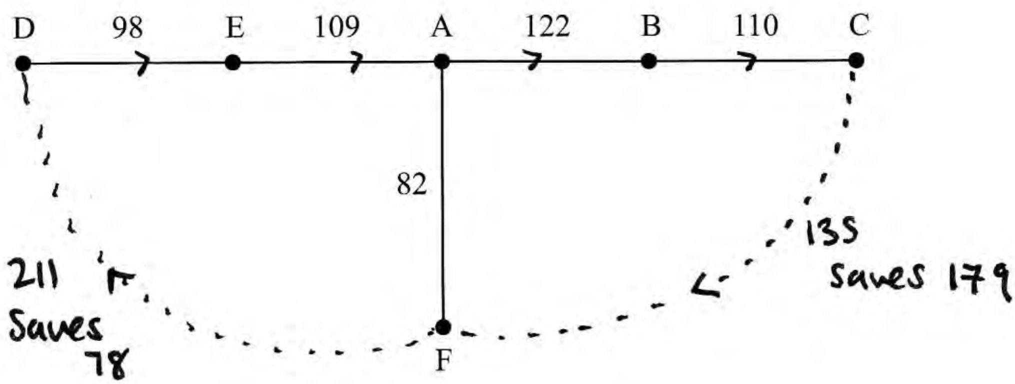
(1)

(Total 8 marks)

2.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

(a)



Initial UB = 2×521 = 1042 (need 233 (or more) in cuts)

CF −179
FD −78
785 = Improved upper bound

b) A − F − E − D − B − C − A
82 113 98 130 110 217

= 750 'better' upper bound.

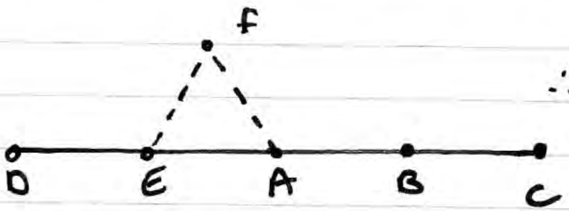
(c)

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

AE 109
ED 98
AB 122
BC 110

RMST = 439
+AF 82
+FE 113

634



∴ not viable.

∴ 634 < optimal length ≤ 750

3. Table 1 below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to four demand points 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	1	2	3	4	Supply
A	22	36	19	37	35
B	29	35	30	36	15
C	24	32	25	41	20
D	23	30	23	38	30
Demand	30	20	30	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.
Table 3 shows some of the improvement indices for this solution.

	1	2	3	4
A	30	5		
B		15	0	
C			20	
D			10	20

Table 2

	1	2	3	4
A	x	x		
B		x	x	
C	8	2	x	1
D	9	2	x	x

Table 3

(a) Explain why a zero has been placed in cell B3 in Table 2. **(1)**

(b) Calculate the shadow costs and the missing improvement indices and enter them into Table 3 in your answer book. **(4)**

(c) Taking the most negative improvement index to indicate the entering cell, state the stepping-stone route that should be used to obtain the next solution. You must state your entering cell and exiting cell. **(3)**

(Total 8 marks)

(a)

	1	2	3	4
A	30	5		
B		15	0	
C			20	
D			10	20

Table 2

Initial solution was degenerate
occupied cells = m+n-1
so a zero is required.

(b)

		22	36	31	46
SC		1	2	3	4
0	A	x	x	-12	-9
-1	B	3	x	x	-9
-6	C	8	2	x	1
-8	D	9	2	x	x

Table 3

(c) *You may not need to use all of these tables*

	1	2	3	4
A	30	5 $-\theta$	$+\theta$	
B		15 $+\theta$	0 $-\theta$	
C			20	
D			10	20

entering cell = A3

$\theta = 0$

exiting cell = A2

	1	2	3	4
A	30	5	0	
B		15		
C			20	
D			10	20

Improved solution

4. Robin (R) and Steve (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Robin.

	S plays 1	S plays 2	S plays 3
R plays 1	2	1	3
R plays 2	1	–1	2
R plays 3	–1	3	–3

Find the best strategy for Robin and the value of the game to him.

(Total 9 marks)

4.

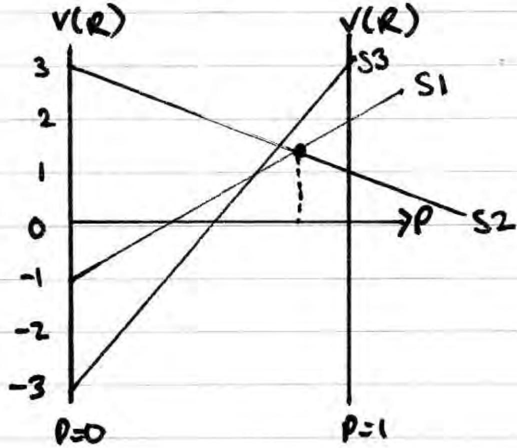
	S plays 1	S plays 2	S plays 3
R plays 1	2	1	3
~~R plays 2~~	~~1~~	~~–1~~	~~2~~
R plays 3	–1	3	–3

R1 dominates R2, R2 can be deleted

R plays 1 prob = p   R plays 3 prob = 1-p

If S plays 1 $V(R) = 2p - 1(1-p) = 3p-1$ *
S plays 2 $V(R) = p + 3(1-p) = -2p+3$ *
S plays 3 $V(R) = 3p - 3(1-p)$ $6p-3$

p=0	p=1
-1	2
3	1
-3	3



$$3p - 1 = -2p + 3$$
$$5p = 4$$
$$p = \frac{4}{5}$$

$\therefore$ R should play 1 prob = $\frac{4}{5}$
R should play 2 NEVER
R should play 3 prob = $\frac{1}{5}$
$V(R) = \frac{7}{5}$

5. A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit, P.
The following tableau is obtained.

Basic variable	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	$-\frac{1}{2}$	10
s	$1\frac{1}{2}$	$2\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	5
z	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	5
P	–5	–10	0	0	0	20	220

(a) Starting by increasing y, perform one complete iteration of the Simplex algorithm, to obtain a new tableau, T. State the row operations you use.

(5)

(b) Write down the profit equation given by T.

(1)

(c) Use the profit equation from part (b) to explain why T is optimal.

(2)

(Total 8 marks)

5.

b.v.	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	$-\frac{1}{2}$	10
s	$1\frac{1}{2}$	$2\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	5
z	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	5
P	–5	–10	0	0	0	20	220

$\theta = 10 \div -\frac{1}{2} = -20$

$\theta = 5 \div 2\frac{1}{2} = 2$ *

$\theta = 5 \div \frac{1}{2} = 10$

(a) *You may not need to use all of these tableaux*

b.v.	x	y	z	r	s	t	Value	Row Ops
r	$\frac{4}{5}$	0	0	1	$\frac{1}{5}$	$-\frac{3}{5}$	1	$+\frac{1}{2}$ new R2
y	$\frac{3}{5}$	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	2	R2 $\times \frac{2}{5}$
Z	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$\frac{3}{5}$	4	$-\frac{1}{2}$ new R2
P	1	0	0	0	4	18	240	+10 new R2

b.v.	x	y	z	r	s	t	Value	Row Ops

b) $P + x + 4s + 18t = 240$

c) $P = 240 - x - 4s - 18t$

$\therefore$ increasing production of x will reduce profit.

6.

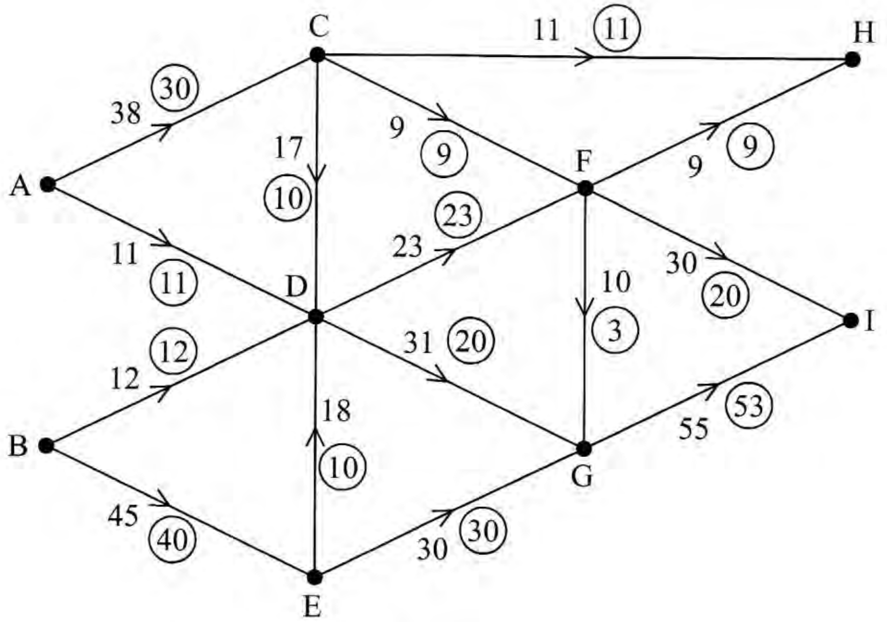


Figure 1

Figure 1 shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) On Diagram 1 and Diagram 2 in the answer book, add a supersource S and a supersink T. On Diagram 1, show the minimum capacities of the arcs you have added.

(2)

(c) Complete the initialisation of the labelling procedure on Diagram 2 in the answer book by entering values on the arcs to S and T and on arcs CD, DE, DG, FG, FI and GI.

(3)

(d) Find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(e) Show your maximum flow on Diagram 3 in the answer book.

(2)

(f) Prove that your flow is maximal.

(2)

(Total 13 marks)

6. (a) Value of initial flow 93

(b) and (c)

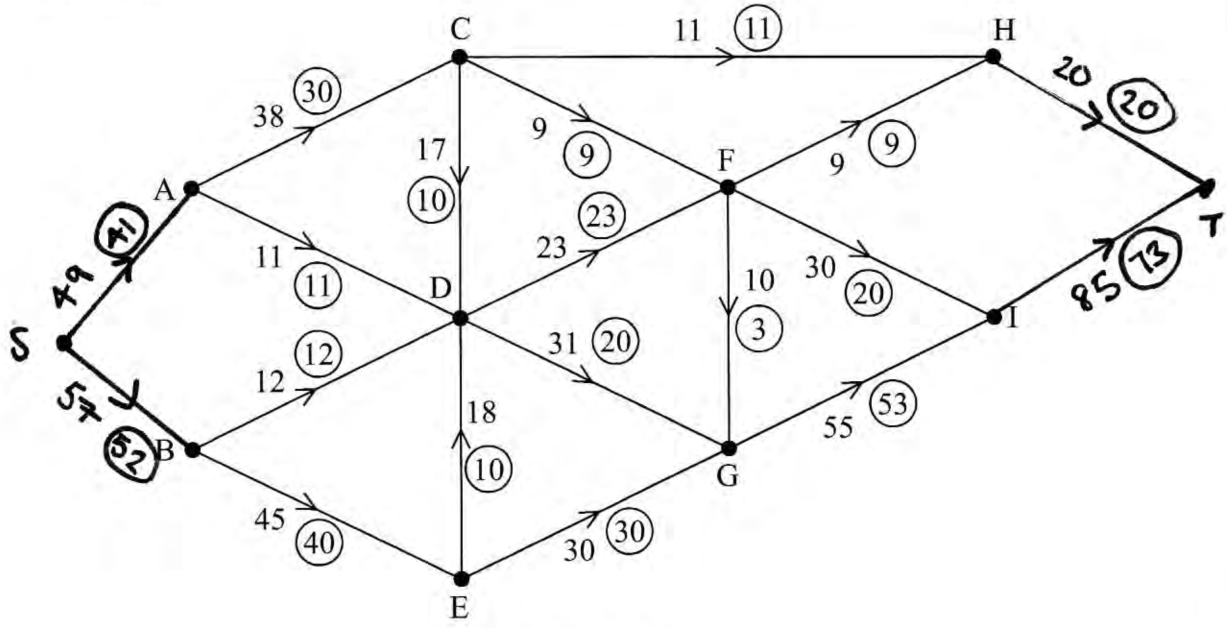


Diagram 1

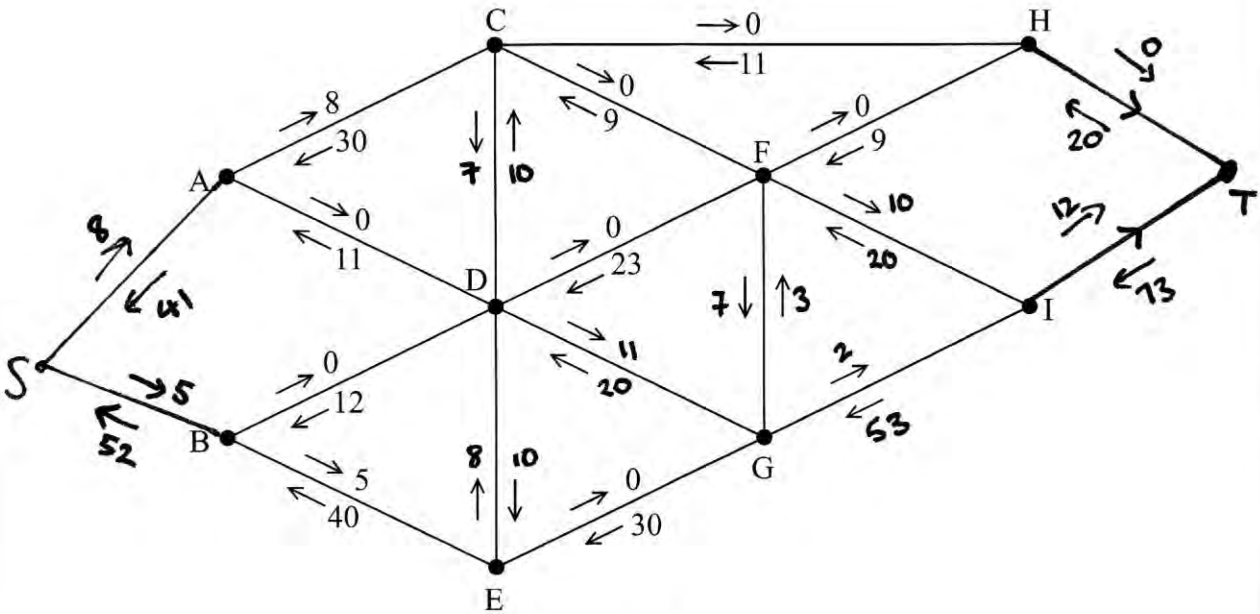


Diagram 2

SBEDG FIT − 3, IT − 2

∴ max flow = 98

(e)

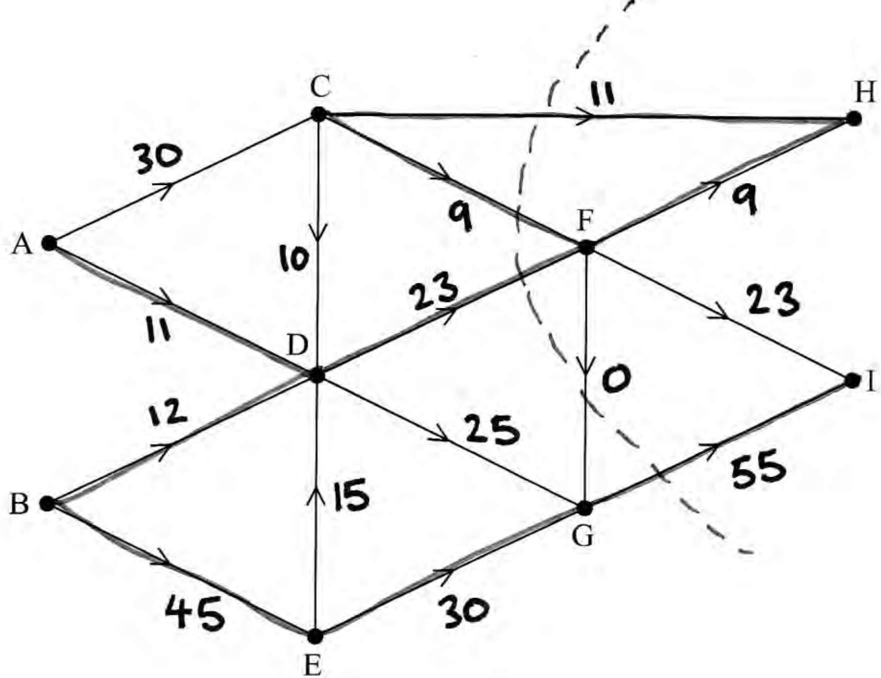


Diagram 3

(f) Cut through saturated arcs to sink CH, CF, DF, GI and empty arc to source FG is possible. ∴ this is min cut of capacity 98

∴ by mincut-max flow theorem flow is maximal.

7. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	1	–3	2
A plays 2	–2	3	–1
A plays 3	5	–1	0

Formulate the game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

7.

	B plays 1	B plays 2	B plays 3
A plays 1	1	–3	2
A plays 2	–2	3	–1
A plays 3	5	–1	0

+ 4 to every value to create new game

	B1	B2	B3
A1	5	1	6
A2	2	7	3
A3	9	3	4

let V = value of new game to A.

let A plays 1,2,3 with prob = p_1, p_2, p_3 respectively. $p_1, p_2, p_3 \geq 0$

objective is to maximise $P = V \Rightarrow P - V = 0$.

Subject to : $V - 5p_1 - 2p_2 - 9p_3 \leq 0$

$V - p_1 - 7p_2 - 3p_3 \leq 0$

$V - 6p_1 - 3p_2 - 4p_3 \leq 0$

$p_1 + p_2 + p_3 \leq 1$

8. A factory can process up to five units of carrots each month.
Each unit can be sold fresh or frozen or canned.
The profits, in £100s, for the number of units sold, are shown in the table.
The total monthly profit is to be maximised.

Number of units	0	1	2	3	4	5
Fresh	0	45	85	120	150	175
Frozen	0	45	70	100	120	130
Canned	0	35	75	125	155	195

Use dynamic programming to determine how many of the five units should be sold fresh, frozen and canned in order to maximise the monthly profit. State the maximum monthly profit.

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Stage	State	Action	Dest.	Value
1	0	0	0	0 *
fresh	1	1	0	45 *
	2	2	0	85 *
	3	3	0	120 *
	4	4	0	150 *
	5	5	0	175 *
2	0	0	0	0 + 0 = 0 *
frozen	1	0	1	0 + 45 = 45 *
		1	0	45 + 0 = 45 *
	2	0	2	0 + 85 = 85
		1	1	45 + 45 = 90 *
		2	0	70 + 0 = 70
	3	0	3	0 + 120 = 120
		1	2	45 + 85 = 130 *
		2	1	70 + 45 = 115
		3	0	100 + 0 = 100
	4	0	4	0 + 150 = 150
		1	3	45 + 120 = 165 *
		2	2	70 + 85 = 155
		3	1	100 + 45 = 145
		4	0	120 + 0 = 120
	5	0	5	0 + 175 = 175
		1	4	45 + 150 = 195 *
		2	3	70 + 120 = 190
		3	2	100 + 85 = 185
		4	1	120 + 45 = 165
		5	0	130 + 0 = 130
3	5	0	5	0 + 195 = 195
Canned		1	4	35 + 165 = 200
		2	3	75 + 130 = 205
		3	2	125 + 90 = 215 *
		4	1	155 + 45 = 200
		5	0	195 + 0 = 195
	fresh	1		
	frozen	1		
	Canned	3		Max Profit = £21500